

# A Model of Endogenous Extreme Events

Lorán Chollete \*

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## Abstract

Our paper addresses the question "What determines susceptibility to extremes and duration of extreme episodes?" Extreme events play a large role in recent socio-economic life, and in economic models, such as those constructed by Barro (2006), Gabaix (2008), and Wachter (2011). It is therefore important to understand the extent to which the likelihood of extremes depends on behavior of economic agents. We model endogeneity in extremes using the concept of frictions due to congestion, as in the transaction cost and public good literature. We construct a parsimonious model of an economy with both financial congestion and positive externalities due to network effects. Our model delivers three main results. First, susceptibility to extremes depends on differences in marginal substitution between net borrowers and lenders. Second, extreme episodes last until marginal substitution rates converge, or expected costs rise. Third, a government policy that taxes resource transfers may decrease or increase the likelihood of extreme events.

**Keywords:** Endogenous Risk; Extreme Event; Financial Congestion; Network; Public Good

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\*Email [loran.g.chollete@uis.no](mailto:loran.g.chollete@uis.no). The author is grateful for support from the Research Council of Norway, Finansmarkedsfondet Grant #185339, and for comments from participants at UiS Business School, the 4th Annual Conference on Extreme Events, Expectations and Decisions, Norwegian School of Economics and Business Administration, Norwegian Central Bank, and Ryerson University.

# 1 Introduction

*'Financial markets...can be quite fragile and subject to crises of confidence. Unfortunately, theory gives little guidance on the exact timing or duration of these crises...'*  
Reinhart and Rogoff (2009b), page xil.

The need to understand extreme events in markets has become compelling. This paper develops a simple model of endogenous extremes, based on positive and negative externalities. We address two salient aspects of modern financial markets: dynamics and endogeneity in extremes. By dynamics, we refer to recurring episodes of 'surprise' extreme events. By endogeneity, we refer to the effect of economic agents on the likelihood of extremes.<sup>1</sup> The economic costs of extreme events can be prohibitive, including widespread risk of default, and an impaired trading process because prices are uninformative. Extreme events also carry large social and psychological costs, such as the risk of spillovers and increased Knightian uncertainty in an unstable economy.<sup>2</sup>

Discussions of extreme economic events often model them as generated exogenously.<sup>3</sup> But sometimes the likelihood of extreme events is affected by behavior of economic agents. Dynamic, endogenous extremes occur in economics and in nature, including the effect of human activity on the likelihood of extreme financial events, and on the natural environment.<sup>4</sup> In this paper, we build on existing literature to explore a possible explanation for endogenous extremes, based on interaction of congestion and network externality effects. Extreme events have externality features, since they affect many individuals in the national or global economic system, even though often precipitated by a small number of individuals. It is well known that externalities cause inefficiency of the price system.<sup>5</sup> Consequently, if extreme events are due to externalities, society may not pay the appropriate price for the extremes that it generates.

**Congestion as a Comprehensive Umbrella.** A number of researchers have analyzed financial extremes and crises, resulting in a variety of different approaches. Rational approaches discuss bubble expectations, agency costs, multiple equilibria, and fire-sale externalities. The financial frictions approach emphasizes that liquidity needs at the investor, bank and individual level have aggregate

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<sup>1</sup>Endogenous extremes may arise due to agents' negligence, bounded rationality, excessive risk taking, or corruption.

<sup>2</sup>See Harris (2003), chapter 9; Weitzman (2007); and Caballero and Krishnamurthy (2008). Phelps (2007) emphasizes that a modern economy features endogenous uncertainty due to unforecastability of innovation. For further discussion of financial innovation, see Rajan (2010).

<sup>3</sup>See Friedman and Laibson (1989); and Barro (2006).

<sup>4</sup>For extremes in economics, see Fisher (1933); Minsky (1982); Grossman (1988); Gabaix et al. (2006); Allen and Gale (2007); Brunnermeier and Pedersen (2009); and Krishnamurthy (2010). For extremes in nature, see Below et al. (2007); and Stern (2007).

<sup>5</sup>For textbook expositions of externalities, see Varian (1992), and Mas-Colell et al. (1995). For aggregate effects of externalities, see Blanchard and Kiyotaki (1987); Harrison (2001); and Gabaix (2010a).

ramifications.<sup>6</sup> Behavioral finance focuses on psychological biases and inefficiencies on the part of banks and individuals. And econophysics research posits that crashes are an emergent property of systems with interacting components.<sup>7</sup> Several of these research frameworks are presented in Table 1. These multifarious approaches are non-nested and often develop independently of each other. It is therefore difficult to test models of extreme events or develop policy aimed at mitigating extremes, in a manner that is credible to economists from different persuasions.

On an empirical level, the various approaches yield mixed results. Figures 1 to 4 illustrate time series data on measures of liquidity and investor confidence in the period surrounding large crashes in the Dow Jones Industrial Average (DJIA). Let us focus on the lower panels, which represent percentage changes. We see in Figure 1 that the various measures all experienced concurrent spikes with the 1987 stock market crash. However, in 1998 during the LTCM crisis there is no strong pattern, as depicted in Figure 2. In Figure 3 we examine the months around Lehman Brothers' bankruptcy in September 2008. Here the liquidity measures display concurrent spikes, but not the confidence measure VIX.<sup>8</sup> Finally, in Figure 4 all measures spike around the date of the flash crash in May 2010. However, the magnitude of changes in this figure (as in all other figures) is very much larger than the DJIA itself, which is somewhat puzzling. In order to explore the empirical relation a bit more formally, Table 2 presents rank correlations of the liquidity and confidence measures with the DJIA. The results are not reassuring. Only the confidence measure VIX is significantly correlated with the DJIA. The liquidity measures are not correlated with the DJIA. While these results are simple, they underscore that focusing on liquidity (or any single factor) alone may not suffice to capture the forces behind market crashes.

What can be done to remedy this situation? A possible approach is just to work within one framework (e.g. behavioral or frictions) at a time, and build slowly to more general results. Another approach, which we find persuasive, is to look for common ground in all of the theories, that is, to answer the question "What, if anything, do the various theories of extreme events have in common?"

**A Common Theme.** From this perspective, at least one common element emerges: congestion in financial trades. All the above theories are based on the idea that extreme events arise when 'too many' individuals start doing the same activity. The activity is typically one of the following: withdrawing money from banks, selling assets that no longer seem valuable, refraining from buying

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<sup>6</sup>For rational bubbles, agency costs and multiple equilibria, see Blanchard (1979), Bernanke and Gertler (1989), and Benhabib and Farmer (1999), respectively. For fire-sale externalities, see Allen and Gale (2007). For financial frictions and liquidity, see Brunnermeier and Pedersen (2009), Pedersen (2009), and Allen et al. (2011).

<sup>7</sup>In behavioral finance, Odean (1999) and Barber and Odean (2000b) provide evidence of excess trading due to overconfidence; Benartzi and Thaler (1995) discuss implications of myopia for the equity premium; herd behavior is examined by Froot et al. (1992); limits to arbitrage are examined by Shleifer and Vishny (1997). For econophysics research, see Montier (2002) and Sornette (2004).

<sup>8</sup>VIX is the implied volatility of an at-the-money 30-day option, which reflects investor confidence.

assets, demanding margins be repaid, holding on to liquid assets so that entrepreneurs cannot borrow to finance projects. However, if a small enough group of individuals did this action, it would not matter. There has to be a critical mass for the system to become congested, which then causes markets to fail or crash.<sup>9</sup> This congestion of the system is consistent with actions that arise because of rational beliefs, behavioral biases or any of the other proposed theories of extremes.

In addition to nesting existing theories, this formulation is important because it delivers an important insight: it clarifies that extreme events occur not just because agents are acting in a certain way, but also highlights the importance of *system capacity*. Just as a large highway can handle more traffic than a small road, a developed financial market can handle more people selling assets, because of another, countervailing effect, network externalities: the more people involved in the market, the greater likelihood of finding counterparties. So market participation brings manifold benefits, especially in the early or middle stages of economic development—entrepreneurs in need of funding for worthwhile projects are allowed to borrow and invest in welfare-enhancing innovations. This latter perspective is *not emphasized* in the literature of extreme events and crises. Therefore, the congestion approach has two advantages. First, it is inclusive and respectful to major frameworks for modeling extreme events. Second, it provides a way of modeling the important tension between costs and benefits of trading resources in financial markets.

How can we formally model congestion in financial markets? There is a mature public finance framework on congestion externalities (Baumol and Oates (1988); Cornes and Sandler (1996)), on which we build in the present paper. Consider an economy with  $I$  agents, indexed by  $i = 1, \dots, I$ . As in the public finance literature we model each agent  $i$  as buying two types of good, a private consumption good  $y_i$  and a congestible public good  $q_i$ . In this case the congestible good is the level of resource transfers (borrowing, lending, investing or trading) by each agent. If the aggregate amount of the congestible public good is  $Q \equiv \sum_i q_i$ , then the *likelihood of congestion* in the system is represented by a function  $C(Q)$ .

**Relation to Existing Approaches.** The congestion function has an immediate interpretation in existing theories. In a rational setting,  $C(\cdot)$  represents likelihood of experiencing high agency costs or ending up in an inefficient equilibrium. In a behavioral setting,  $C(\cdot)$  captures the possibility that overconfidence or herd behavior is strong enough to generate excessive trading. And in the frictions literature,  $C(\cdot)$  measures the likelihood of large, simultaneous liquidity needs in a significant portion of market participants. In sum, our approach attributes extreme events to *any* major economic factor that affects the likelihood of congestion. Therefore, it accounts for endogenous extreme events regardless of the source—illiquidity, behavioral factors, or rational causes.

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<sup>9</sup>Put differently, as long as the system has enough financial slack to absorb the effect of agents' actions, financial markets can continue to function smoothly. It is when the actions cumulate to a large enough degree (relative to system capacity) that extreme events occur.

## 1.1 Congestion and Network Effects in Markets

As discussed above, a prominent characteristic of endogenous extremes is congestion: too many investors try to perform the same strategy (e.g. selling an unattractive asset) at the same time, which subsequently prevents anyone from using the market. The importance of congested strategies has been explicitly modelled in the literature on herding, liquidity, and information complementarities.<sup>10</sup> However, the existence of many investors in a market also has a *positive* network effect, since it raises the likelihood of finding someone with whom to trade.<sup>11</sup> This effect constitutes a social benefit of using the financial system to transfer risk and resources. We therefore consider both congestion and network effects that may arise due to the presence of market participants.

**A Highway Metaphor.** It is useful to think of markets as being like highways. In the case of a physical highway, its use allows us to work, consume, invest and live in different places. Consequently, the highway enhances employment of resources for all of society, since it is easier for resources to go where they are most needed instead of being locally restricted. Nevertheless, over-use of the highway also leads to congestion, pollution and other external effects. Highway use therefore confers both positive and negative social effects. In the case of markets, they are a virtual highway whose use enhances welfare when agents participate, but which is sometimes prone to financial congestion. Use of this virtual highway, therefore, has both positive and negative social effects. Since congestion externalities have been studied extensively in public economics, we use a simple public good framework in our economic model.

As the recent financial crisis has demonstrated, markets can become congested with frightening speed and intensity, which precipitates extreme events. In anticipation of Section 2, we use  $C$  to denote the likelihood of extreme events or 'tail risk', which can affect both real and financial sectors. Previous research has typically modeled this tail risk as an exogenous probability (Barro (2006); Gabaix (2008); Wachter (2011)). It is evidently valuable for both academics and policymakers to understand more about determinants of tail risk. One approach (Shin (2009)) is to consider the likelihood  $C$  of extreme events to depend on a reasonable economic variable  $x$ , such as excess borrowing: that is, the researcher assumes  $C = C(x)$ , based on economic intuition. While appealing as a starting point, this approach runs the risk of allowing the researcher freedom to choose a tractable but potentially inaccurate characterization of tail risk. As we show below (in Propositions 4 and 5

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<sup>10</sup>For early references, see Fisher (1933) and Keynes (1936), Chapter 12. For herding research, see Camerer (1989); Banerjee (1992); Bikhchandani et al. (1992); and Froot et al. (1992). For liquidity, see Pedersen (2007); Brunnermeier and Pedersen (2009); Pedersen (2009); and Wagner (2011). For complementarities, see Morris and Shin (1998); Cooper (1999); Vives (2008); and Veldkamp (2011).

<sup>11</sup>For network externalities, see Katz and Shapiro (1985); Shapiro and Varian (1998); and Easley and Kleinberg (2010).

and Corollaries 1 and 2), the optimal  $C(\cdot)$  that arises from a micro-founded approach has surprising properties and policy implications, which might not be evident based on a priori reasoning.

Our analysis highlights public good features of financial markets. Negative public good aspects have been examined by a number of authors.<sup>12</sup> As discussed above, market participation also confers positive externalities. We therefore focus on a public good setting with *congestion* and *network effects*, to summarize both negative frictions and positive external effects present in modern financial markets. This framework captures two relevant aspects of financial markets. First, during times of crisis, many investors wish to perform the same type of resource transfer, such as selling bad stocks or diversifying. This results in crowded trades, and what is individually rational becomes impossible for anyone to do (Pedersen (2009); Duffie (2010)). Second, and perhaps more important for economic development, in normal times increased use of financial markets is welfare-enhancing, since it loosens intertemporal budget constraints and permits profitable projects to be undertaken. It is only when markets have too large demands placed on them (excess trading, complex securities, etc.) that congestion effects dominate. We use the term *financial congestion* to denote such situations. As shown in Table 1, financial congestion springs from a variety of sources, including liquidity and frictions, consumer sentiment, excess leverage, and uncertainty about complex securities or new technology. The congestion-network mechanism is always in place, it just has a different effect depending on the quality of market use. Since the same forces are at work in normal and bad times they may in principle be manipulated by public policy.

How does this formulation of endogenous extremes help us? It does so in three ways. First, it allows us to understand the origin of some extremes (the endogenous ones), thereby giving us insight into which we can plausibly try to avert. Second, it gives banks and regulatory authorities an additional set of tools from public economics—subsidies, property rights, and so on—that may help to address extreme events before and during their occurrence. Finally, as noted by Allen and Gale (2007), modern economics does not always show a specific market failure that central banks and regulators can correct by their intervention.<sup>13</sup> Our formulation provides a perspective on the role of government, since it identifies a clear market failure, namely the (positive and negative) externalities from agents' neglect to consider their impact on the likelihood of extremes.

## 1.2 Anatomy of Financial Congestion

Financial congestion arises from various sources, including illiquidity, behavioral biases, excess borrowing and investor sentiment. Focusing on one source alone may not be enough to detect

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<sup>12</sup>See Ibragimov et al. (2009); Acharya et al. (2010a); Acharya et al. (2010c); Shin (2010); Brunnermeier and Sannikov (2011); and Ibragimov et al. (2011).

<sup>13</sup>See Bernanke and Gertler (2001) for a related discussion.

imminent extreme events. In order to illustrate how financial congestion manifests itself and to fix ideas, we briefly discuss two simple case studies of extreme events, in 1987 and 2007.

**Crash of October 1987.** The US stock market crash on October 19, 1987 is a very clear example of what financial markets behave like under congestion. Grossman (1988) demonstrated that excess use of synthetic securities resulted in excess trading which precipitated such crashes. The crash is one of the most significant drops ever seen in stock markets. The size of the drop was tremendous, reflected in all major indices. The DJIA fell 508 points, or 22.6%. The S&P500 also lost more than 20% and the Nasdaq fell around 15%. The effect was widespread and affected all types of stocks. Standard large companies like IBM lost around 25% of their value. At the same time, technology stocks like Apple and Intel dropped by around 20% each. Congestion was apparent in the shifting directions of aggregate buys and sells. In particular the following pattern was observed:<sup>14</sup>

- *9:30am.* DJIA falls 200 points from 2246 to 2046, shortly after the opening bell.
- *10:00 am.* DJIA rises to 2100. This pattern continues throughout the day...
- *2:30 pm.* DJIA again at approximately 2100.
- *2:45 pm.* Selloff begins, removing 300 more points.
- *4:00 pm.* DJIA at approximately 1800 points.

The heavy trading volume was too difficult for computers at the NYSE to keep pace with, so it took considerable time to cumulate all the losses. Eventually the total loss exceeded 500 points, nearly one-quarter of the DJIA's value. This and other important market crashes are summarized in Table 3.<sup>15</sup>

**2007 Subprime Scare.** In the spring and summer of 2007, the aftershock from the subprime market, a relatively small part of US financial markets, reached over to touch hedge funds and international markets. In the US, credit spreads widened ominously, even for safer debt, and the housing market reached record breaking levels. In Britain the interbank rate reached its highest level in 9 years. One of the more outstanding examples occurred in July and August of 2007, when hedge funds suffered such severe losses that Goldman Sachs, in a one-of-a-kind intervention, had to infuse US\$3 billion into one of its funds, Global Equity Opportunities. This fund lost 30 per cent of its value in the week between August 3 and August 10, 2007. A major reason cited for the severe hedge fund losses was that the extremes occurring in markets were '25 standard deviation' events (New York

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<sup>14</sup>This account of the 1987 crash summarizes material in Sornette (2004).

<sup>15</sup>We include only the USA and Hong Kong, because both countries have liberal capital markets, with few restrictions on investing and transferring capital overseas. Thus the extreme events are not due to government intervention but to competitive market forces.

Times, August 13, 2007). The incidents are puzzling because hedge funds did not seem directly exposed to heavy enough risk to warrant such drops in value. Moreover, most large investors have risk management systems that are stress tested against extreme market events such as terrorism risk, banking crises, and interest rate changes. So what sort of event could surprise such respected investors enough to lose as much as one-third of their value? A potential answer is that our approach to understanding surprise extreme events is incomplete. This incompleteness may stem from the fact that both information economics and current risk management are generally silent about endogeneity in the likelihood of extremes.

In light of the preceding observations, we analyze extreme events that are dynamic and endogenous. It should be noted that a type of endogeneity is recognized in certain spheres of risk analysis. Information economics acknowledges that individual agents' behavior can affect individual outcomes in settings such as insurance markets. However, this framework is usually restricted to individual agents or sectors, and typically requires asymmetric information between borrowers and lenders.<sup>16</sup> In our model, we illustrate that endogenous risk effects can spill over to other sectors even in the absence of asymmetric information. A graphical depiction of our approach is in Table 4, which shows that our view of endogenous probability nests that of moral hazard. The difference is that we consider broader settings, encompassing spillovers and general information structures.

### 1.3 Related Literature

Our research relates to existing work on extreme events, coordination externalities, liquidity, and systemic risk. Regarding extreme events, Mandelbrot (1963) and Fama (1965) show that US stock returns are not gaussian and have heavy tails. Fama (1965) also documents that stock crashes occur more frequently than booms. Jansen and de Vries (1991) investigate the distribution of extreme stock prices in *S&P500* stocks, and document that the magnitude of 1987's crash was somewhat exceptional, occurring once in 6 to 15 years. Susmel (2001) uses extreme value theory to investigate univariate tail distributions for international stock returns. He documents that Latin American markets have significantly heavier left tails than developed economies. Empirically, it has been shown that investors and assets tend to behave similarly during extreme periods, which also motivates the congestion approach. In this regard, Longin and Solnik (2001) use a parametric multivariate approach to derive a general distribution of extreme correlation. The authors examine equity index data for G5 economies and document that correlations are significant in the left tail of returns. They also show that correlations increase during market downturns. Hartmann et al. (2003) doc-

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<sup>16</sup>The 2007-2009 financial crisis, however, affected numerous sectors and regions. Moreover, for subprime mortgages it is difficult to apply standard information economics. To do so, one must argue that poor quality of the subprime borrowers was 'hidden information', and that lenders did not understand the potential for default when supplying loans to borrowers with poor credit history or no collateral.



ument that Latin American currencies have less extreme dependence than in east Asia, and that developing markets often have a smaller likelihood of joint extremes than do industrialized nations. Hartmann et al. (2004) report that stock markets crash together in one out of five to eight crashes, and that G5 markets are statistically dependent during crises. Poon et al. (2004) model multivariate tails of stock index returns from G5 markets. They document that in only 13 of 84 cases is there evidence of asymptotic dependence. They argue that the probability of systemic risk may be overestimated in financial literature. Chollete et al. (2009) use general dependence measures to model portfolios of international stock returns from the G5 and Latin America. They find that an empirical model that allows for asymmetric dependence outperforms standard models, and improves Value-at-Risk computations. In a comprehensive study of more than 800 years of financial crises, Reinhart and Rogoff (2009b) conclude that the biggest factors in crises are excessive debt and sudden shifts of confidence. During booms, governments, banks or corporations increase borrowing, and underestimate aggregate risk. The authors emphasize a "This time is different" mentality, where market participants downplay the likelihood of extreme events during the boom period preceding crises. Adrian and Brunnermeier (2010) construct a measure, CoVaR, that summarizes the conditional likelihood of an institution's experiencing a tail event, given that other institutions are in distress. They estimate CoVaR for commercial banks, investment banks and hedge funds in the US, and document statistically significant spillover risk across institutions. Theoretical work on extreme events also supports the idea of endogenous extremes due to congestion. In this regard, Montier (2002) discusses the notion that crashes and outliers are endogenous, perhaps due to a preponderance of sellers relative to buyers. Danielsson and Shin (2003) model a scenario where unanticipated coordination of agents' behavior leads to an endogenous increase in risk. Liu et al. (2003) demonstrate in a jump diffusion setting that consideration of rare events discourages individual investors from holding leveraged positions. Bazerman and Watkins (2004) suggests that certain 'surprise' events in modern society are predictable, since there may exist sufficient information to know that these events are imminent. Liu et al. (2005) develop an equilibrium model of asset prices with rare events. The authors document that aversion to rare events can ameliorate option mispricing. Gabaix et al. (2006) develop a theory of stock volatility, where the driving force is trading by large investors, during illiquid markets. Barro (2006) builds a representative agent economy that incorporates the risk of a rare disaster, modelled as a large drop in the economy's wealth endowment. When this model is calibrated to the global economy, it can explain the equity premium and low risk free rate puzzles, and helps account for stock market volatility. Weitzman (2007) constructs a Bayesian model of asset returns. He discovers that when agents consider the possibility of extremes, there is a reversal of all major asset pricing puzzles. Gabaix (2008), Gabaix (2010b), and Wachter (2011) generalize the Barro framework to account for dynamic probability of extreme events. These latter models can explain outstanding macroeconomic and finance puzzles as well as the behavior of stock volatility. Bollerslev and Todorov (2011) use high frequency options data to construct an index of

implicit disaster fears among investors. The authors' method helps to explain patterns in the equity premium and stock market variance. These papers all underscore the importance of accounting for extreme events in markets. None of the papers, however, analyzes the endogenous causes of dynamic extreme events. This lack of research on endogenous extremes serves as a primary motivation for our paper.

Regarding coordination externalities, Bikhchandani et al. (1992) and Banerjee (1992) build models of how agents may coordinate and disregard their private information. Such herd behavior has effects on the stock market, as examined by Froot et al. (1992). Allen and Gale (1998) construct a model of banking panics that are related to the business cycle. In this model depositors may rationally fear low returns on their deposits after observing that leading economic indicators suggest a downturn, and therefore optimally withdraw all funds from banks. The authors show that bank runs can be efficient, although this result does not hold when runs lower asset returns, nor in the presence of a stock market. In their Theorem 5 and Corollary 5.1, the authors formalize the inefficiency of bank runs, which motivates central bank intervention. Aggregate consequences of externalities have also been modelled by Blanchard and Kiyotaki (1987); Cooper (1999); Harrison (2001); and Veldkamp and Van Nieuwerburgh (2010). A related literature on bubbles suggests extreme events can be caused by rational indeterminacy, behavioral factors, or new technology, as discussed by Blanchard (1979), Abreu and Brunnermeier (2003), and Hong et al. (2008), respectively. Allen and Gale (2007) show that bubbles may be precipitated by incentive and limited liability issues, which reduce the costs of individual risk taking. In chapter 7 of Allen and Gale (2007) the authors suggest there is not always a clear market failure for regulators to correct, in the case of market instability. This latter point motivates our paper to develop a framework that formalizes market failures inherent in extreme events. We recently became aware of papers by Bianchi (2010), Bianchi (2011) and Bianchi and Mendoza (2011), who analyze dynamic equilibrium models where excess borrowing increases financial system fragility. The authors show that these negative externalities raise the likelihood of extreme events, and entail significant welfare costs in terms of foregone consumption. Bianchi (2011) suggests that increased borrowing costs during tranquil periods may correct the externalities. The above papers do not consider the interplay between negative and positive externalities from resource transfers, which motivates our paper.

Regarding liquidity, Brunnermeier and Pedersen (2009) demonstrate that liquidity needs at the market and funding level can be self-reinforcing. During bad times, such liquidity needs can precipitate financial crises. Pedersen (2009) describes a situation where major market players all require liquidity, which ends up causing a shutdown in markets. Similar frictions can arise in markets where capital is deployed in a sluggish manner, and when there is a discrepancy between sophisticated and naive investors, see Duffie (2010) and Stein (2009). Wagner (2011) demonstrates that when investors face liquidation risk in multiple assets, they hedge by selecting heterogeneous portfo-

lios with lower diversification benefits. These papers correctly emphasize the importance of transaction costs and liquidity in market outcomes. However, as shown in Tables 1 and 5, extreme events may occur because of other factors such as excessive government debt and confidence shifts (Reinhart and Rogoff (2009b), and Keynes (1936)) or over-borrowing in the financial sector (Fisher (1933)). Moreover, liquidity is notoriously very difficult to measure (Aitken and Comerton-Forde (2003); Sadka and Koracjczyk (2008); and Goyenko et al. (2009)). Finally, the results in Table 2 show that liquidity does not always convincingly relate to stock market changes during extreme periods. Thus, a focus on liquidity may ignore other important determinants of endogenous extreme events. Expanding the focus of the liquidity literature (with respect to extreme events) provides a further motivation for a general congestion approach.

Regarding systemic risk, Danielsson and Zigrand (2008) construct an equilibrium model where asset prices are determined in the presence of systemic risk. The authors argue that while regulation can reduce the likelihood of systemic risk, it carries costs, such as increased risk premia and volatility, and the possibility of non-market clearing. Acharya et al. (2010a) describe the causes of the financial crisis of 2008, arguing that a key catalyst was excessive leverage, which created systemic tail risk. Acharya et al. (2010b) construct a measure of systemic risk tendency, SES, based on comovement of expected shortfall of individual institutions and the aggregate financial system. They demonstrate ex ante predictive power of SES for various companies during the period 2007-2009. Acharya et al. (2010c) develop an approach to regulating systemic risk based on SES. They propose that financial firms be taxed proportionally to their expected loss in the event of a systemic crisis. On the theoretical side, Embrechts et al. (2002) and Ibragimov and Walden (2007) show that when portfolio distributions are heavy-tailed with nonlinear dependence, they may result in limited diversification. Shin (2009) demonstrates a wedge between individual risk and systemic risk, based on the tendency of agents to coordinate during extreme periods. A similar result based on heavy tails is proven by Ibragimov et al. (2009); and Ibragimov et al. (2011). Thus, there are aggregate economic ramifications for heavy tailed assets, since individuals' diversification decisions yield both individual benefits and aggregate systemic costs. If systemic externality costs are severe, the economy may require intervention to improve resource allocation. Morris and Shin (2011) develop a model where adverse selection is amplified across market participants when agents cannot compute maximal expected losses. None of these papers examines endogeneity of tail risk where both positive and negative externalities are accounted for. This provides a final important motivation for our paper.

## **1.4 Contribution of our Paper**

Our paper contributes to the literature in several respects. First, unlike previous research, we characterize the endogenous probability of extreme events, using a micro-founded approach. Specifically,

we derive mathematical expressions to characterize the 'signature' of dynamic, endogenous extremes. Second, we account for both positive and negative externalities in financial markets, within the same model. Third, our model demonstrates the conditions where government intervention is and is not justified in the face of extreme events. More generally, our framework allows us to discuss an economic approach to extreme events, using the lens of public economics. The remainder of the paper is organized in the following manner. Section 2 constructs a simple model of endogeneity of  $C$ , using a congestible public good framework. Section 3 develops this model to characterize dynamic, endogenous extremes in a model of resource transfers between agents. Section 4 concludes.

## 2 Endogenous Extremes: Financial Congestion and Networks

*A larger network means a smaller world*

Delta advertisement

We model endogeneity of extremes as arising due to congestion effects. Extreme probabilities can be exogenous or endogenous, each with a different policy response.<sup>17</sup> Exogenous extremes arrive from outside the economic system and are truly acts of nature, from the perspective of the domestic economy. For example, in a crop-based economy, the probability  $p$  of extreme changes in crop value could depend on exogenous swings in weather.<sup>18</sup> Since weather is generally unpredictable beyond a few days, and exogenous to an individual farmer, in this case the probability of extremes is essentially random. Endogenous extremes, by contrast, are generated and amplified within the economic system, by agents' activity and interaction. This activity persists because extremes have externality-like attributes, and therefore agents 'over-produce' the amount of extremes in the system. Several alternative approaches are presented in Table 5. For example, stock market crashes and banking panics may stem from excessive risk taking and borrowing of a segment of the economy (Fisher (1933)), excessive credit creation (Allen and Gale (2000)), and excessive reliance on computer-based trading (Grossman (1988)).<sup>19</sup> Since each agent has an incentive to borrow or risk too much from the social point of view, competition leads to overproduction of extremes. As discussed above, this congestion effect balances against a countervailing network effect, since market

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<sup>17</sup> In practice, there is a spectrum of extremes, with some being a mixture of exogenous and endogenous. The tools developed herein help us to assess the dominant influence on extremes.

<sup>18</sup> Other causes of exogenous extremes include foreign wars, natural catastrophes, and uncertainty about new technology.

<sup>19</sup> The above authors consider some form of extreme event or crisis, but vary in their emphasis on endogeneity. Our paper seems to be the first to use this framework explicitly in a general setting.

participation increases social welfare. Hence, the probability of extremes may no longer be random. We develop the relevant expression for endogenous extremes below.

While exogenous extremes are statistically unrelated to the economic environment, endogenous extremes (since they are generated by economic agents) should reflect optimizing or equilibrium behavior of agents. We focus on a canonical form of economic interaction, namely transfer of resources.<sup>20</sup> The heart of the externality is as follows. A key feature of modern financial markets is that they enhance agents' ability to *transfer resources*, which involves either trading commodities and assets or moving assets across time. This transfer of resources can aid or harm other individuals not party to the transfer. For example, massive stock sales by some investors can decrease the stock price, thereby increasing market volatility and diminishing portfolio values of all other investors who own that stock.<sup>21</sup> In similar vein, excessive borrowing by a relatively small set of investors can increase the likelihood of a systemwide market crash.<sup>22</sup> On the positive side, increased market participation makes it easier for investors and borrowers to find counterparties due to thick market effects. This is true domestically as well as internationally, since emerging economies with excess savings can channel their surplus to developed countries and loosen their budget constraints. Hence the behavior of individual agents inherently affects the wellbeing of others without being reflected in a price—the definition of an externality. In sum, modern markets confer ability to transfer financial resources easily, but may bear hidden costs and benefits in the form of externalities.<sup>23</sup><sup>24</sup> Even though agents realize their collective behavior affects market thickness and the likelihood of extremes, they may persist in myopic behavior, since they do not bear all costs and benefits.

## 2.1 A Model of Financial Congestion

We consider a stylized model where agents use financial markets to transfer idle resources to the present from the future, or from one resource-plenty agent to a resource-scarce agent. Agents partic-

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<sup>20</sup>Resource transfers include such activities as borrowing funds, and trading assets. Resource transfers are essential functions of financial markets, see pp. 4-7, Goetzmann and Rouwenhorst (2005).

<sup>21</sup>See Gabaix et al. (2006).

<sup>22</sup>Fisher (1933), Minsky (1982), Montier (2002), and Allen and Gale (2007) discuss the fact that large asset price and output fluctuations for the entire economy may result from various forms of resource transfers within specific sectors—increased trading, increased desire to liquidate assets, and increased borrowing.

<sup>23</sup>The externality costs of large resource transfers for an individual depend on the dominant social attitude towards transfers at the particular time. Thus, there might be zero or even negative perceived costs of transferring resources during the upswing in asset cycles. There is also evidence of different attitudes by the same individuals at different stages of their life cycles, see Agarwal et al. (2007). For related ideas, see Minsky (1982); Kiyotaki and Moore (1997); Baker and Wurgler (2007); and Bansal and Shaliastovich (2010). For research on the leverage cycle, see Fostel and Geanakoplos (2008).

<sup>24</sup>We focus on the likelihood of extremes. For work on the structure of specific extreme events, see Abreu and Brunnermeier (2003); Hong et al. (2008); and Reinhart and Rogoff (2009a). For work discussing rational individuals' perception of extreme risk, see Weitzman (2007).

ipate in the financial market, which has elements of the commons, but also another aspect, individual contribution: participating in the market generates more thickness and a better chance of matching and diversification.<sup>25</sup> Therefore an agent who participates in resource transfers faces 3 effects. She enhances her own utility; she contributes to decreased performance of the system whenever she excessively coordinates with others' trading strategies (negative congestion externality); and she helps improve functioning of the system by enlarging trade possibilities for others (positive network externality).

Hence at the aggregate level resource transfers result in a tension between enhancing thick markets versus avoiding congestion. We therefore build a simple model of financial congestion, where some use of financial intermediaries yields positive network effects but excess use results in reduction or removal of ability to trade. To formalize our approach we build on the public economics literature on congestion (Baumol and Oates (1988); Cornes and Sandler (1996); Myles (1995)) and use two distribution functions: the *congestion function*  $C(\cdot)$ , which measures the distribution of congested trades in financial markets, and the *network function*  $N(\cdot)$ , which summarizes enhanced trading opportunities due to thicker markets.

**The Environment.** The **economy**  $E$  is defined as a collection of goods  $(q, y)$ , prices  $\pi$  and preferences  $U$ , that is,  $E = (q, y, \pi; U(\cdot))$ . Specifically,  $U(\cdot)$  is a utility function,  $y$  is a consumption good,  $q$  represents resource transfers; and  $\pi$  represents average prices. The environment comprises  $I \geq 2$  identical individuals of whom we consider one representative individual. Each agent  $i$  buys a consumption good  $y^i$  and also trades  $q^i$  units of her resources. The total amount of trading is  $\sum_{i=1}^I q^i \equiv Q$ , which affects the likelihood of congestion or friction in financial markets. Thus  $Q$  is a public good that negatively affects utility, since it raises the likelihood of socially harmful extreme events. Each agent has an exogenous income  $W^i$ , and takes as given the prices of  $y$  and  $q$ , namely  $\pi_y$  and  $\pi_Q$ . We normalize the price of  $y$  to  $\pi_y = 1$ .

**Assumptions and Definitions.** Agent's preferences are represented by a neoclassical utility function  $U$  as in Allen and Gale (2007).  $U(\cdot)$  is therefore quasi-concave, increasing and continuously differentiable.<sup>26</sup>  $U$  depends on goods, congestion  $C$  and network effects  $N$ , that is,  $U = U(y, q, C(\cdot), N(\cdot))$ . As in the public economics literature (Baumol and Oates (1988); Cornes and Sandler (1996)) congestion depends on aggregate usage  $Q$ , that is,  $C = C(Q)$ . Thus, the likelihood of congested trades depends on the level of resource transfers in the economy. Throughout, we restrict attention to interior optima for illustrative purposes. Derivatives are denoted with a subscript. For example,  $U_y \equiv \frac{\partial U}{\partial y}$ ,  $U_q \equiv \frac{\partial U}{\partial q}$ , and  $U_c \equiv \frac{\partial U}{\partial c}$ . Similarly,  $C_Q \equiv \frac{\partial C}{\partial Q}$ , and  $N_Q \equiv \frac{\partial N}{\partial Q}$ .

<sup>25</sup>For work on thin markets, see Rostek and Weretka (2010) and Rostek and Weretka (2011).

<sup>26</sup>For definitions and implications of quasi-concavity, see Mas-Colell et al. (1995).

*Assumption 1.* The derivatives of the congestion and network functions satisfy  $N_Q \geq 0$  and  $C_Q \geq 0$ . Thus, an increase in total resource transfers always (weakly) increases both the network effect and the likelihood of congestion.

*Assumption 2.* The derivatives of the utility function satisfy  $U_N \geq 0$  and  $U_c \leq 0$ . Thus, an increase in market thickness weakly increases utility, while an increase in congestion weakly decreases utility.

*Assumption 3.* For congestion to occur, at least two agents must use the markets. Thus, in a two-person economy,  $C(0, \cdot) = C(\cdot, 0) = 0$ . This rules out cases where an autocrat can shut down markets unilaterally.

**Definition 1:** An **extreme event** is a large drop in average prices  $\pi$ .

**Definition 2:** An **exogenous** extreme event occurs when a large fall in asset prices is unrelated to any economic variable  $v \in E$ .

**Definition 3:** An **endogenous** extreme event occurs when a large fall in asset prices is related to some economic variable  $v \in E$ .

Since endogenous extremes arise because of congestion in our model, the likelihood of an endogenous extreme event is given by the derivative of the distribution function, that is, by  $C_Q$ . Below, we consider 2 cases in turn: markets exhibit only congestion effects, and markets have both congestion and positive network effects.

### 2.1.1 Financial Markets Exhibit Only Congestion Effects

In this setting, there is no aggregate benefit from market participation, there is only individual benefit to the agent. Thus, the market is just a facility like a recreation area that yields pleasure to each individual that uses it, but accords no systematic benefit for society as a whole. Moreover, there are negative external effects—sometimes the market gets congested, which causes extreme events that negatively affect all society. The representative individual has a quasi-concave utility function

$$U^i(\cdot) = U^i(y^i, q^i, C(Q)), \quad (1)$$

where  $C(Q)$  is a congestion function with the properties  $C_Q \geq 0$ , and  $U_c \leq 0$ . That is, the likelihood of congestion increases as resource transfers increase; and the agent's utility falls if congestion becomes more prevalent.<sup>27</sup>

*Private Optimum:* The representative individual chooses her level of resource transfers, in a Nash-Cournot setting, where she takes  $\hat{Q} = \sum_{j \neq i} q^j$  as exogenously given. Her problem is

$$\max_{(y^i, q^i)} U(y^i, q^i, C(\hat{Q} + q^i))$$

subject to

$$y^i + \pi_Q q^i = W^i.$$

The first order necessary conditions for interior optima imply  $\frac{U_q^i + U_c^i C_Q}{U_y^i} = \pi_Q$ , which can be expressed in terms of the likelihood of congestion:

$$C_Q = \frac{\pi_Q - \frac{U_q^i}{U_y^i}}{\frac{U_c^i}{U_y^i}}. \quad (2)$$

According to (2), the likelihood of congestion is proportional to the difference in the marginal cost and benefit of an additional resource transfer. We now turn to the social optimum.

*Social Optimum:* If we consider a simple equal weighting of utilities, the corresponding social problem is

$$\max_{(y^i, q^i)} \sum_i U^i(y^i, q^i, C(Q))$$

subject to

$$\sum_i y^i + \pi_Q \sum_i q^i = \sum_i W^i,$$

The first-order necessary conditions are  $U_y^i - \lambda = 0$  and  $U_q^i + \left(\sum_j U_c^j\right) C_Q - \lambda \pi_Q = 0$ . These combine to yield  $\frac{U_q^i}{U_y^i} = \pi_Q - \sum_j \left(\frac{U_c^j}{U_y^i}\right) \tilde{C}_Q$ , or

$$\frac{U_q^i + U_c^i \tilde{C}_Q}{U_y^i} = \pi_Q - \sum_{j \neq i} \left(\frac{U_c^j}{U_y^i}\right) \tilde{C}_Q.$$

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<sup>27</sup>This formulation of congestion is standard in public economics, see Baumol and Oates (1988); and Cornes and Sandler (1996).  $q$  can be, for instance, the amount of public good related to trading in credit transfers by financial market participants.



The above expression is intuitive: at the optimum agent  $i$ 's marginal valuation of additional resource transfers  $q$  equals the marginal private cost  $\pi_Q$  plus the social cost associated with increasing extreme congestion in the financial system. To focus on extremes, we rewrite the above expression as  $\sum_j \left( \frac{U_C^j}{U_y^i} \right) \tilde{C}_Q = \pi_Q - \frac{U_q^i}{U_y^i}$ , which implies

$$\tilde{C}_Q = \frac{\pi_Q - \frac{U_q^i}{U_y^i}}{\sum_j \left( \frac{U_C^j}{U_y^i} \right)}. \quad (3)$$

Equation (3) says that at the social optimum, the marginal likelihood of congestion  $\tilde{C}_Q$  equals the marginal private cost minus the marginal benefit agent  $i$  places on additional (congestion-causing) resource transfers.

When is the likelihood of congestion larger? By comparing (2) and (3), we see that the private solution  $C_Q$  is larger. This is true because the denominator does not account for all economy-wide marginal rates of transformation. Intuitively, when she does not have to account for her extra congestion effects, each agent will over-use the financial system.<sup>28</sup> Thus for a given level of resources, the likelihood of congestion is larger when agents act competitively and do not consider congestion externalities. This result provides an asset market counterpart to the banking result of Allen and Gale (1998) (Corollary 5.1). We summarize it in the following Proposition.

**Proposition 1.** *In an economy with congestion effects due to resource transfers, the likelihood of congestion in a competitive market is larger than in the social optimum.*

**Proof:** By inspection of (2) and (3). ■

### 2.1.2 Financial Markets Exhibit Congestion and Network Effects

We now consider a more realistic environment, which is the focus of our analysis in the paper. The environment is one in which individual and social welfare are affected by both congestion and positive externalities. The positive externalities reflect ease of finding counterparties and diversification when markets are thicker, and also reflect the overall quality of the financial network  $N$  available (see Shapiro and Varian (1998); Easley and Kleinberg (2010)). The overall quality of markets depends on total trading activity  $Q$ , which is contributed to by each new participant that comes to the marketplace to conduct resource transfers. In order to capture this network effect we include a *network function*  $N(Q)$ . The representative utility function may be written as  $U^i(y^i, q^i, C(Q), N(Q))$ ,

<sup>28</sup>Overuse involves excess trading, or any economic activity that generates excessively high informational demands on the system.

where the derivatives satisfy  $N_Q \geq 0$ , and  $U_N \geq 0$ . Thus, an increase in resource transfers increases network quality, and utility increases with network quality. We can now expand the above discussion of negative externalities to consider optimal social benefits that come from thicker markets, enhanced liquidity and trading opportunities, due to investors' participation in the financial system.

*Private Optimum:* The representative individual takes  $\hat{Q} = \sum_{j \neq i} q^j$  as exogenously given, so her problem is

$$\max_{(y^i, q^i)} U(y^i, q^i, C(\hat{Q} + q^i), N(\hat{Q} + q^i))$$

subject to

$$y^i + \pi_Q q^i = W^i.$$

The first order conditions are  $U_y^i - \lambda = 0$  and  $U_q^i + U_C^i C_Q + U_N^i N_Q - \lambda \pi_Q = 0$ , which combine to give

$$\pi_Q^p = \frac{U_q^i + U_N^i N_Q}{U_y^i} + \frac{U_C^i}{U_y^i} C_Q^p, \quad (4)$$

or

$$C_Q^p = \frac{\pi_Q^p - \frac{U_q^i + U_N^i N_Q}{U_y^i}}{\frac{U_C^i}{U_y^i}}, \quad (5)$$

where the superscript  $p$  denotes a private solution.  $C_Q^p$  measures the likelihood of congestion-caused extreme events in a competitive economy, and is an important quantity which we shall use in Section 3 below.

*Social Optimum:* The corresponding social optimum solves the following program:

$$\max_{(y^i, q^i)} \sum_i U^i(y^i, q^i, C(Q), N(Q))$$

subject to

$$\sum_i y^i + \pi_Q \sum_i q^i = \sum_i W^i.$$

The necessary first order conditions are  $U_y^i - \lambda = 0$  and  $U_q^i + \sum_j U_C^j C_Q + \sum_j U_N^j N_Q - \lambda \pi_Q = 0$ , which together yield

$$\pi_Q^s = \frac{U_q^i + \sum_j U_N^j N_Q}{U_y^i} + \sum_j \left( \frac{U_C^j}{U_y^i} \right) C_Q^s, \quad (6)$$

or

$$C_Q^s = \frac{\pi_Q^s - \frac{U_q^i + \sum_j U_N^j N_Q}{U_y^i}}{\sum_j \left( \frac{U_C^j}{U_y^i} \right)}, \quad (7)$$

where the superscript  $s$  denotes the social optimum.  $C_Q^s$  is the likelihood of congestion-caused extreme events in an economy where social costs are considered. Importantly for both the social and competitive optimum, the likelihood of extremes is negatively related to network effects, because both congestion and networks depend (in opposite ways) on aggregate resource transfers  $Q$ . We summarize this in the following Proposition.

**Proposition 2.** *Consider an economy with external congestion and network effects due to resource transfers. At an interior optimum, the (private and social) likelihood of extreme events increases as the marginal network effect falls. That is,  $C_Q$  is inversely and monotonically related to the network effect  $N_Q$ .*

**Proof:** By inspection of (5) and (7). ■

While the result in Proposition 2 follows directly from the calculus of networks and congestion, it is instructive because it explicitly relates the countervailing forces that have to be reckoned with when designing public policies aimed at increasing market thickness and liquidity, in order to reduce exposure to extremes. Focusing on the competitive equilibrium (5), we see that the relation between congestion  $C_Q^p$  and networks  $N_Q$  is nonlinear in general. Consequently, the probability of extremes depends on the level of financial market participation in a *monotone*, but potentially nonlinear way.

When do private markets yield inefficiently high levels of endogenous extreme events? Comparison of conditions (5) and (7) reveals that the private likelihood of congestion  $C_Q^p$  exceeds the social optimum  $C_Q^s$  if the following inequality holds:

$$\pi_Q^p \frac{U_y^i}{U_c^i} - \frac{U_q^i + U_N^i N_Q}{U_c^i} > \pi_Q^s \sum_j \left( \frac{U_y^i}{U_c^j} \right) - \frac{U_q^i + \sum_j U_N^j N_Q}{\sum_j U_c^j}.$$

Given the assumption of  $I$  identical individuals, superscripts can be removed to yield  $U^i = U^j = U$ , and  $\sum_j U_c^j = IU_C$ . Therefore the above inequality can be rewritten as

$$\frac{\pi_Q^p U_y - U_q - U_N N_Q}{U_C} > \frac{\pi_Q^s U_y - U_q - IU_N N_Q}{IU_C},$$

or

$$\frac{I\pi_Q^p U_y - IU_q - IU_N N_Q - \pi_Q^s U_y + U_q + IU_N N_Q}{IU_C} > 0,$$

which simplifies to

$$\frac{I\pi_Q^p - \pi_Q^s}{I - 1} > \frac{U_q}{U_y}.$$

For a large economy, this converges to the following inequality:

$$\pi_Q^p > \frac{U_q}{U_y}. \quad (8)$$

This expression depends on the relative size of network and congestion effects. To see this, return to equation (4), which in the identical individual case is

$$\pi_Q^p = \frac{U_q}{U_y} + \frac{U_N N_Q + U_C C_Q}{U_y}. \quad (9)$$

The second term on the right of (9) determines whether the private or social optimum dominates. Since  $U_N \geq 0$  and  $U_C \leq 0$  by Assumption 2, this term can be either positive or negative, depending on the relative size of congestion and network effects. If network effects  $U_N N_Q$  are relatively large, the likelihood of congestion-caused extreme events is higher in the private equilibrium. Intuitively, when benefits from network externalities are large, it may be optimal for individuals to put up with a higher likelihood of extremes. This leads us to the following Proposition.

**Proposition 3.** *Consider a large economy  $E$  with both external congestion and network effects from resource transfers. In this economy, the equilibrium likelihood of extreme events  $C_Q^p$  is smaller than that of the social optimum  $C_Q^s$  if and only if  $U_N N_Q < |U_C C_Q|$ , that is, if the congestion effects dominate marginal network benefits.*

**Proof.** We have to show that  $C_Q^p < C_Q^s$  if and only if  $U_N N_Q < |U_C C_Q|$ . By comparing equilibrium and socially optimum congestion as above, we obtain the necessary and sufficient condition for  $C_Q^p > C_Q^s$  as (8). Reversing the sign, we have that  $C_Q^p < C_Q^s$  if  $\pi_Q^p < \frac{U_q}{U_y}$ . By inspection of (9), this latter inequality obtains under the maintained assumptions if  $U_N N_Q < |U_C C_Q|$ , as was to be shown. ■

Importantly, the likelihood of extreme events is dynamic to the extent that network effects  $U_N N_Q$  change over time. This raises the question of when network effects are likely to be largest. Intuitively, the marginal benefit from participation is larger when only few agents participate in markets. Perhaps more subtly, network effects can also arise in a market with high demand for borrowing, when financial innovation or education attracts additional individuals with excess savings.

Can public policy err in that targeting extreme events? The following Corollary demonstrates how a policy response in the face of endogenous extreme events can have unintended, adverse consequences.

**Corollary 1:** *Consider a large economy  $E$  with both congestion and network effects from resource transfers. In this economy, if the congestion effect  $U_C C_Q$  dominates the network effect  $U_N N_Q$ , a tax on financial congestion will raise the likelihood of extreme events and tail risk.*

**Proof.** Suppose that  $|U_C C_Q| > U_N N_Q$ , and consider that a policymaker imposes the optimal tax  $t$ . We have to show that imposing  $t$  will raise the competitive likelihood of extremes  $C_Q^p$  to a new level  $C_Q^{New} > C_Q^p$ . By definition, the optimal tax  $t$  is such that when agents pay the price  $\hat{\pi} = \pi_Q^p + t$ , their maximizing behavior leads to the socially optimum congestion level.<sup>29</sup> Thus, the new likelihood of extremes is  $C_Q^{New} = C_Q^s$ . But by Proposition 3 above, if  $U_N N_Q > U_C C_Q$ , then  $C_Q^s > C_Q^p$ . Therefore  $C_Q^{New} > C_Q^p$ , as was to be shown. ■

Corollary 1 tells us that if the goal of public policy is to reduce endogenous extreme events, it must account for network effects or run the risk of precipitating further extreme events.<sup>30</sup> It can be seen as an extension of the results of Allen and Gale (1998) (Theorem 5 and Corollary 5.1) to asset markets that exhibit both negative congestion effects and positive network externalities.

### 3 Endogenous Extremes and Resource Transfers

In the previous section we constructed a basic model of endogenous extremes due to congestion. We now develop the model to explore the timing and duration of extremes, in light of questions raised by Reinhart and Rogoff (2009a). We consider an economy populated by 2 types of representative agents with differentiated resource or wealth endowments. These agents use the financial system to transfer resources between themselves for 1 period. Suppose that types 1 and 2 transfer resources to each other in the amounts  $q_1$  and  $q_2$ , recognizing that these transfers might raise the likelihood  $C_Q^p$  of extreme events. A key idea is that agents know the congestion function, of the form derived in Section 2. They are not subject to irrational behavior or asymmetric information about the likelihood of extremes. Below, we show that, under fairly moderate assumptions, the likelihood of extremes in this economy may overshoot or undershoot the socially efficient level.

As modelled in Section 2 above, the more agents engage in resource transfers such as excessive borrowing or investing in risky securities, the more likely it is that markets experience congestion and asset prices reach extreme levels, affecting the entire system. Thus, let  $C(q_1, q_2)$  be the distribution function of such extreme events, with the likelihood of extremes given by  $C_Q^i \equiv \frac{\partial C(q_1, q_2)}{\partial q_i} \geq 0$ ,  $i = 1, 2$ . Let  $k_1$  and  $k_2$  be the costs of extreme events, net of interest, for Agents 1 and 2, respectively.<sup>31</sup> With probability  $1 - C_Q(\cdot)$  there is no extreme event and each agent receives 0 net.

<sup>29</sup>For a derivation of the optimal tax in a congestion setting, see Cornes and Sandler (1996), pp. 275-276.

<sup>30</sup> Similar results are found in public economics, where the public good is *over*-supplied under private provision. See Buchanan and Kafoglis (1963), and Diamond and Mirrlees (1973).

<sup>31</sup>Financial costs of extremes include risk of default and an impaired financial system. Social and psychological costs include increased Knightian uncertainty in an unstable economy.

For concreteness, the two main agents each conduct only one type of resource transfer—only selling and buying. We call these agents sellers and buyers, respectively. The transfer of resources may affect other agents in the economy, including other buyers, sellers, banks and investors, domestically and internationally. We denote these other agents by  $O$ , for other. In the following analysis we use subscripts 0, 1 and 2 to index variables pertaining to other, sellers and buyers, respectively.

We now develop this setting to incorporate time. Sellers and buyers are both in the market for transferring resources. Effective supply of resources by sellers is  $q_1$  and demand for resources by buyers is  $q_2$ . The framework is a two-period economy, where the first period is  $t$  and the second period is  $t + 1$ , in order to distinguish subscripts that refer to time from those that refer to agents. In the first period sellers and buyers interact and transfer resources. In the second period, sellers are repaid with interest  $q_{1,t} \cdot (1 + i)$ , and buyers repay the resources,  $q_{2,t} \cdot (1 + i)$ , where  $i$  is the prevailing interest rate. For simplicity, assume that agents receive all their wealth and make all their repayments in the second period.<sup>32</sup> Thus, the seller's and buyer's wealth levels in the first period completely derive from resource transfers:  $w_{1,t} = -q_{1,t}$ , and  $w_{2,t} = q_{2,t}$ , respectively. In the second period  $t + 1$ , the buyer and seller receive exogenous wealth endowments  $\bar{w}_1$  and  $\bar{w}_2$ , respectively.

We focus on representative sellers and buyers with neoclassical utility functions  $u_1$  and  $u_2$ , respectively, which depend on wealth:  $u_i = u_i(w_i)$ , where  $u'_i(w_i) > 0$ ,  $i = 1, 2$ . To control for contemporaneous costs, we consider utility to be net of current costs. Each agent knows there is a possibility of systemwide extreme events, captured by the probability  $C_Q$ , whose functional form is common knowledge. There is no asymmetric information regarding the likelihood of extremes.<sup>33</sup> As described in Section 2, the probability of future extreme events increases with the average level of current resource transfers,  $C_{t+1}(\cdot) = C_{t+1}(q_{1,t}, q_{2,t})$ , where  $C_{Q,t+1}^i \equiv \frac{\partial C_{t+1}(\cdot)}{\partial q_{i,t}} \geq 0$ ,  $i = 1, 2$ .<sup>34</sup> If an extreme event occurs in the future, agent  $i$  incurs a positive cost  $k_{i,t+1}$ ,  $i = 0, 1, 2$ . Our results depend on properties of  $k$ , so we analyze two cases, finite  $k$  and potentially infinite  $k$ .

<sup>32</sup>This timing allows us to model the use of financial markets to transfer wealth over time.

<sup>33</sup>Similar assumptions occur in many other economic contexts, such as price taking, competitive agents in Arrow and Debreu (1954) and Debreu (1959), even though the demand of each agent will affect price to some extent. Such myopic behavior can be found in other rational settings: investors with log utility decide their portfolios without reference to future investment opportunities, see Ingersoll (1987), chapter 11.

<sup>34</sup>This summarizes the intuition that excessive resource transfers are destabilizing, without emphasizing the particular channel of destabilization. Channels through which resource transfers lead to increased likelihood of extremes are explored by a number of authors, including Fisher (1933), Allen and Gale (2000), and Krishnamurthy (2010).

### 3.1 Large but Finite Costs $k$ of Extreme Events

Suppose  $0 < k_{1,t+1} < \infty$  and consider the seller's problem. Given an interest rate  $i$ , at period  $t$  the seller decides how much resources to transfer this period by maximizing utility subject to the following wealth constraint, which accounts for the possibility of costly extreme events:

$$w_{1,t+1} \geq \bar{w}_1 + C_{t+1}(q_{1,t}, q_{2,t})[q_{1,t} \cdot (1+i) - k_{1,t+1}] + [1 - C_{t+1}(q_{1,t}, q_{2,t})][q_{1,t} \cdot (1+i)].$$

Given locally nonsatiated preferences, this constraint holds as an equality, which simplifies to  $w_{1,t+1} = \bar{w}_1 + q_{1,t} \cdot (1+i) - C_{t+1}(q_{1,t}, q_{2,t}) \cdot k_{1,t+1}$ . Using  $\beta$  to denote the discount factor, the seller's problem is:

$$\begin{aligned} \max_{q_1} u_1(w_{1,t}) + \beta u_1(w_{1,t+1}), \text{ s.t.} \\ w_{1,t} &= -q_{1,t} \\ w_{1,t+1} &= \bar{w}_1 + q_{1,t} \cdot (1+i) - C_{t+1}(q_{1,t}, q_{2,t}) \cdot k_{1,t+1}. \end{aligned}$$

After substituting the constraints into the utility arguments, first order conditions for an interior solution are  $-u'_1(w_{1,t}) + \beta u'_1(w_{1,t+1})[(1+i) - \frac{\partial C_{t+1}(q_{1,t}, q_{2,t})}{\partial q_{1,t}} \cdot k_{1,t+1}] = 0$ , which can be rewritten as

$$C_{Q,t+1}^1 \equiv \frac{\partial C_{t+1}(q_{1,t}, q_{2,t})}{\partial q_{1,t}} = -\frac{u'_1(w_{1,t})}{\beta u'_1(w_{1,t+1}) \cdot k_{1,t+1}} + \frac{1+i}{k_{1,t+1}}. \quad (10)$$

Equation (10) says that optimally the probability of extremes is related to the marginal rate of substitution for transferring resources between periods  $t$  and  $t+1$ , discounted by expected costs. Since the first term of the right hand side of (10) depends on  $q_{1,t}$  via the budget constraint, it follows that extreme probabilities respond to variables affecting the level of resource transfers.

Similarly, the buyer's problem is

$$\begin{aligned} \max_{q_2} u_2(w_{2,t}) + \beta u_2(w_{2,t+1}), \text{ s.t.} \\ w_{2,t} &= q_{2,t} \\ w_{2,t+1} &= \bar{w}_2 - q_{2,t} \cdot (1+i) - C_{t+1}(q_{1,t}, q_{2,t}) \cdot k_{2,t+1}, \end{aligned}$$

which yields first order conditions that can be rewritten as

$$C_{Q,t+1}^2 \equiv \frac{\partial C_{t+1}(q_{1,t}, q_{2,t})}{\partial q_{2,t}} = \frac{u'_2(w_{2,t})}{\beta u'_2(w_{2,t+1}) \cdot k_{2,t+1}} - \frac{1+i}{k_{2,t+1}}. \quad (11)$$

As in equation (10), the above expression implies that the future probability of extremes is potentially dynamic, and depends on the current level of resource transfers.

**Equilibrium:** In equilibrium, the effective demand and effective supply of resource transfers will be equal,  $q_1 = q_2 \equiv q$ . For illustrative purposes, consider a symmetric equilibrium where buyers and sellers have identical utility functions and costs,  $u_1 = u_2 = u$ , and  $k_1 = k_2 = k$ . Now equate optimality conditions for the seller and buyer in (10) and (11):  $-\frac{u'(w_{1,t})}{\beta u'(w_{1,t+1}) \cdot k_{t+1}} + \frac{1+i}{k_{t+1}} = \frac{u'(w_{2,t})}{\beta u'(w_{2,t+1}) \cdot k_{t+1}} - \frac{1+i}{k_{t+1}}$ . This implies

$$1 + i = \frac{1}{2\beta} \left[ \frac{u'(w_{1,t})}{u'(w_{1,t+1})} + \frac{u'(w_{2,t})}{u'(w_{2,t+1})} \right].$$

Substituting this expression in equation (11) and simplifying, we obtain that in equilibrium, extreme probabilities  $C_{Q,t+1}$  satisfy

$$C_{Q,t+1} \equiv \frac{\partial C_{t+1}}{\partial q_t} = \frac{1}{2\beta k_{t+1}} \left[ \frac{u'(w_{2,t})}{u'(w_{2,t+1})} - \frac{u'(w_{1,t})}{u'(w_{1,t+1})} \right] \quad (12)$$

Equation (12) constitutes the *signature* of endogenous extremes. Somewhat surprisingly, the responsiveness of extreme probability to resource transfers is proportional to the differential in marginal rates of substitution for agents in the corresponding market. When there is a big difference in marginal rates of substitutions between borrowers and lenders, the susceptibility to extreme events is higher. As before, the extreme probability is dynamic: it depends directly on the expected costs of extremes, and indirectly (with indeterminate sign) on the equilibrium level of resource transfers via the budget constraint. If extremes were truly exogenous, there would be no statistical relation between extreme probability and  $q$ , and  $\frac{\partial C_{Q,t+1}}{\partial q_t} = 0$ . Thus, the distance of the right side of (12) from zero gives a sense of the error from assuming extremes are exogenous, when they are in reality endogenous.

Importantly, there are 2 main countervailing effects. First is the  $k_{t+1}$  cost term in the denominator: the larger it is, the less likelihood of congestion since it negatively affects utility. Second is the difference in marginal rates of substitution  $\frac{u'(w_{2,t})}{u'(w_{2,t+1})} - \frac{u'(w_{1,t})}{u'(w_{1,t+1})}$ : the larger this is, the more attractive for individuals to transfer. In addition, there is a latent effect of resource transfers  $q_t$ , which operate through the budget constraint.

We summarize the results from (12) in the following Proposition.

**Proposition 4.** *In an economy with symmetric preferences and nonzero social costs of extremes, the likelihood of extremes  $C_Q$  is potentially dynamic.  $C_Q$  depends indeterminately on equilibrium resource transfers; decreases with expected costs of extreme events  $k_{t+1}$ ; and increases with the divergence between agents' marginal rates of substitution.*

**Proof:** By examination of (12). ■



An immediate result of the above proposition is that the likelihood of extremes becomes arbitrarily low when marginal rates of substitution are equated. This gives us information about timing and expected duration of extreme events, as summarized in the following Corollary.

**Corollary 2.** In an economy with symmetric preferences and nonzero social costs of extremes, extreme episodes will persist until the marginal rates of substitution are equated, or if expected costs of extremes rise high enough .

**Proof:** By examination of (12). ■

**Social Optimum.** We now consider optimality issues. To see that the likelihood of extremes is inefficient in competitive equilibrium, suppose the seller considers the effect of her selling on other agents  $O$ , and therefore internalizes costs  $k_{0,t+1}$ . Her problem is similar to that preceding equation (10), except that the second budget constraint becomes

$$w_{1,t+1} = \bar{w}_1 + q_{1,t} \cdot (1 + i) - C_{t+1}(q_{1,t}, q_{2,t}) \cdot (k_{0,t+1} + k_{1,t+1}).$$

Solving the first order conditions and rewriting as before, we obtain the counterpart of equation (10) for a socially optimal level of extremes:

$$\frac{\partial C_{t+1}(q_{1,t}, q_{2,t})}{\partial q_{1,t}} = -\frac{u'_1(w_{1,t})}{\beta u'_1(w_{1,t+1}) \cdot (k_{0,t+1} + k_{1,t+1})} + \frac{1 + i}{k_{0,t+1} + k_{1,t+1}}. \quad (13)$$

The quantities in equations (10) and (13) will differ in general. Thus, when the resource seller takes into account the future costs of other agents, optimizing behavior delivers a different likelihood of extremes. It is in this sense that competitive markets may lead to endogenous, inefficient probability of crashes.<sup>35</sup> We are not just saying there is a link between excessive resource transfers and extremes. Instead, we are showing that even *without* asymmetric information or irrationality, extreme events may arise as an equilibrium phenomenon, to the extent that  $\frac{\partial C_{t+1}(q_{1,t}, q_{2,t})}{\partial q_{1,t}} \neq 0$ . This phenomenon occurs due to the failure of *both* resource sellers and buyers to internalize an important externality, the expected costs from congestion, which affect the probability of systemwide, future financial extremes.

Importantly, as in Section 2, we cannot determine a priori whether the likelihood of extremes is larger in the competitive equilibrium or the social optimum. We summarize our results from equations (10), and (13) in the following Proposition:

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<sup>35</sup>Optimality will not necessarily entail complete elimination of extreme events. Rather, the extreme probability is adjusted to the point where marginal benefit to sellers of an additional unit of the externality-generating activity,  $u'_1(q)$ , equals marginal cost to other agents,  $-u'_0(q)$ .

**Proposition 5.** *In an economy with symmetric preferences and nonzero social costs of extremes, the equilibrium level of extreme probability is in general not socially optimal. Moreover, the likelihood of extremes may be smaller or larger in competitive markets.*

**Proof:** By comparison of (10) and (13). ■

### 3.2 Potentially Infinite Costs $k$ of Extreme Events

Recent financial events have potentially catastrophic consequences that are difficult to quantify ex ante. These events include the costs of the Japan earthquake and nuclear meltdown in January 2011, the US debt default scare in early August 2011, the Greece debt crises of April 2010 and June 2011, and the financial crisis inception in September 2008. In order to capture such events, we consider the possibility of infinite expected costs of extremes for each type of agent. Specifically, suppose  $k_{1,t+1} = \infty$  and consider the seller's problem as in section 3.1 above. Once again, her maximization program is

$$\begin{aligned} \max_{q_1} \quad & u_1(w_{1,t}) + \beta u_1(w_{1,t+1}), \text{ s.t.} \\ & w_{1,t} = -q_{1,t} \\ & w_{1,t+1} = \bar{w}_1 + q_{1,t} \cdot (1 + i) - C_{t+1}(q_{1,t}, q_{2,t}) \cdot k_{1,t+1}. \end{aligned}$$

However, now the agent's program involves optimizing over a quantity of infinite magnitude, which often leads to corner solutions. To see this, substitute the constraints into the maximand, which becomes

$$u_1(-q_{1,t}) + \beta u_1(\bar{w}_1 + q_{1,t} \cdot (1 + i) - C_{t+1}(q_{1,t}, q_{2,t}) \cdot k_{1,t+1}) \quad (14)$$

Since the second term involves minus infinity, the only solution for a risk-averse individual is to set  $q_{1,t} = 0$ , so that  $C_{t+1}(\cdot)$  is as close to zero as possible. A similar logic applies to the seller. We remove the subscript on  $k$  for simplicity and summarize this result in the following Proposition.

**Proposition 6.** *Consider an economy where agents transfer resources, and in which there is common knowledge about the likelihood of extreme events, captured by the function  $C(Q)$ . If the expected cost of extreme events  $k$  is infinite, then risk-averse agents will not transfer resources.*

**Proof.** The agent's problem involves choosing transfers  $q_1$  to maximize (14). If  $k = \infty$  then the second term becomes  $-\infty$ . This implies unbounded negative utility for a risk-averter, unless  $C_{t+1}(\cdot) = 0$ . The agent will therefore set  $q_1 = 0$ , which according to Assumption 3 yields  $C_{t+1}(\cdot) = 0$ .  $q_1 = 0$  corresponds to non-transferral of resources. ■

According to Proposition 6, if agents face extreme events with arbitrarily large consequences, the only way they will trade is if they are risk neutral or risk-loving.<sup>36</sup>

### 3.3 Summary and Implications

The preceding results have implications for regulatory policy and risk management. Proposition 4 cautions risk managers against the assumption that exposure to extreme events is constant over time. Further, Proposition 4 suggests possible warning signals for regulators and risk managers—low expected costs and a large gap between agents’ desires to transfer resources over time.<sup>37</sup> Proposition 5 and Corollary 1 suggest, in principle, a role for regulators and central banks to intervene and prevent excessive financial extremes. However, these latter results also caution regulators that their intervention might inadvertently increase the likelihood of extremes, in a manner that has not previously been examined.<sup>38</sup>

More tentatively, the results may also have relevance for current financial issues in the US and Euro zone. Proposition 4 suggests that the increased likelihood of extremes in today’s markets have been precipitated by low expected costs previously (e.g. on the part of Greek debtors) and by a large gap between the marginal rates of substitution for borrowers and lenders of capital. After expected costs rise and marginal rates of substitution converge, then the elevated extremes will begin to play out.

## 4 Conclusions

Our paper develops a simple approach to endogenous extreme events. We suggest that the probability of extremes may vary systematically over time, and be explained on the basis of financial congestion and network effects. Moreover, we distinguish exogenous from endogenous extremes, the latter of which can be understood in a public good framework. This distinction has immediate policy implications: for truly exogenous extremes, we must focus on ex post protection, while for endogenous extremes, we can in principle use economic incentives to entice agents to reduce extremes themselves. We have three main contributions. First, we develop the ‘signature’ of en-

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<sup>36</sup>If markets are thin, then even risk-loving agents will be unable to transfer resources.

<sup>37</sup>More generally, Proposition 4 predicts that developments to enhance resource transfers will affect the likelihood of extremes. These developments include many financial innovations and loose interest rates.

<sup>38</sup>Theoretically, regulators could tax ‘excessive’ transfers. However, this requires extensive monitoring of investor positions. A more realistic approach involves combining regulation with enhanced education about costs of extreme events, and the role of individuals and institutions in precipitating these costs. This approach is similar to recent education about other externalities such as effects of drunk driving, cigarette smoking, and human impact on the natural environment.

ogenous extremes, and provide insight on their incidence: According to Proposition 4, economies are more susceptible to extremes if expected costs are low and there is a large discrepancy between marginal rates of substitutions for resource borrowers and lenders. Second, extreme episodes last until marginal rates of substitution converge, or expected costs rise. Third and perhaps most interesting, our approach (Corollary 1) suggests limits on the role for central bank and regulatory intervention. In tackling issues related to economic instability, regulators can ameliorate the likelihood of extremes, but only if they account for countervailing network effects.

While our paper describes a method for understanding patterns in the likelihood of extremes, it does not aim to predict all possible extreme events. The aim is to show that, far from being random, the probability of endogenous extremes may have similar patterns. We accomplished this aim using a broad congestion-network approach, which allows us to relate the likelihood of extremes to economic variables: resource transfers, expected costs and marginal rates of substitution. Our paper may be seen as a step towards incorporating dynamic, endogenous, extremes into standard economic analysis. Acknowledgement of endogenous extremes may also be helpful for risk management. Important extensions include estimating financial congestion, identifying dynamic extremes in particular markets, and exploring various channels of endogenous extremes encountered in practice. Such refinements present an exciting task for future research.

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Table 1: Different Faces of Congestion

Form of Congestion	Researchers	Main Effect
Excess borrowing	Fisher (1933); Reinhart and Rogoff (2009b)	Preponderance of debt $\Rightarrow$ economy susceptible to shocks
Excessive speculation	Keynes (1936); Minsky (1982)	Sudden shifts in confidence $\Rightarrow$ large changes in market prices
Lack of liquidity	Allen and Gale (2007); Pedersen (2009); Brunnermeier and Pedersen (2009)	Simultaneous needs for liquidity $\Rightarrow$ fragile credit markets
Over-confidence	Odean (1999); Barber and Odean (2000a)	Excessive trading $\Rightarrow$ market inefficiency

Figure 1: Congestion Factors around the Crash of October 1987

The figure shows the behavior of liquidity and sentiment during the two months before and after October 1987. Source: Author’s calculations. Spreads, turnover and VIX data are obtained from Wharton Research Data Services (WRDS). Dow-Jones data are obtained from the Federal Reserve Bank of St. Louis website.

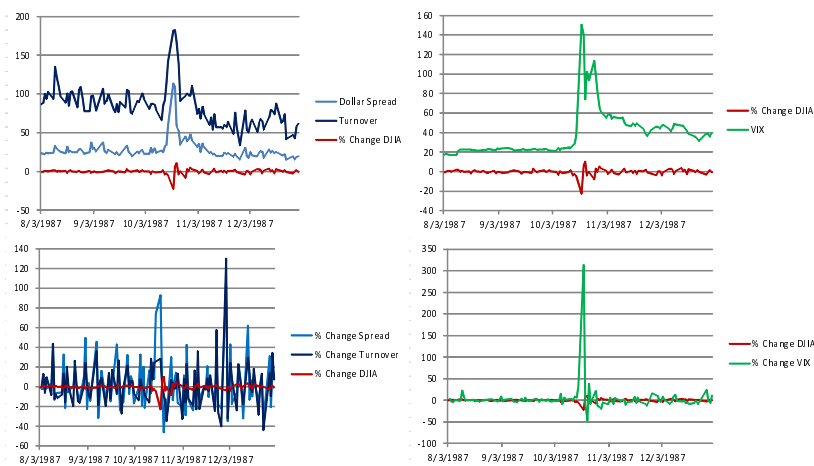


Figure 2: Congestion around the LTCM crisis of August 1998

The figure shows the behavior of liquidity and sentiment during the two months before and after August 1998. Source: Author's calculations. Spreads, turnover and VIX data are obtained from Wharton Research Data Services (WRDS). Dow-Jones data are obtained from the Federal Reserve Bank of St. Louis website.

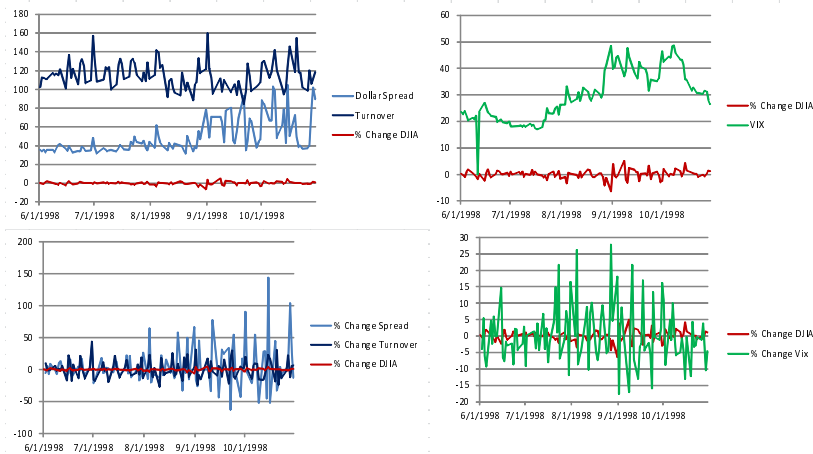


Figure 3: Congestion around the Lehman Brothers Bankruptcy of September 2008

The figure shows the behavior of liquidity and sentiment during the two months before and after September 2008. Source: Author's calculations. Spreads, turnover and VIX data are obtained from Wharton Research Data Services (WRDS). Dow-Jones data are obtained from the Federal Reserve Bank of St. Louis website.

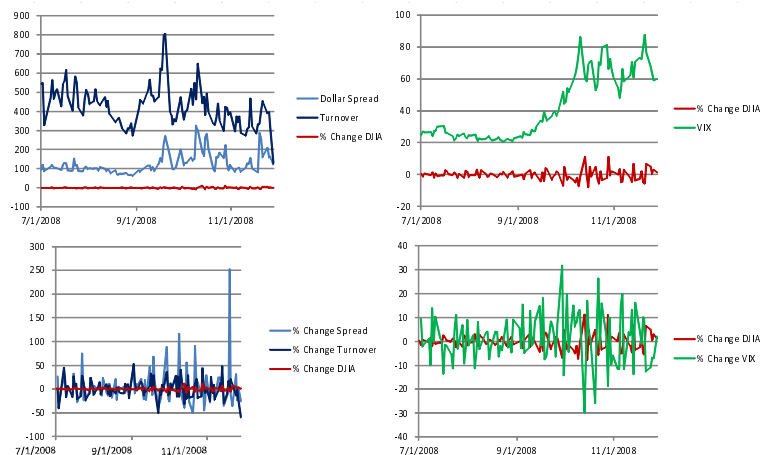


Figure 4: Congestion around the Flash Crash of May 2010

The figure shows the behavior of liquidity and sentiment during the two months before and after the Flash Crash of May 2010. Source: Author's calculations. Spreads, turnover and VIX data are obtained from Wharton Research Data Services (WRDS). Dow-Jones data are obtained from the Federal Reserve Bank of St. Louis website.

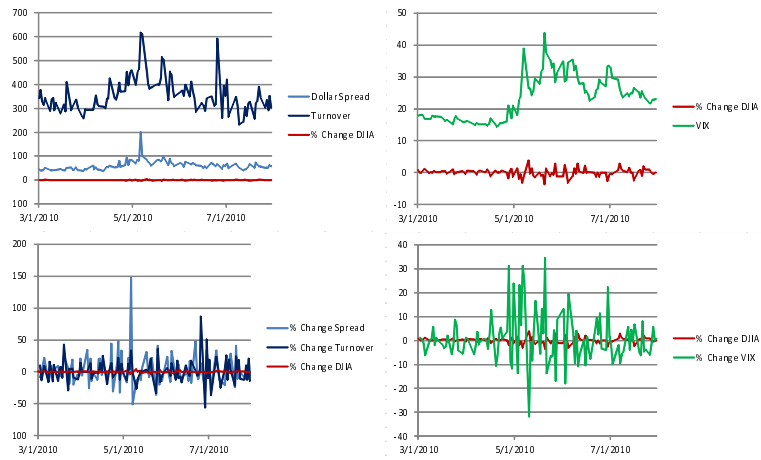


Table 2: Rank Correlations for the Dow Jones, Liquidity and VIX

The table presents the Spearman (rank) correlations of the liquidity and confidence measures with the Dow Jones Industrial Average. APLIQ denotes the market average of the Amihud (2002) liquidity measure. SPREAD and TURNOVER denote the average dollar bid-ask spread and turnover, respectively. VIX is the implied volatility of an at-the-money 30-day option, which reflects investor confidence. All variables are in *percentage changes* relative to the previous trading day. The time period for each extreme event comprises the four months surrounding the month of the event: for the October 1987 crash, the sample period is August 1, 1987 to December 31, 1987. For the August 1998 LTCM event, the sample period is June 1, 1998 to October 31, 1998. For the September 2008 episode around the Lehman Brothers Bankruptcy, the sample period is July 1, 2008 to November 30, 2008. And for the May 2010 flash crash the period is March 1, 2010 to July 31, 2010. P-values are in parentheses.

<b>Panel A: Correlations around October 1987</b>					
	DJIA	ALIQ	SPREAD	TURNOVER	VIX
ALIQ	-0.133 (0.176)	1.000			
SPREAD	-0.037 (0.704)	-0.155 (0.112)	1.000		
TURNOVER	0.143 (0.143)	-0.240 (0.013)	0.716 ( $<.0001$ )	1.000	
VIX	-0.659 ( $<.0001$ )	0.123 (0.210)	0.210 (0.031)	0.080 (0.416)	1.000

<b>Panel B: Correlations around August 1998</b>					
	DJIA	ALIQ	SPREAD	TURNOVER	VIX
ALIQ	0.108 (0.268)	1.000			
SPREAD	-0.126 (0.199)	-0.145 (0.139)	1.000		
TURNOVER	0.125 (0.203)	-0.215 (0.027)	0.197 (0.043)	1.000	
VIX	-0.837 ( $<.0001$ )	-0.135 (0.169)	0.229 (0.018)	-0.032 (0.742)	1.000

<b>Panel C: Correlations around September 2008</b>					
	DJIA	ALIQ	SPREAD	TURNOVER	VIX
ALIQ	-0.004 (0.966)	1.000			
SPREAD	-0.035 (0.720)	0.118 (0.228)	1.000		
TURNOVER	0.155 (0.113)	-0.139 (0.156)	0.525 ( $<.0001$ )	1.000	
VIX	-0.866 ( $<.0001$ )	-0.015 (0.881)	0.088 (0.369)	-0.032 (0.748)	1.000

<b>Panel D: Correlations around the May 2010 Flash Crash</b>					
	DJIA	ALIQ	SPREAD	TURNOVER	VIX
ALIQ	-0.060 (0.536)	1.000			
SPREAD	-0.152 (0.117)	0.169 (0.081)	1.000		
TURNOVER	-0.201 (0.038)	0.071 (0.467)	0.560 ( $<.0001$ )	1.000	
VIX	-0.802 ( $<.0001$ )	-0.004 (0.967)	0.210 (0.030)	0.275 (0.004)	1.000

Table 3: Snapshot of Extreme Events in Financial Markets

Event	Context	Peak	Trough	% change
<b>Panel A: US Crashes</b>				
October 28, 1929	Smoot-Hawley Tariff debate	301.22	260.64	-13.47%
October 19, 1987	Algorithmic Trading	2246.74	1738.74	-22.61%
August 31, 1998	LTCM, Russian Default	8051.68	7539.06	-6.37%
September 17, 2001	9/11	9605.51	8920.70	-7.13%
September 29, 2008	Lehman Brothers bankruptcy	11143.13	10365.45	-6.98%
October 15, 2008	Emergency Econ. Stability Act	9310.99	8577.91	-7.87%
May 6, 2010	Flash Crash	10868.12	10520.32	-3.20%
August 8, 2011	US Debt Downgrade	11444.61	10809.85	-5.55%
<b>Panel B: Hong Kong Crashes</b>				
October 19-26, 1987	US spillover	3362.4	2241.7	-33.3%
February 4 - March 3, 1994	Bubble	12157.6	9802	-19.4%
October 17-28, 1997	Bubble	13601	9059.9	-33.4%

Source for Panel A: author calculations, based on data from the Federal Reserve Bank of St. Louis.  
 Source for Panel B: Sornette (2004). US data are for the Dow-Jones Industrial Average. Hong Kong data are for the Hang Seng Index.

Table 4: Two Examples of Endogenous Probabilities

	<i>Effects felt in one market or transaction</i>	<i>Spillover effects in many markets</i>
<i>Asymmetric Information</i>	<b>Moral Hazard</b>	
<i>Symmetric or Asymmetric Information</i>		<b>Endogenous Extremes</b>

Table 5: Excessive Resource Transfers and Extreme Events

Form of Resource Transfer	Author
Over-borrowing, leverage	Fisher (1933); Minsky (1982)
Excessive credit creation	Allen and Gale (2000)
Large sales due to confidence shifts	Reinhart and Rogoff (2009b); Keynes (1936)
Increased Trading	Montier (2002); Gabaix et al. (2006)
Fire-sale asset liquidation	Allen and Gale (2007)



Table 6: Summary of Results

<b>Result</b>	<b>Explanation</b>
<i>Proposition 1</i>	If only congestion effects are present, competitive markets yield higher $C_Q(\cdot)$
<i>Proposition 2</i>	If both congestion and network effects, equilibrium congestion inversely related to network effects.
<i>Proposition 3</i>	If marginal congestion dominates network effect, then competitive $C_Q(\cdot)$ is <i>smaller</i> than social optimum
<i>Corollary 1</i>	If congestion dominates network effect, then tax policy can end up <i>raising</i> tail risk
<i>Proposition 4</i>	In an economy with trade, likelihood of extremes increases with gap in MRS between buyers and sellers, decreases with expected cost of extremes
<i>Corollary 2</i>	In an economy with trade, sufficient conditions for the end of an extreme episode are that MRS are equated, or expected costs of extremes rise high enough
<i>Proposition 5</i>	In an economy with trade, the likelihood of extremes may be larger or smaller in competitive markets
<i>Proposition 6</i>	If expected costs of extremes are infinite, there will be no trade, except for risk-loving investors