Asymmetric Dependence in the US Economy: Application to Money and the Phillips Curve

Lorán Chollete and Cathy Ning[*]

January 12, 2012

Abstract

A central role for economic policy involves reducing the incidence of systemic downturns, when key economic variables experience joint extreme events. In this paper, we empirically analyze such dependence using two approaches, correlations and copulas. We document four findings. First, linear correlations and copulas disagree substantially about the nation's dependence structure, indicating complexity in the US economy. Second, money appears to be neutral during both normal and extreme economic conditions. Third, GDP exhibits asymmetric dependence with employment and consumption, with relatively greater dependence during downturns. Finally, there is asymmetric dependence between inflation and employment, in the left tail only. This latter finding suggests that historical Phillips curve relations are a property mainly of systemic downturns.

Keywords: Asymmetric Dependence; Copula; Extreme Event; Money Neutrality; Phillips Curve; Systemic Downturn

JEL Classification: C14, E20, E30, E40

1   Introduction and motivation

When a national economy experiences high dependence across important sectors after a negative shock, this indicates a severe downturn. The experience of crises in the 1990s and 2000s has stimulated researchers' interest in measuring dependence of extreme events in the US economy.[1] A further aspect of macroeconomic dependence is that it amplifies the impact of surprise events. For example, the collapse of a major lending institution affects many households' spending and therefore reduces aggregate demand.[2] The lack of empirical research on such "simultaneous hard times" means that individuals and society are not prepared, when such preparation matters most. Dependence is also important from a theoretical perspective, since it indicates strategic complementarities.[3] Macroeconomists have devoted considerable research to examine dependence of key national economic variables. However, most empirical and theoretical studies consider average dependence, which is appropriate if the true dependence structure is linear. When dependence is nonlinear, it is important to use robust measures.[4] Copulas are an important class of robust dependence measures, which have been applied successfully in banking and finance, but with little comparable research on a national economy. In light of the above considerations, we investigate dependence in the US macroeconomy, using both correlations and a parsimonious set of copulas. We also discuss implications for economic modeling and policy.

The main goal of this paper is to assess the dependence structure of major economic variables in the US economy. The recent history of the US economy is interesting in itself, due to the economic crisis, increasingly globalized markets, and spillovers between financial and labor or product markets. A secondary focus of our paper is the relation between dependence and systemic stability. In general, systemic instability increases with the degree of dependence, as observed by Caballero and Krishnamurthy (2008); Ibragimov et al. (2009), and Shin (2009). Systemic instability may also be exacerbated when different dependence measures give conflicting or inaccurate signals. It is therefore vital for households, banks and policymakers to have accurate estimates of dependence. There are several

---

[1]We use the term dependence to denote situations where two or more economic variables move together, as in the statistical literature of Drouet Mari and Kotz (2001); and Embrechts et al. (2002). We adopt this practice because numerous words are used in economics (coherence, correlation, concordance, co-dependency, co-movement, and procyclicality), and we wish to use a general term.

[2]See Hall (2011).

[3]See Wilson (1975); Bikhchandani et al. (1992); Cooper (1999); Veldkamp and Wolfers (2007); and Vives (2008).

[4]See Granger (2001); Hamilton (2001); and Embrechts et al. (2002).

measures available in economics, including the traditional correlation and copulas. While each approach has advantages and disadvantages, they rarely have been compared in the same empirical study. Such reliance on one measure prevents easy assessment of the degree of dependence, and how it differs over time or across sectors. When economic variables exhibit asymmetric dependence, then multiple sectors are endangered at the same time. This makes it especially difficult for economic policy to stimulate a weak economy.[5] Thus, there are aggregate ramifications for elevated levels of asymmetric dependence. These policy considerations are absent from most previous empirical research on nonlinear dependence of economic variables, and provide a further motivation for our paper.

There is a long literature examining dependence in the macroeconomy, including output-inflation tradeoffs, money neutrality, consumption-income relations, business cycle co-movements, investment and taxes, and policy effectiveness.[6] Dependence is rarely innocuous. It is appealing in the case of valuable policy tradeoffs such as the original Phillips curve. Alternatively, it can be unappealing when indicating economic fragility or inefficiency.[7] Despite the clear policy and academic relevance, little existing research examines nonlinear dependence in macroeconomics.[8] Therefore our research fills a much-needed role, by documenting the type of dependence in the US economy during normal and extreme periods.

1.1 Four Reasons to Examine Dependence in the Macroeconomy

In this section we summarize four major reasons to analyze the dependence structure of US economic variables. In order to illustrate the fundamental nature of dependence, we focus on macroeconomic relations that have been studied for a relatively long time in the economics profession.

1. Theoretical Considerations. Existing macroeconomic models such as Phelps (1968) and Lucas (1972) are based on second moments. To the extent that there is dependence beyond the second moment (e.g. tail dependence, or asymmetric dependence), existing models will ignore potentially valuable information. Thus, from an academic perspective it is important to assess asymmetric dependence at the macroeconomic level.

---

[5]For an assessment of policy effectiveness, see Ramey (2011).

[6]See Keynes (1939); Phillips (1958); Burns and Mitchell (1946); Hall and Jorgenson (1967); Friedman (1968); Phelps (1968); Lucas and Rapping (1969); Kydland and Prescott (1982); and Hansen (1985).

[7]See Keynes (1936); Samuelson (1967); and Feldstein and Horioka (1980).

[8]Two exceptions are Granger et al. (2006); and Dowd (2008), discussed in Section 2 below.

2. Economic Policy. As discussed above, elevated levels of tail dependence across macro variables are symptomatic of systemic economic downturns. During such periods, government policy is more likely to be ineffective because there are multiple problem areas to deal with. From a policy perspective, therefore, there is an important need for quantitative evidence on the degree of dependence across important macro variables.

3. Empirical Testing. The empirical frameworks for many macroeconomic models are based on second moments, in particular linear dependence. For example, the labor supply curve of Lucas and Rapping (1969) is expressed as

$$U_t = \alpha + \beta_1 \ln \frac{w_t}{w_{t-1}} + \beta_2 \ln \frac{P_t}{P_{t-1}} + \beta_3 U_3 t + \epsilon_t, \tag{1}$$

where U_t is unemployment, w_t is wages, and P_t is the price index, all in period t, and ϵ is an error term. Models of the form (1) are typically tested using some version of least squares. A significantly negative β_2 is taken to mean that inflation and unemployment are negatively related, in support of the Lucas-Rapping model. However, β_2 is a *linear* measure of dependence, and will not capture nonlinear relations between unemployment and inflation. In particular, theoretical research shows that higher moments matter for macroeconomic models.[9] Higher-moment dependence is completely ignored in the regression approach to (1). By contrast, nonlinear dependence can be detected using general dependence measures such as copulas.

As another example, macroeconomic theory suggests that consumption is positively related to income, according to the permanent income and life cycle hypotheses.[10] This relation is often tested using a regression of current consumption C_i on the appropriate income measure Y_i:

$$C_i = a + \beta Y_i + \epsilon_i, \tag{2}$$

where ϵ is an error term. However, $\beta = \frac{Cov(Y,C)}{Var(Y)}$, so β is biased toward 0 whenever the denominator is large, that is, during periods of big changes in Y.[11] As demonstrated in Section 3 below, covariance-based measures like correlations will fail to detect even basic nonlinear dependence. Recent research has demonstrated that second moments do not capture all risks–in particular tail risk, the eventuality that multiple sectors are affected during down

---

[9]See Barro (2006); Barro and Jin (2011); and Barro and Ursua (2011).

[10]See Modigliani and Brumberg (1954); Friedman (1957); and Romer (2001), Chapter 7.

[11]For documentation of big changes in Y, see Reinhart and Rogoff (2009); Barro and Jin (2011); and Barro and Ursua (2011).

markets and economic crises.[12] As we discuss in Section 3, the copula approach allows us to take a snapshot of how macro variables measures comove not only in normal times but also during extremes, via the tail dependence coefficient.

4. Nonstationarity and Extreme Events. Many quantitative approaches to macroeconomics rely on stationarity. Theoretical models such as Lucas (1972) require stationarity in order to set up and solve dynamic programs. Therefore these models are not equipped to handle nonstationarities such as regime shifts and extreme events, when several macro variables receive large shocks. By contrast, robust dependence measures are less affected by extreme events. In particular, robust dependence measures like copulas are *invariant* to large deviations that are monotonic transforms of the underlying data.[13] Therefore, a copula approach can detect the true dependence structure between macro variables, which will inform macroeconomists about which type of dependence prevails in the US economy. Such knowledge is evidently valuable for building both theoretical and empirical models of the economy.

1.2 Contributions of Our Paper

Our paper contributes to the literature in 3 ways. First, we analyze macroeconomic dependence for the Phillips curve, and money relationships, using both correlation and nonlinear copula approaches. Second, we establish new stylized facts about macroeconomic dependence during extreme periods. These results may be valuable for modelling business cycles and other features of the aggregate economy. Third, unlike most dependence research, our paper builds on specific macroeconomic theories to study tail dependence in a particular national economy, the USA. We test for joint downturns and upturns in important macro variables. Our results may therefore inform policy on systemic tendencies of the US economy.

More generally, our paper provides a modern examination of key implications from theoretical macroeconomics, to see whether the predictions hold differentially in normal and extreme times. If dependence during extremes is pronounced, this is particularly challenging for policymakers. Practically, one source of confusion in current economic policy is

---

[12]See Chollete et al. (2009); Acharya et al. (2010); and Adrian and Brunnermeier (2010).

[13]For invariance of copulas, see Schweizer and Wolff (1981). To be precise, the copula approach generally requires stationarity, but is invariant to increasing monotonic transformation. Thus, when we monotonically transform nonstationary series (e.g. prices) to stationary time series (log price changes or inflation), we do not change their dependence relationship. See Section 3 for more details.

4

the lack of robust documentation of dependence in financial risk during extreme periods. Our paper appears to be the first[14] to use robust dependence methods to analyze the Phillips curve and money-income relations for the US economy. Since we isolate tail dependence for these relations, our approach provides a new perspective on traditionally important macroeconomic policy variables.

The remaining structure of the paper is as follows. In Section 2 we review literature on macroeconomics and dependence. In Section 3 we compare and contrast dependence measures used in economics. Section 4 summarizes our empirical methodology. Section 5 discusses our data and main results, and Section 6 concludes.

2 Macroeconomic Dependence and Economic Policy

We focus on dependence in the Phillips curve and money comovements. The Phillips curve involves negative dependence between inflation and unemployment. The monetary comovement that we investigate concerns the positive relation between money and real variables such as output and consumption. We allow for asymmetric dependence in order to detect whether dependence is pronounced during extreme periods and whether there is a difference in extreme upturns versus downturns.

It has long been observed that macroeconomic variables exhibit dependence. Lucas (1977) emphasizes that macroeconomic cycles concern "co-movements among various aggregative time series".[15] This feature is so pronounced that "with respect to ... co-movements among series, business cycles are all alike".[16] Similarly, Long and Plosser (1983) state that "The term 'business cycles' refers to the joint time-series behavior of a wide range of economic variables such as prices, outputs, employment, consumption and investment".[17] A central macroeconomic precept is therefore that business cycles exhibit a recognizable dependence structure between key variables.[18]

---

[14]Related papers are Granger et al. (2006); and Dowd (2008).

[15]Lucas (1977), page 9.

[16]Lucas (1977), page 10.

[17]Long and Plosser (1983), page 39. Both Lucas (1977) and Long and Plosser (1983) state that the dependence structure of macroeconomic variables is inherent in the definition of business cycles. In similar fashion Sargent (1979), page 212, states that "an important feature of business-cycle phenomena is high pairwise coherences at low business cycle frequencies..." Here coherence denotes statistical dependence.

[18]Aspects of this precept have been examined by Keynes (1936); Burns and Mitchell (1946); Phillips (1958); Phelps (1968); Lucas and Rapping (1969); Lucas (1972); Lucas (1977); Minsky (1982); and

An important caveat, noted as early as Keynes (1936), concerns nonlinearities in dependence, such as the liquidity trap, and asymmetric booms and busts. These nonlinearities are not only of theoretical interest, they also impinge on the effectiveness of macroeconomic policy. This dual importance of dependence structure motivates our use of nonlinear dependence models in our empirical analysis.

When economic variables have substantial nonlinear dependence in their tails, correlations and standard regression techniques may be biased and inefficient. That is, correlations do not accurately represent the true dependence structure.[19] From an economic perspective, nonlinearities are very important. Keynes (1936) underscores the need for avoiding such nonlinearities and, anticipating the modern Phillips curve, Keynes (1939) discusses the lack of consensus on the dependence structure of real wages and output. Hamilton (2001) shows that nonlinearities are important for explaining the Phillips curve. More broadly, Granger (2001) and Phelps (2007) emphasize the likelihood of subtle, fundamental nonlinearities in market economies. Thus, from the inception of modern macroeconomics to the present, it has been acknowledged that nonlinear dependence in macro variables presents an important academic and policy issue. However, that discussion has a gap: it generally stops short of examining multivariate ($n > 2$) dependence, asymmetric dependence, and the practical difficulty of estimating nonlinear dependence on empirical data. The use of copulas is one way to fill this gap.

In light of the above discussion, it appears worthwhile to check whether conventional macroeconomic dependence relations break down at extremes, which is what we do in this paper. Our empirical results will allow us a closer look at dependence across macro variables studied by Keynes (1939), Lucas (1972), and others.

2.1 Related Literature in Macroeconomics and Dependence

Macroeconomic Literature. We focus on two key results, the Phillips curve and money comovements. First, the *Phillips curve* concerns a central macroeconomic relation, the dependence between inflation and unemployment. Phillips (1958) documents negative dependence between unemployment rates and changes in wages in the UK. He argues that it supports the hypothesis that in general (except for extreme events when import prices

---

Long and Plosser (1983). More recent theoretical models of dependence rely on strategic complementarities, see Bikhchandani et al. (1992); Cooper (1999); and Vives (2008).

[19]See Samuelson (1967); Granger (2001); Hamilton (2001); and Embrechts et al. (2005).

rise enough to start a wage-price spiral), levels and changes in unemployment explain the change in wages. The equation he estimates is of the form

$$\log y = \alpha + b \log x,$$

where y represents the rate of wage change and x denotes percentage unemployment.[20] The author estimates $b = -1.394$, thereby documenting a negative relation. Since the log function is convex, the dependence structure differs at the center versus the extremes. This finding was later extended to inflation and unemployment and named the Phillips curve. Tail dependence is often expressed in terms of joint extremes, so in the following, we consider inflation and employment, in order to discuss high inflation-employment and low inflation-employment pairs.

To understand the strong policy implications of the *dependence structure* in the Phillips curve, suppose inflation and employment have *symmetric* tail dependence. This suggests an attractive tradeoff, since extremely high inflationary periods are offset with high employment, and vice versa. Thus, economic policy has equal effects during upturns and downturns.[21] By contrast, if dependence is asymmetric and more pronounced in the left tail, then extremely low inflation coincides with low employment, but high inflation does not coincide with high employment. This latter situation is consistent with extreme stagflation. In related research, Laxton et al. (1999) show that standard empirical techniques are not powerful enough to identify convexity of the Phillips curve. In addition, Hamilton (2001) demonstrates that accounting for nonlinear dependence is important to identify the Phillips curve. Thus, empirically, nonlinearity is crucial in this macroeconomic relation.[22] Such nonlinearity also has theoretical content. Phelps (1968) develops a theoretical model for the Phillips curve, based on labor market frictions, imperfect information, and adaptive expectations. He shows that if there are money-wage rigidities the observed Phillips curve will occur for large unemployment rates.[23] However, for very small unemployment levels, the dependence structure will diverge, in the context of a disequilibrium wage-price spiral. Phelps' theoretical results therefore suggest asymmetric dependence between inflation and unemployment. A testable implication of the Phelps (1968) result is therefore examining

---

[20]Phillips (1958), page 290.

[21]Tail dependence denotes dependence of economic variables during extreme periods. See equation (6) below.

[22]More generally, Granger (2001) suggests that nonlinearity in macroeconomic variables is subtle, and not detectable without robust techniques. See also Rothman et al. (2001).

[23]See Phelps (1968), equation (33).

whether asymmetric dependence characterizes the inflation-employment data, which we do in Section 5 below. To the best of our knowledge this type of test has not been conducted before.

In other work on the Phillips curve, Lucas and Rapping (1969) develop a theoretical model of aggregate labor supply. They derive an unemployment function,

$$U_t = \alpha + \beta_1 \ln \frac{w_t}{w_{t-1}} + \beta_2 \ln \frac{P_t}{P_{t-1}} + \beta_3 U_3 t + \epsilon_t,$$

and show that β_1 and β_2 are negative. This finding is empirically upheld using US data from 1930 to 1965. Thus, there is negative dependence between unemployment, wage growth, and inflation. Lucas (1973) examines aggregate macroeconomic data from eighteen countries from 1951-1967. He documents mixed evidence of dependence between inflation and output, and stronger evidence for countries with stable prices, such as the US economy.

Second, regarding *money* and the real economy, Lucas (1972) analyzes positive dependence between inflation and real GDP, which he considers to be "a central feature of the modern business cycle".[24] He constructs an economy where money is neutral, and delivers the empirically observed positive dependence. Lucas (1977) builds an equilibrium model to account for the dependence between prices and other variables in the macroeconomy. Kydland and Prescott (1982) develop an aggregate model with adjustment lags in production to explain dependence between output and other economic variables. Long and Plosser (1983) construct a frictionless equilibrium model of the business cycle. Their model reproduces positive dependence across economic sectors, in accordance with empirical patterns of business cycles. King and Plosser (1984) extend the Lucas (1977) model to include monetary and banking considerations. They analyze a model where exchange in the real economy is enhanced via transaction services from the financial industry. Their model delivers zero dependence between money and output growth, and positive dependence between money and prices. Veldkamp and Van Nieuwerburgh (2006) analyze the fact that business cycles are asymmetric–downturns are typically short while upturns are smooth and gradual. The authors construct an equilibrium model where agents take time to learn about aggregate productivity. They document for US macro data from 1952 to 2002, positive dependence between output and variables such as investment, employment, and consumption. Their model is able to replicate much of the observed dependence.

---

[24]Lucas (1972), page 103.

Dependence Literature. The above research emphasizes on theoretical and practical grounds the importance of isolating dependence of macro variables in order to learn about systemic downturns and national economic performance during extreme periods. Previous research on asymmetric dependence has tended to be in international economics or finance, falling into either correlation or copula frameworks. The literature in each area is vast and growing, so we summarize only some key contributions.[25] With regard to correlation, Longin and Solnik (1995), and Ang and Bekaert (2002) find that international stock correlations tend to increase over time. Cappiello et al. (2006) document that international stock and bond correlations increase in response to negative returns, although part of this apparent increase may be due to an inherent volatility-induced bias.[26] Regarding copulas, Mashal and Zeevi (2002) show that equity returns, currencies and commodities exhibit tail dependence. Patton (2004) examines dependence between small and large-cap US stocks, and finds evidence of asymmetric dependence in stock returns. Patton (2006) examines Deutschemark and Yen series, and documents strong evidence of asymmetric dependence in exchange rates. Jondeau and Rockinger (2006) utilize a model of returns that incorporates a skewed-t GARCH for the marginals, along with a dynamic gaussian and student-t copula for the dependence structure. Rosenberg and Schuermann (2006) analyze the distribution of bank losses using copulas to represent the aggregate expected loss from market risk, credit risk, and operational risk. Granger et al. (2006) examine the relation between US income and consumption. They document that a business-cycle factor has information for the joint distribution of income and consumption, but only through the individual series, not in the copula. Rodriguez (2007) constructs a copula-based model for equity prices Latin American and East Asian countries. His model allows for regime switches, and yields enhanced predictive power for international financial contagion. Dowd (2008) uses copulas to shed new light on Gibson's paradox in the UK, a positive correlation between interest rates and prices. The author finds that the best-fitting copula has lower tail dependence which means that the Gibson paradox relates mainly to low price-interest rate combinations. Okimoto (2008) finds evidence of asymmetric dependence between stock indices from the US and UK. Ning (2008) examines the dependence of stock returns from North America and East Asia. She finds asymmetric, dynamic tail dependence in many countries. Ning (2008) also documents that dependence is higher intra-continent than across continents. Chollete et al. (2009) use general vine copulas in order to model portfolios of international stock returns from the G5 and Latin America. They find that the model

---

[25]For summaries of copula literature, see Cherubini et al. (2004), Embrechts et al. (2005), Jondeau et al. (2007), and Patton (2009).

[26]See Forbes and Rigobon (2002).

9

outperforms dynamic gaussian and student-t copulas. Ning (2010) analyzes dependence between stock markets and foreign exchange, and discovers significant upper and lower tail dependence between these two asset classes. These papers all contribute to the mounting evidence on significant asymmetric dependence in economic variables. However, none of the papers analyzes dependence for the Phillips curve and money in the US economy, which motivates our paper.

3 Measuring Dependence in the Macroeconomy

Dependence is assessed with various measures. If two economic variables have relatively low dependence, it suggests related limited exposure to systemic downturns. We estimate dependence in two ways, using correlations and copulas. The extent of discrepancy between the two can suggest complexity, since the two statistics give different signals. The discrepancy can also be informative to understand the mistakes from using correlations alone. Throughout, we consider X and Y to be two random variables, with joint distribution $F_{X,Y}(x,y)$, and continuous marginals $F_X(x)$ and $F_Y(y)$, respectively.

3.1 Consequences of Measuring Dependence by Correlation

Most of the previous section's macroeconomic results were formulated with some type of covariance. However, if we wish to isolate asymmetric dependence, covariances and correlations are not enough. Covariance measures average linear dependence,[27] which differs from dependence of the distribution. For example, consider two variables X and Y. X is zero-mean and non-skewed: $E[X] = \bar{X} = 0$ and $E[X^3] = 0$. Furthermore, Y satisfies a simple nonlinear relation with X, namely $Y = X^2$. Then the covariance between X and Y is

$$
\begin{aligned}
cov(X,Y) &= E[(X - \bar{X})(Y - \bar{Y})] \\
&= E[(X - 0)(X^2 - \bar{Y})] \\
&= E[X^3 - X\bar{Y}] \\
&= E[X^3] - \bar{Y}E[X] \\
&= 0.
\end{aligned}
$$

---

[27]See Casella and Berger (1990), Chapter 4; and Embrechts et al. (2002).

Evidently X and Y have a perfect deterministic relation, but covariance cannot account for it. The reason is that covariance captures only linear and not distributional dependence. Thus, covariance cannot detect dependence in even the simplest continuous nonlinear relation, $Y = X^2$. Similar reasoning applies to any statistical measure that builds on covariance, such as linear regression.

We now explain why correlation is misleading as a signal of dependence. The Pearson **correlation** coefficient ρ for two random variables X and Y is the covariance divided by the product of the standard deviations:

$$\rho = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \cdot \text{Var}(Y)}}. \tag{3}$$

Advantages of Correlations. Correlations are the most familiar measures of dependence in economics. If properly specified, correlations tell us about average dependence over the distribution. Moreover, a body of theoretical macroeconomic research delivers important results expressed in terms of correlations (see Section 2 above.) Thus, there is a well-developed link between theory and empirics in the case of correlations, which is evidently valuable for theoretical and applied macroeconomists.

Disadvantages of Correlations. There are a number of shortcomings. First, correlation is not invariant to nonlinear monotonic transformations. Thus, the correlation of two economic series may differ from the correlation of the squared or log series. Second, there is evidence of infinite variance in economic data.[28] From equation (3), if either X or Y has infinite variance, the estimated correlation gives little information on dependence, since it will be undefined or close to zero. A third drawback concerns estimation bias: by definition the conditional correlation is biased and spuriously increases during volatile periods.[29] Fourth, correlation is a linear measure and therefore may overlook important nonlinear dependence. It does not distinguish between dependence during up and down markets.[30] Whether these shortcomings matter in practice for the US economy is an empirical question that we tackle in this paper.

---

[28]See Mandelbrot (1963); Fama (1965); Gabaix et al. (2003); and Rachev (2003).

[29]See Forbes and Rigobon (2002). After adjusting for such bias, Forbes and Rigobon (2002) document that prior findings of international dependence (contagion) are reversed.

[30]Such nonlinearity may be substantial, as illustrated by Ang and Chen (2002) in the domestic context. These researchers document significant asymmetry in downside and upside correlations of US stock returns.

More broadly, the fact that correlations cannot detect dependence in even simple nonlinear relations applies to any statistical measure that builds on correlation, such as linear regression. Such fragility of correlation is of practical importance in economic *research* and *policy*. Regarding research, linear approximation is attractive for parsimony. However, a linear correlation approach can mask theoretically important nonlinearities, as demonstrated by Granger (2001), Hamilton (2001), and Mogstad and Wiswall (2009). Regarding policy, it is crucial to understand the dependence patterns of key economic variables during upturns versus downturns.[31]

Alternatives to Correlation. Before discussing copulas, we briefly mention four alternatives to correlation. As we see in Section 3.3, these alternatives are all nested in the copula framework. First, is the **rank** (or Spearman) **correlation**, ρ_S. This is more robust than the traditional correlation. ρ_S measures dependence of the ranks,[32] and can be expressed as $\rho_S = \frac{\text{Cov}(F_X(x), F_Y(y))}{\sqrt{\text{Var}(F_X(x))\text{Var}(F_Y(y))}}$. The rank correlation is useful when analyzing data with a number of extreme observations, since it is independent of the levels of the variables, and therefore robust to outliers. A related measure is **Kendall's** τ, which measures the difference between positive and negative dependence:

$$\tau(X, Y) = P[(X - \tilde{X})(Y - \tilde{Y}) > 0] - P[(X - \tilde{X})(Y - \tilde{Y}) < 0], \tag{4}$$

where the tildes denote independent copies of the relevant random variable. A third nonlinear dependence measure may be termed **downside risk**, $d(u)$. This function measures the conditional probability of an extreme event beyond some threshold u. For simplicity, normalize variables to the unit interval $[0, 1]$. Hence

$$d(u) \equiv \Pr(F_X(x) \leq u \mid F_Y(y) \leq u). \tag{5}$$

The final nonlinear measures are left and right **tail dependence**, $\lambda_l(u)$ and $\lambda_r(u)$. $\lambda_l(u)$ is the limit of downside risk $d(u)$, while $\lambda_r(u)$ is the limit of upside gains:

$$\lambda_l(u) \equiv \lim_{u \downarrow 0} \Pr(F_X(x) \leq u \mid F_Y(y) \leq u). \tag{6}$$

$$\lambda_r(u) \equiv \lim_{u \uparrow 1} \Pr(F_X(x) \geq u \mid F_Y(y) \geq u). \tag{7}$$

---

[31]For related literature on economic asymmetries, see De Long and Summers (1986); and Veldkamp and Van Nieuwerburgh (2006).

[32]See Cherubini et al. (2004), page 100.

Intuitively, left tail dependence λ_l refers to the relative amount of mass in the lower quantile. This quantity is key for macroeconomic policy, since it captures dependence across macro variables during economic downturns. It turns out that tail dependence is inherent in copulas, as we discuss below. Economic examples of tail dependence include the liquidity trap of Keynes (1936) and the nonlinear Phillips curve of Phelps (1968), discussed in Section 2 above.

3.2   Measuring Dependence with Copulas

Copulas have been used successfully in international economics and finance.[33] A **copula** $C(\cdot)$ is a joint distribution with uniform marginals. In the bivariate case, that means

$$C(u,v) = \Pr[U \leq u, V \leq v], \tag{8}$$

where U and V are uniformly distributed.

Intuitively, copulas "couple" or join marginals into a joint distribution. Copulas often have convenient parametric forms, and summarize the dependence structure between variables.[34] Specifically, for any joint distribution $F_{X,Y}(x,y)$ with marginals $F_X(x)$ and $F_Y(y)$, we can write the distribution as

$$F_{X,Y}(x,y) = C(F_X(x), F_Y(y)). \tag{9}$$

The usefulness of (9) is that it simplifies analysis of dependence in a distribution $F_{X,Y}(x,y)$ by studying instead a copula C. Since copulas represent dependence of arbitrary distributions, in principle they allow us to examine asymmetric dependence for important macroeconomic structures such as the Phillips Curve and money-income relations. This permits us a closer look at key macroeconomic relationships that have been studied since Keynes (1936) and Lucas (1972).

If we knew the entire joint distribution of macroeconomic variables, we could summarize all relevant dependence and therefore all potential for systemic downturns in the macroeconomy. Suppose we examine money (X) and income (Y), then all dependence is contained in their joint density $f_{X,Y}(x,y)$. This information is often not available, because there might be no simple parametric joint density that characterizes the relationship. Moreover, there is a great deal of estimation and mis-specification error in attempting to find the

---

[33]See Ane and Kharoubi (2001); Patton (2006); Rosenberg and Schuermann (2006); and Okimoto (2008).
[34]Sklar (1959) proves that copulas uniquely characterize continuous distributions.

density parametrically. An alternative to measuring dependence in this setting is the copula function $C(u, v)$ described above. It is often convenient to differentiate equation (9) and use a corresponding density version

$$f(x, y) = c(F_X(x), F_Y(y)) \cdot f_X(x) \cdot f_Y(y), \tag{10}$$

where $f(x, y)$ and $c(F_X, F_Y)$ are the joint and copula densities, respectively.[35] Equation (10) is valuable because it empowers the researcher to separate out the joint distribution from the marginals. For example, if we are interested in whether the relation between money and income is different during extreme and normal times, we would want to focus directly on the dependence between the two variables. We estimate (10) in Section 5, for different copula specifications.

Advantages of Copulas. There are four main advantages of using copulas in macroeconomics. First, they are a convenient choice for modeling potentially nonlinear macroeconomic dependence, such as systemic downturns. This aspect of copulas is especially attractive since they nest the other robust dependence measures, as described in Section 3.3 below. In particular, different copulas possess different *tail dependence*, as shown in Table 2. Thus, by estimating copulas we obtain a sense of dependence at extremes.

Second, copulas can aggregate variables from disparate sources, see Rosenberg and Schuermann (2006). In a related sense, copulas permit one to model *joint* dependence in the macroeconomy without specifying the distribution of individual variables in the system.[36] A third advantage is **invariance**: the copula is unchanged by increasing transforms of the data. That is, the copula extracts the way in which X (money) and Y (income) comove, regardless of the scale used to measure them. Given the importance of this result, we elaborate. Suppose that the copula for X and Y is C^*, that is, the joint distribution of money and income satisfies $H_{X,Y}(x, y) = C^*(F_X(\cdot), F_Y(\cdot))$. Now consider any monotone transforms γ_1 and γ_2 of X and Y, respectively. It can be shown that C is still the copula for $\gamma_1(X)$ and $\gamma_2(Y)$. That is,

$$H_{\gamma_1(X), \gamma_2(Y)}(\gamma_1(x), \gamma_2(y)) = C^*(F_X(\cdot), F_Y(\cdot)). \tag{11}$$

---

[35]Specifically, $f(x, y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y}$, and similarly $c(F_X(x), F_Y(y)) = \frac{\partial^2 C(F_X(x), F_Y(y))}{\partial x \partial y}$. The terms $f_X(x)$ and $f_Y(y)$ are the marginal densities.

[36]For example, if we model the joint distribution of money and income (or unemployment and inflation) as bivariate normal, this automatically restricts both the individual money and income series to be univariate normal. The semi-parametric copula approach avoids this restriction by using empirical marginal distributions, based on ranks of the data.

Intuitively, a monotone transform does not alter the ranks of data, so the copula (which is based on ranks) is unaffected.[37] This result is evidently important for assessing dependence in data that features extreme events and other nonstationarities.

Fourth, since copulas are rank-based and can incorporate asymmetry, they are also natural dependence measures from a theoretical perspective. The reason is that a growing body of research recognizes that individuals care a great deal about ranks and downside performance when evaluating economic decisions.[38]

Drawbacks to Copulas. There are two drawbacks to using copulas. First, from an economic perspective, a potential disadvantage is that many copulas do not have moments that are directly related to macroeconomic models based on correlations or variances. This is not a big issue for our study, since we report and compare Pearson and rank correlations. Further, our models include a *t* copula, which contains a correlation parameter. Second, from a statistical perspective, it is not easy to say which parametric copula is most appropriate for a given dataset, because some copulas may fit better near the center and others near the tails. This issue is not strongly relevant to our paper, since our approach to the macroeconomic research in Section 2 focuses on asymmetry and tail dependence. Thus the emphasis is on the shape of copulas, rather than on a specific copula. Further, we use several specification checks, and include the SJC copula, which is a flexible model.

3.3 Relationship of dependence measures

We briefly outline the relationship of the dependence measures discussed above.[39] If the true joint distribution is bivariate normal, then the copula and traditional correlation give the same information. Once we move far away from normality, there is no clear relation between correlation and the other measures. However, all the other, more robust measures of dependence are pure copula properties, and do not depend on the marginals. We describe relationships for rank correlation ρ_S, Kendall's τ, downside risk $d(u)$, and tail dependence $\lambda(u)$ in turn. The relations of copulas to rank correlation and Kendall's τ are given by

$$\rho_S = 12 \int_0^1 \int_0^1 C(u,v)dC(u,v) - 3 \tag{12}$$

---

[37]For a formal proof, see Schweizer and Wolff (1981); and Cherubini et al. (2004), Chapter 2.

[38] See Kahneman and Tversky (1979); Benartzi and Thaler (1995); Barberis et al. (2001); Polkovnichenko (2005); and Rostek (2010).

[39]For further details, see Cherubini et al. (2004); Embrechts et al. (2005); and Jondeau et al. (2007).

and

$$\tau = 4 \int_0^1 \int_0^1 C(u,v) dC(u,v) - 1. \tag{13}$$

Thus, if we know the correct copula, we can recover rank correlation and Kendall's τ. Therefore, rank correlation and Kendall's τ are pure copula properties. Regarding downside risk, it can be shown that $d(u)$ satisfies

$$
\begin{aligned}
d(u) &\equiv \Pr(F_X(x) \le u \mid F_Y(y) \le u) \\
&= \frac{\Pr(F_X(x) \le u, F_Y(y) \le u)}{\Pr(F_Y(y) \le u)} \\
&= \frac{C(u,u)}{u},
\end{aligned}
\tag{14}
$$

where the third line uses definition (8) and the fact that since $F_Y(y)$ is uniform, $\Pr[F_Y(y) \le u] = u$. Thus downside risk is also a copula property and does not depend on the marginals at all. Since tail dependence is the limit of downside risk, it follows from (6) and (14) that left tail dependence is defined in terms of copulas:

$$\lambda_l(u) = \lim_{u \downarrow 0} \frac{C(u,u)}{u}. \tag{15}$$

To summarize, the nonlinear dependence measures are directly linked to copulas, and ρ and the normal copula give the same information when the data are jointly normal. While the above discussion describes how to link the various concepts in theory, there is little empirical work comparing the different dependence measures for the US economy. This provides a further rationale for our empirical study.

4 Empirical Methodology

4.1 Estimation method for copulas

One advantage of the copula approach is that it separates the dependence structure from the marginals, with dependence completely captured in the copula function.[40] Since our focus is on the dependence between macroeconomic variables, rather than their marginals, we specify a parametric copula function but make no assumptions on the marginal distribu-

---

[40]See Sklar (1959); Embrechts et al. (2005); and Patton (2006).

tions. Therefore, the approach is free of specification errors for marginals.[41] The estimation procedure comprises two steps. In the first step, the marginal distribution function $G(.)$ is estimated non-parametrically via its rescaled empirical cumulative distribution function (ECDF)

$$\widehat{F}(x_t) = \frac{1}{T+1} \sum_{t=1}^{T} 1\{X_t < x\}. \tag{16}$$

The ECDF is rescaled to ensure that the first order condition of the copula's log-likelihood function is well defined for all finite T.[42] By the Glivenko–Cantelli theorem, $\widehat{F}_X(x_t)$ converges to its theoretical counterpart $F_X(x_t)$ uniformly.

In the second step, given the non-parametrically estimated ECDF, $\widehat{F(x_t)}$ and $\widehat{G}(y_t)$, we estimate the copula parameters θ_c parametrically by maximum likelihood, with

$$\widehat{\theta}_c = \arg\max_{\theta_c} \widetilde{L},$$
$$\text{where } \widetilde{L}(\theta_c) = \frac{1}{T} \sum \log c(\widehat{F}(x_t), \widehat{G}(y_t); \theta_c),$$

where c(.) is the copula density function. Joe (1997) proves that under a set of regularity conditions, the two-step estimator is consistent, asymptotically normal, and often highly efficient. In addition, as indicated in Patton (2006), this method has the benefit of being computationally tractable. Chen and Fan (2006) establish asymptotic properties for this semi-parametric estimator. Copula estimation requires that the series be i.i.d. Since many of our macro series are not i.i.d., thus we filter the variables with various ARMA-GARCH models.[43] We then compute the ECDFs of the filtered variables, which are used in the second-stage maximum likelihood estimation.

4.2 Empirical Dependence Model

We utilize three basic copula structures, namely, the Gaussian, Student-t, and Clayton. The functional forms are displayed in Table 1. They possess different tail dependence properties: the Gaussian has no tail dependence, the Student-t has symmetric dependence, and the Clayton has asymmetric dependence, as shown in Table 2. The Gaussian specification

---

[41]See Joe (1997), and Cherubini et al. (2004). Statistical properties of this approach are highlighted in the simulation studies of Fermanian and Scaillet (2003).

[42]See Genest et al. (1995), and Chen and Fan (2006) for further discussion on this methodology.

[43]Details of the filtering procedure are available from the authors, upon request.

is a natural benchmark, as the most common distributional assumption in economics, with zero tail dependence.[44] The Student-t is important since it has symmetric tail dependence and nests the normal copula. The Clayton copula is useful because it has asymmetric tail dependence–the mass in its left tail greatly exceeds the mass in its right tail. Practically, these copulas capture the most important shapes for macroeconomics, and are a subset of those frequently used in recent empirical papers. The copulas are estimated by maximum likelihood.

We would like to allow for symmetric or asymmetric dependence, and therefore utilize a flexible copula, the Symmetrized Joe Clayton (SJC) copula of Patton (2006). The SJC copula is defined as

$$C_{SJC}(u,v|\lambda r, \lambda_l) = 0.5 \times (C_{JC}(u,v|\lambda_r, \lambda_l) + C_{JC}(1-u, 1-v|\lambda_l, \lambda_r) + u + v - 1), \quad (17)$$

where $C_{JC}(u,v|\lambda_r, \lambda_l)$ is the Joe-Clayton copula. The Joe-Clayton copula is in turn defined as

$$C_{JC}(u,v|\lambda_r, \lambda_l) = 1 - (1 - \left\{ \left[1 - (1-u)^k\right]^{-r} + \left[1 - (1-v)^k\right]^{-r} - 1 \right\}^{-1/r})^{1/k},$$

where $k = 1/log_2(2 - \lambda_r)$ and $r = -1/log_2(\lambda_l)$, and λ_l and $\lambda_r \in (0,1)$. By construction, the SJC copula is symmetric when $\lambda_l = \lambda_r$. This copula is very flexible since it allows for both asymmetric upper and lower tail dependence, with symmetric dependence as a special case.

5   Data and Results

The data that we use comprise both monthly and quarterly data from the Federal Reserve Bank of St. Louis. Our motivation for the choice of variables is based on Section 2's discussion. Monthly data are from January 1964 to December 2008, and include the following variables: the riskfree rate, price (measured in consumer price index, CPI), inflation, employment rate, wage, consumption, money supply, and GDP. Inflation is computed as the log difference of the consumer price index (CPI) in the past twelve months. Quarterly data are from January 1964 to October 2008, and include investment in addition to all other vari-

---

[44]Tail dependence refers to dependence at extreme quantiles as in expression (6). See de Haan and Ferreira (2006).

ables in the monthly data. Quarterly data on wages, money supply, interest rate, consumer price index and employment rate are not available since the Federal Reserve is currently updating these series. Therefore we compute these by taking the average of three months' data. The macroeconomic variables, including GDP, wage, consumption and investment, are in real terms. GDP is not available at monthly frequency, so we use the Industrial Production Index as an approximation. Since all macro variables are nonstationary, we estimate the dependence of the log differences of all variables, which are stationary.

5.1 Estimates of macroeconomic dependence: correlations

We first examine dependence at normal times using linear and rank correlations. Table 3 displays correlations between GDP and other macro variables. Panel A shows monthly correlations. GDP has significant positive correlations with the interest rate, employment rate, real wages, and real consumption while it has negative correlation with the price level CPI. The rank correlations and Kendall's tau have the same sign as the linear correlations and are strongly statistically significant as well. Thus our linear and rank correlation results of GDP and other macro variables agree with in normal, non-extreme situations. It is instructive to consider the highest and lowest correlations. The highest linear correlation is between GDP and employment, at 0.3591. This is also true for the rank correlations, Spearman's rho and Kendall's tau, at 0.3461 and 0.2453 respectively. The smallest positive linear correlations are between GDP and real wages. There are some discrepancies between linear correlations and rank correlations. For example, the linear correlation between GDP and interest rate exceeds that between GDP and consumption. However, the rank correlations show the opposite order for these two sets of variables. Therefore, empirically linear and rank correlations do not always agree with each other.

Panel B displays results from quarterly data. Again, significant positive dependence is found in all except the GDP-Price pair, which has significant negative linear and rank correlations. The strength of dependence is generally stronger than in monthly data. For example, the linear, Spearman's rho and Kendall's tau correlations for the GDP-consumption pair increases from 0.2379, 0.2427, 0.1662 in monthly data to 0.6690, 0.5824,0.4253 in quarterly data respectively. Further, the maximum dependence is now for GDP-consumption, instead of the pair GDP-employment as in monthly data.

Next, in Table 4, we examine the dependence between money supply and other macro variables. Panel A shows the monthly estimates. The linear correlation for the money-employment pair is insignificant, which is consistent with money neutrality. The rank correlations for this pair are also insignificant. For money and GDP, surprisingly, the linear correlation is significantly negative, which contradicts both money neutrality and the new Keynesian model. This result may be due to the inherent drawbacks of linear correlation. For example, linear correlation is only appropriate for measuring dependence in elliptical distributions, and these variables may not be elliptical.[45] The rank correlations are statistically insignificant, consistent with money neutrality. Panel B presents results from the quarterly data, where all of the dependence measures are statistically insignificant. Therefore, our results indicate neutrality of money, that is, neither employment nor output exhibits dependence with money supply. Moreover, in monthly data the linear and rank correlations disagree, indicating correlation complexity.

In Table 5 we present the dependence between inflation and other variables. In Panel A, the monthly data all display significant positive dependence for both inflation-employment and inflation-GDP. These results hold for both linear and rank correlations. Since employment is inversely related to unemployment, the positive dependence for inflation-employment implies a negative dependence for inflation-unemployment. These findings are consistent with the traditional Phillips curve. Interestingly, if measured with linear correlation, dependence is larger for inflation-GDP than for inflation-employment. However, if measured in rank correlations, the degree of dependence is larger for inflation-employment than for inflation-GDP. Thus, empirically a greater linear correlation between two macro variables occurs with a relatively smaller *rank* correlation. This important discrepancy is also reflected in the quarterly results from Panel B. Specifically, we find significant positive linear correlations for inflation-GDP. However, in contrast to linear correlations, the rank correlations are statistically insignificant. Such lack of conformity in dependence measures is further evidence of complexity in the US macroeconomy.

5.2 Estimates of macroeconomic dependence: copulas

We now present our tail dependence estimates. The estimation methodology is described in Section 4 above. Table 6 presents our results on dependence between GDP and other macro variables. Panel A displays tail dependence estimates for monthly data, where we

---

[45]See Samuelson (1967); Chamberlain (1983); and Embrechts et al. (2002).

find that only employment and consumption have tail dependence with GDP, in both tails. Interestingly, tail dependence in the GDP-interest rate pair is insignificant. This implies that, at extreme economic times, interest rates do not comove with GDP. Similarly, during economic booms, interest rates do not increase with GDP. There exists significant left and right tail dependence for the GDP-employment pair, with left tail dependence (0.1952) much higher than right tail dependence (0.1132). Hence, extremely low GDP and low employment rates tend to coincide during economic crises, while extremely high GDP and high employment rate are likely to occur together during economic booms. These tendencies are asymmetric, because the GDP-employment pair is more likely to be extremely low during extreme economic downturn than to be jointly high during economic upturns. We find no tail dependence for the GDP-real wage pair. Regarding positive dependence between consumption and GDP, we find significant dependence in both left and right tails. Again, there is strong asymmetry. Left tail dependence is 0.1864, while right tail dependence is 0.0090. Thus during economic downturns, low GDP tends to coincide with low consumption, and vice versa. This tendency is asymmetric, and more pronounced during economic downturns than upturns.

Panel B presents quarterly results. These generally agree with the monthly results, but with higher values and statistical significance. The main differences are as follows. First, the extreme dependence for the GDP-employment and GDP-consumption pairs are much stronger than those from monthly data. This reinforces the asymmetric dependence for the GDP-employment and GDP-consumption pairs. Second, there exists significant left tail dependence for the GDP-interest rate pair. This implies a possible policy ineffectiveness of the Fed's interest rate management , during economic downturns GDP falls significantly even when the interest rate is at a very low level. This supports the liquidity trap hypothesis. Finally, left tail dependence for the GDP-real wage pair is significantly positive, indicating significant decreases in real wage during extreme economic downturns.

In light of the above discussion, we summarize our results as follows in the following three points. First, our results support the view of liquidity traps during extreme economic times. Second, during economic downturn when GDP drops, the employment rate, real wages, and real consumption are likely to decrease as well. However, during economic upturns when GDP rises, employment and real consumption also tend to increase, but with relatively lower magnitude. Third, real wages tend not to increase with GDP during economic upturns.

Table 7 presents dependence between money, employment and GDP during extreme economic conditions. We find that both left and right tail dependence coefficients are always statistically insignificant. Thus, money is neutral at extremes. This result is robust to both the monthly and quarterly data, and is consistent with our previous findings from linear and rank correlations.

In Table 8, we evaluate tail dependence in inflation-GDP, and in inflation-unemployment. In both monthly and quarterly results, inflation-GDP shows insignificant dependence, which is different from the positive inflation-GDP dependence from correlations in Table 5 Thus dependence under extreme economic situations differs from dependence under normal economic situations for the inflation-GDP pair. Inflation-employment exhibits significant, positive left tail dependence, but no right tail dependence. Since employment is inversely related to unemployment, this is consistent with the Phelps (1968) conjecture that unemployment and inflation are asymmetrically dependent at extremes. In quarterly data, none of the pairs has dependence, which contradicts the strong correlations in Table 5.

To summarize the tail dependence results, our most striking finding is that GDP is asymmetrically related to employment and consumption, from Table 6. This indicates that during big downturns in economic activity, employment, consumption fall, and do not rise as much during big upturns. From Table 6, we also find evidence of liquidity traps during economic downturns. From Table 7, we find evidence of money neutrality during extreme economic conditions. From Table 8, we observe that inflation is asymmetrically related to employment. That is, employment is dependent with inflation at the left tail during economic downturns, but is not dependent with inflation at the right tail, during upturns. According to Table 8, there is some evidence for the Phillips curve during extreme periods. In particular, inflation and employment exhibit significant tail dependence, although only in the left tail of the distribution.

5.3   Comparing correlations and copulas

In terms of comparison, both correlations and copulas show diversity in the dependence structure of the US macroeconomy. The two approaches agree that GDP is highly dependent with investment and employment. Both approaches also show evidence of money neutrality. However, they do not agree with each other on the dependence of many other pairs. For example, GDP is linearly dependent with the interest rate and price level, but

not tail dependent with the price level. Inflation is linearly correlated with employment and GDP, but not tail dependent with the latter. The fact that copulas and correlations disagree, and the asymmetric dependence in some series, suggest an important degree of uncertainty which impinges on US economic policy.

Our documentation of significant asymmetric dependence in GDP-interest rates and inflation-employment suggests that the US economy is susceptible to simultaneous shocks in key macro variables. This result may be valuable to build into theoretical macroeconomic models. More generally, the fact that correlations and copulas disagree suggests correlation complexity. Hence, clear information about the economy's state is not always readily available for individuals, banks, and policymakers.

6   Conclusions

In this paper, we examine the dependence structure of important US macroeconomic variables. Motivated by theoretical and empirical considerations, we assess the tendency of macro variables to move together during extreme periods. Interestingly, GDP exhibits linear dependence with interest rates and prices, but no extreme dependence with the latter. This suggests existence of liquidity traps during economic downturns. In addition, we document four significant findings. First, correlations and copulas disagree substantially, which indicates complexity in the dependence structure of the US economy. Second, money appears to be neutral at all times. Third, GDP exhibits asymmetric extreme dependence with employment, and consumption, with relatively greater dependence in economic downturns. Fourth, the Philips curve relation between inflation and employment manifests not during upturns, but only during periods of low inflation and employment.

Our results add to the body of stylized facts about the US economy, by describing its dependence structure during both normal and extreme periods. Such policy-relevant information has not been documented for monetary and Phillips curve relations in the US economy. Most significantly, our findings indicate that the US economic system exhibits tail dependence in important macroeconomic quantities. From a policy perspective, our findings underscore the importance of using techniques that are robust to different economic situations, when measuring dependence in important macroeconomic and policy variables.

References

Acharya, A., Cooley, T., Richardson, M., Walter, I., 2010. Manufacturing tail risk: A perspective on the financial crisis of 2007-2008. Foundations and Trends in Finance 4 (4), 247–325.

Adrian, T., Brunnermeier, M., 2010. Covar: A systemic risk contribution measure. Tech. rep., Princeton University.

Ane, T., Kharoubi, C., 2001. Dependence structure and risk measure. Journal of Business 76 (3), 411–438.

Ang, A., Bekaert, G., 2002. International asset allocation with regime shifts. Review of Financial Studies 15 (4), 1137–87.

Ang, A., Chen, J., 2002. Asymmetric correlations of equity portfolios. Journal of Financial Economics 63 (3), 443–94.

Barberis, N., Huang, M., Santos, T., 2001. Prospect theory and asset prices. Quarterly Journal of Economics CXVI (1), 1–53.

Barro, R., 2006. Rare disasters and asset markets in the twentieth century. Quarterly Journal of Economics 121 (3), 823–866.

Barro, R., Jin, T., 2011. On the size distribution of macroeconomic disasters. Econometrica forthcoming.

Barro, R., Ursua, J., 2011. Rare macroeconomic disasters. Annual Review of Economics forthcoming.

Benartzi, S., Thaler, R., 1995. Myopic loss aversion and the equity premium puzzle. Quarterly Journal of Economics 110 (1), 73–92.

Bikhchandani, S., Hirschliefer, D., Welch, I., 1992. A theory of fads, fashion, custom, and cultural change as informational cascades. Journal of Political Economy 100, 992–1026.

Burns, A., Mitchell, W., 1946. Measuring Business Cycles. National Bureau of Economic Research.

Caballero, R., Krishnamurthy, A., 2008. Collective risk management in a flight to quality episode. Journal of Finance LXIII (5), 2195–2230.

Cappiello, L., Engle, R. F., Sheppard, K., 2006. Asymmetric dynamics in the correlations of global equity and bond returns. Journal of Financial Econometrics 4 (4), 537–572.

Casella, G., Berger, R., 1990. Statistical Inference. Duxbury Press.

Chamberlain, G., 1983. A characterization of the distributions that imply mean-variance utility functions. Journal of Economic Theory 29, 185–201.

Chen, X., Fan, Y., 2006. Estimation and model selection of semiparametric copula-based multivariate dynamic models under copula misspecification. Journal of Econometrics 135, 125–154.

Cherubini, U., Luciano, E., Vecchiato, W., 2004. Copula Methods in Finance. Wiley, West Sussex, UK.

Chollete, L., Heinen, A., Valdesogo, A., 2009. Modeling international financial returns with a multivariate regime-switching copula. Journal of Financial Econometrics 7 (4), 437–480.

Cooper, R., 1999. Coordination Games: Complementarities and Macroeconomics. Cambridge Press, New York.

de Haan, L., Ferreira, A., 2006. Extreme Value Theory: An Introduction. Springer, New York.

De Long, B., Summers, L., 1986. Are business cycles symmetrical? In: Gordon, R. (Ed.), The American Business Cycle: Continuity and Change. University of Chicago Press, Chicago, pp. 166–179.

Dowd, K., 2008. Copulas in macroeconomics. Journal of International and Global Economic Studies 1 (1), 1–26.

Drouet Mari, D., Kotz, S., 2001. Correlation and Dependence. World Scientific Publishing Company.

Embrechts, P., McNeil, A., Frey, R., 2005. Quantitative Risk Management: Concepts, Techniques and Tools. Princeton Series in Finance. Princeton University Press, New Jersey.

Embrechts, P., McNeil, A., Straumann, D., 2002. Correlation and dependence in risk managament: Properties and pitfalls. In: Dempster, M. (Ed.), Risk Management: Value at Risk and Beyond. Cambridge University Press, Cambridge, UK, pp. 176–223.

Fama, E., 1965. The behavior of stock market prices. Journal of Business 38, 34–105.

Feldstein, M., Horioka, C., 1980. Domestic saving and international capital flows. Economic Journal 90, 314–329.

Fermanian, J.-D., Scaillet, O., 2003. Nonparametric estimation of copulas for time series. The Journal of Risk 5 (4), 25–54.

Forbes, K., Rigobon, R., 2002. No contagion, only interdependence: Measuring stock market comovements. Journal of Finance 57 (5), 2223–61.

Friedman, M., 1957. A Theory of the Consumption Function. New Jersey. Princeton University Press.

Friedman, M., 1968. The role of monetary policy. American Economic Review 58, 1–17.

Gabaix, X., Gopikrishnan, P., Plerou, V., Stanley, H., 2003. A theory of power-law distributions in financial market fluctuations. Nature 423, 267–270.

Genest, C., Ghoudi, K., Rivest, L.-P., 1995. A semiparametric estimation procedure of dependence parameters in multivariate families of distributions. Biometrika 82 (3), 543–552.

Granger, C., 2001. Overview of nonlinear macroeconometric empirical models. Macroeconomic Dynamics 5 (4), 466–481.

Granger, C., Terasvirta, T., Patton, A., 2006. Common factors in conditional distributions for bivariate time series. Journal of Econometrics 132, 43–57.

Hall, R., 2011. The high sensitivity of economic activity to financial frictions. Economic Journal 121, 351–378.

Hall, R., Jorgenson, D., 1967. Tax policy and investment behavior. American Economic Review 57, 391–414.

Hamilton, J., 2001. A parametric approach to flexible nonlinear inference. Econometrica 69 (3), 537–573.

Hansen, G., 1985. Indivisible labor and the business cycle. Journal of Monetary Economics 16 (3), 309–327.

Ibragimov, R., Jaffee, D., Walden, J., 2009. Non-diversification traps in catastrophe insurance markets. Review of Financial Studies 22 (3), 959–993.

Joe, H., 1997. Multivariate models and dependence concepts. Chapman and Hall/CRC, New York.

Jondeau, E., Poon, S., Rockinger, M., 2007. Financial Modeling under Non-Gaussian Distributions. Springer, London.

Jondeau, E., Rockinger, M., 2006. The copula-garch model of conditional dependencies: An international stock market application. Journal of International Money and Finance 25, 827–853.

Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decision under risk. Econometrica 47, 263–291.

Keynes, J., 1936. The General Theory of Employment, Interest and Money. Harcourt, Brace, and Company, New York.

Keynes, J., 1939. Relative movements of real wages and output. Economic Journal 49 (193), 34–51.

King, R., Plosser, C., 1984. Money, credit, and prices in a real business cycle. American Economic Review 74 (3), 363–380.

Kydland, F., Prescott, E., 1982. Time to build and aggregate fluctuations. Econometrica 50 (6), 1345–1370.

Laxton, D., Rose, D., Tambakis, D., 1999. The us phillips curve: The case for asymmetry. Journal of Economic Dynamics and Control 23, 1459–1485.

Long, J., Plosser, C., 1983. Real business cycles. Journal of Political Economy 91 (1), 39–69.

Longin, F., Solnik, B., 1995. Is the correlation in international equity returns constant: 1960-1990? Journal of International Money and Finance 14 (1), 3–26.

Lucas, R., 1972. Expectations and the neutrality of money. Journal of Economic Theory 4, 103–124.

Lucas, R., 1973. Some international evidence on output-inflation tradeoffs. American Economic Review 63, 326–334.

Lucas, R., 1977. Understanding business cycles. Carnegie-Rochester Conference Series on Public Policy 5, 7–29.

Lucas, R., Rapping, L., 1969. Real wages, employment, and inflation. Journal of Political Economy 77, 721–754.

Mandelbrot, B., 1963. The variation of certain speculative prices. Journal of Business 36, 394–419.

Mashal, R., Zeevi, A., October 14 2002. Beyond correlation: Extreme co-movements between financial assets. Tech. rep., Columbia University.

Minsky, H., 1982. Can it Happen Again? Essays on Instability and Finance. M. E. Sharpe, New York.

Modigliani, F., Brumberg, R., 1954. Utility analysis and the consumption function: an interpretation of cross-section data. In: Kurihara, K. (Ed.), Post-Keynesian Economics. Rutgers University Press, pp. 388–436.

Mogstad, M., Wiswall, M., 2009. Family size and children's education: How linear models can mask a nonlinear relationship. Tech. rep., New York University.

Ning, C., 2008. Extreme dependence of international stock market. Tech. rep., Ryerson University.

Ning, C., 2010. Dependence structure between the equity market and the foreign exchange market–a copula approach. Journal of International Money and Finance 29 (5), 743–759.

Okimoto, T., 2008. New evidence on asymmetric dependence structures in international equity markets. Journal of Financial and Quantitative Analysis 43 (3), 787–815.

Patton, A., 2004. On the out-of-sample importance of skewness and asymmetric dependence for asset allocation. Journal of Financial Econometrics 2 (1), 130–168.

Patton, A., 2006. Modelling asymmetric exchange rate dependence. International Economic Review 47 (2), 527–556.

Patton, A., 2009. Copula-based models for financial time series. In: Andersen, T., Davies, R., Kreiss, J., Mikosch, T. (Eds.), Handbook of Financial Time Series. Springer, New York, pp. 767–785.

Phelps, E., 1968. Money-wage dynamics and labor market equilibrium. Journal of Political Economy 76, 678–711.

Phelps, E., 2007. Macroeconomics for a modern economy. American Economic Review 97 (3), 543–561.

Phillips, A., 1958. The relation between unemployment and the rate of change of money wage rates in the united kingdom, 1861-1957. Economica XXV, 283–299.

Polkovnichenko, V., 2005. Household portfolio diversification: A case for rank-dependent preferences. Review of Financial Studies 18 (4), 1467–1501.

Rachev, S., 2003. Handbook of Heavy Tailed Distributions in Finance. North Holland, Amsterdam, Netherlands.

Ramey, V., 2011. Can government purchases stimulate the economy? Journal of Economic Literature 49 (3), 673–685.

Reinhart, C., Rogoff, K., 2009. This Time is Different: 800 Years of Financial Folly. Princeton Press.

Rodriguez, J., 2007. Measuring financial contagion: A copula approach. Journal of Empirical Finance 14, 401–423.

Romer, D., 2001. Advanced Macroeconomics, Second Edition. McGraw-Hill, New York, NY.

Rosenberg, J., Schuermann, T., 2006. A general approach to integrated risk management with skewed, fat-tailed risks. Journal of Financial Economics 79, 569–614.

Rostek, M., 2010. Quantile maximization in decision theory. Review of Economic Studies 77, 339–371.

Rothman, P., van Dijk, D., Franses, P., 2001. Multivariate star analysis of money-output relationship. Macroeconomic Dynamics 5 (4), 506–532.

Samuelson, P., 1967. General proof that diversification pays. Journal of Financial and Quantitative Analysis March, 1–13.

Sargent, T., 1979. Macroeconomic Theory. Academic Press.

Schweizer, B., Wolff, E. F., 1981. On nonparametric measures of dependence for random variables. The Annals of Statistics 9 (4), 879–885.

Shin, H., 2009. Securitisation and system stability. Economic Journal 119, 309–322.

Sklar, A., 1959. Fonctions de repartition a n dimensions et leurs marges. Publications de l'Institut de Statistique de L'Universite de Paris 8, 229–231.

Veldkamp, L., Van Nieuwerburgh, S., 2006. Learning asymmetries in real business cycles. Journal of Monetary Economics 53, 753–772.

Veldkamp, L., Wolfers, J., 2007. Aggregate shocks or aggregate information? costly information and business cycle comovement. Journal of Monetary Economics 54, 37–55.

Vives, X., 2008. Information and Learning in Markets: The Impact of Market Microstructure. Princeton Press.

Wilson, R., 1975. Informational economies of scale. Bell Journal of Economics 6, 184–195.

Table 1: Distribution of various copulas

Copula	Distribution	Parameter Range	Complete Dependence	Independence
Normal	$C_N(u,v;\rho) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v))$	$\rho \in (-1,1)$	$\rho = 1$, or-1	$\rho = 0$
Student-t	$C_t(u,v;\rho,d) = t_{d,\rho}(t_d^{-1}(u), t_d^{-1}(v))$	$\rho \in (-1,1)$	$\rho = 1$, or-1	$\rho = 0$
Clayton	$C_c(u,v) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}$	$\theta \geq 0$	$\theta = \infty$	$\theta = 0$

$\Phi_\rho(x,y)$ and $t_{\nu,\rho}(x,y)$ denote the standard bivariate normal and Student-*t* cumulative distributions, respectively: $\Phi_\rho(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} \frac{1}{2\pi|\Sigma|} \exp\{-\frac{1}{2}(x\ y)\Sigma^{-1}(x\ y)'\}dxdy$, and
$t_{\nu,\rho}(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} \frac{\Gamma(\frac{\nu+2}{2})}{\Gamma(\nu/2)(\nu\pi)|\Sigma|^{1/2}}\{1 + (s\ t)\Sigma^{-1}(s\ t)'/\nu\}^{\frac{-(\nu+2)}{2}} dsdt$. The correlation
matrix is given by $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$.

Table 2: Tail Dependence and Kendall's τ for various Copulas

	Left Tail Dependence	Right Tail Dependence	Kendall's τ
Gaussian	0	0	$\frac{2}{\pi}\arcsin\rho$
Student-*t*	$2t_{d+1}\left(-\sqrt{\frac{(d+1)(1-\rho)}{1+\rho}}\right)$	$2t_{d+1}\left(-\sqrt{\frac{(d+1)(1-\rho)}{1+\rho}}\right)$	$\frac{2}{\pi}\arcsin\rho$
Clayton	$2^{-\frac{1}{\theta}}$	0	$\frac{\theta}{\theta+2}$

The table presents analytical formulas for tail dependence and Kendall's τ, defined in equations (6), (7), and (4) of the text. θ denotes the dependence parameter of the Clayton copula, and d denotes the degrees of freedom for the Student-t copula.

Table 3: Correlations between GDP and Other Macro Variables

Panel A: Monthly data					
	GDP-Interest	GDP-Price	GDP-Emp.	GDP- Wage	GDP-Cons.
Linear correlation	0.2564**	-0.2344**	0.3591**	0.1254**	0.2379**
	(< 0.0001)	(< 0.0001)	(< 0.0001)	(0.0035)	(< 0.0001)
Rank correlation	0.1623**	-0.1754**	0.3462**	0.1151**	0.2427**
	(0.0002)	(< 0.0001)	(< 0.0001)	(0.0075)	(< 0.0001)
Kendall's tau	0.1086**	-0.1204**	0.2453**	0.0799**	0.1662**
	(0.0002)	(< 0.0001)	(< 0.0001)	(0.0055)	(< 0.0001)
Panel B: Quarterly data					
	GDP-Interest	GDP-Price	GDP-Emp.	GDP- Wage	GDP-Cons.
Linear correlation	0.3390**	-0.2970**	0.5752**	0.2188**	0.6690**
	(< 0.0001)	(< 0.0001)	(< 0.0001)	(0.0033)	(< 0.0001)
Rank correlation	0.2499**	-0.2396**	0.5060**	0.2316**	0.5824**
	(0.0008)	(0.0013)	(< 0.0001)	(0.0019)	(< 0.0001)
Kendall's tau	0.1742**	-0.1659**	0.3611**	0.1569**	0.4253**
	(0.0005)	(0.0010)	(< 0.0001)	(0.0018)	(< 0.0001)

Emp. and Cons. denote the employment rate and consumption, respectively. P-values
are in parentheses. ** stands for significance at the 5% level. Data are for the period
1964-2008, from the Federal Reserve Bank of St. Louis.

Table 4: Correlations between Money and Other Macro Variables

Panel A. Monthly data		
	$M1 - Emp.$	$M1 - GDP$
Linear correlation	-0.0101	-0.1342**
	(0.8157)	(0.0018)
Rank correlation	0.0200	-0.0465
	(0.6431)	(0.2816)
Kendall's tau	0.013	-0.0308
	(0.6621)	(0.2856)
Panel B. Quarterly data		
	$M1 - Emp.$	$M1 - GDP$
Pearson corr.	-0.0427	0.0017
	(0.5703)	(0.9817)
Spearman rho	0.0590	0.0875
	(0.4326)	(0.2440)
Kendall's tau	0.0380	0.0530
	(0.4511)	(0.2924)

Emp. denotes the employment rate. P-values are
in parentheses. ** stands for significance at the
5% level. Data are from 1964-2008, from the
Federal Reserve bank of St. Louis.

Table 5: Correlations between Inflation and Other Macro Variables

Panel A. Monthly data		
	Inflation-Emp.	Inflation-GDP
Linear correlation	0.1176**	0.1321**
	(0.0063)	(0.0021)
Rank correlation	0.0990**	0.0925**
	(0.0216)	(0.0318)
Kendall's tau	0.0676**	0.0613**
	(0.0228)	(0.0333)
Panel B. Quarterly data		
	Inflation-Emp.	Inflation-GDP
Pearson corr.	0.2609**	0.1355**
	(0.0004)	(0.0706)
Spearman rho	0.1797**	0.0199
	(0.0161)	(0.7918)
Kendall's tau	0.1249**	0.0134
	(0.0132)	(0.7914)

Emp. denotes the employment rate. P-values are in parentheses.
p-values. ** denotes significance at the 5% level. Data are
from 1964-2008, from the Federal Reserve Bank of St. Louis.

Table 6: Tail dependence: GDP and Other Macro Variables

Panel A: Monthly data					
	GDP-Interest	GDP-Price	GDP-Emp.	GDP- Wage	GDP-Cons.
λ_l	0.2044	0.0819	0.1952**	0.0519	0.1864**
	(0.6609)	(0.8565)	(0.0581)	(0.0454)	(0.0493)
λ_r	0.0000	0.0000	0.1132**	0.0303	0.009**
	(1.0376)	(1.2951)	(0.063)	(0.0418)	(<0.0001)
AIC	-9.0829	-1.7335	-57.056	-14.5406	-36.4694
BIC	-0.5035	6.8459	-48.4766	-5.9612	-27.8899
Panel B: Quarterly data					
	GDP-Interest	GDP-Price	GDP-Emp.	GDP- Wage	GDP-Cons.
λ_l	0.2691**	0.1724	0.4905**	0.1833**	0.4392**
	(0.0847)	(0.7861)	(0.0573)	(0.0903)	(0.0704)
λ_r	0.0426	0.0000	0.2479**	0.0591	0.4235**
	(0.1021)	(1.2302)	(0.1098)	(0.1006)	(0.0756)
AIC	-19.5044	0.7241	-70.2723	-11.3581	-81.718
BIC	-13.1296	7.0989	-63.8975	-4.9833	-75.3433

Emp. and Cons. denote the employment rate and consumption, respectively.
Standard errors are in parentheses. ** denotes significance at the 5% level.
Data are from 1964-2008, from the Federal Reserve Bank of St. Louis.

Table 7: Tail dependence: Money and Other Macro Variables

Panel A: Monthly data		
	M1-Employment	M1-GDP
λ_l	0.0000	0.0000
	(5.0561)	(5.0554)
λ_r	0.0000	0.0000
	(5.0561)	(5.0554)
AIC	4.1881	5.0198
BIC	12.7675	13.5992
Panel B: Quarterly data		
	M1-Employment	M1-GDP
λ_l	0.0000	0.027
	(8.8119)	(1.4356)
λ_r	0.0000	0.0000
	(4.6804)	(1.3089)
AIC	3.9651	3.7581
BIC	10.3399	10.1329

Employment denotes the employment rate. Standard errors are in parentheses. ** denotes significance at the 5% level. Data are from 1964-2008, from the Federal Reserve Bank of St. Louis.

Table 8: Tail dependence: Inflation and Other Macro Variables

Panel A: Monthly data		
	Inflation-Emp.	Inflation-GDP
λ_l	0.036**	0.1589
	(< 0.0001)	(1.0736)
λ_r	0.0000	0.0000
	(< 0.0001)	(1.3347)
AIC	-0.358	0.9597
BIC	8.2214	9.5391
Panel B: Quarterly data		
	Inflation-Emp.	Inflation-GDP
λ_l	0.4090	0.0958
	(0.6516)	(0.7665)
λ_r	0.0000	0.0000
	(1.0372)	(1.1077)
AIC	-1.7116	2.0904
BIC	4.6632	8.4652

Emp. denotes the employment rate. standard errors are in parentheses. ** denotes significance at the 5% level. Data are for the period 1964-2008, from the Federal Reserve Bank of St. Louis.