# Crime and punishment: When tougher antitrust enforcement leads to higher overcharge[*]

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February 18, 2013

## Abstract

The economics of crime and punishment postulates that higher punishment leads to lower crime levels, or less severe crime. It is however hard to get empirical support for this rather intuitive relationship. This paper offers a model that can contribute to explain why this is the case. We show that if criminals can spend resources to reduce the probability of being detected, then a higher general punishment level can increase the crime level. In the context of antitrust enforcement, the model shows that competition authorities who attempt to fight cartels by means of tougher sanctions for all offenders may actually lead cartels to increase their overcharge when leniency programs are in place.

**Keywords:** antitrust enforcement; leniency programs; economics of crime

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# 1  Introduction

Apparently, the prospects for punishment can deter crime. One particular kind of punishment is a fine. Since this is a costless transfer, it has been argued that the optimal fine is the maximal one (equal to the individual's wealth). This is simply because it will lead to the highest possible deterrence without causing any costs. This intuitive and straight forward argument was first introduced in Becker (1968). In this article we contribute to the debate concerning optimal punishment by elaborating on a particular mechanism that can explain why a tougher punishment may lead to a more severe crime. When the punishment becomes tougher, criminals are encouraged to use more resources to avoid being detected. This will reduce the probability of detection which will, in turn, affect the severeness of the crime that might be committed. We apply our model to antitrust enforcement, where penalties for cartel infringements have increased significantly during the last decade. For this particular offence, it is shown that a higher fine for price fixing can lead to a higher overcharge instead of a higher deterrence rate. Our model predicts that this unintended effect of higher fines is present only when a leniency program is in force.

Avoidance activities are common for many agents involved in criminal activities. For example, many persons install radar detectors to avoid speeding tickets. Firms involved in antitrust violations may also spend resources to avoid being detected. It is claimed that efforts to conceal illegal cartels, in particular to hide information about meetings where they fix prices, have become even more sophisticated and elaborate with the escalation of competition law enforcement in Europe and the US.[1] This suggests that the toughness of the punishment, for example the level of the fine, will influence agents' efforts to hide criminal activities. This mechanism is at the heart of

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[1]See Stephan (2009), where several examples of avoidance activities for cartel members are listed. For example, they communicate through private email accounts and unregistered mobile phones using encrypted messages; they avoid any contact through secretaries or administrative staff; they hold meetings in foreign countries. Concerning cartels in EU, it is found that the firms often have illegal meetings in Switzerland.

our analysis.

Our starting point is a standard modeling approach where crime is profitable as such and there is a probability of detection followed by a punishment, typically a fine. We model this as a repeated game, where each agent in a group must decide whether or not to take part in committing a crime in each period. In addition, if a crime is committed the agent has the option to apply for leniency, i.e., informing the police and pay a lower fine. We assume that the agents make two simultaneous decisions. Besides deciding on how severe crime to commit, they can also decide to invest in costly avoidance activity that leads to a lower probability of crime detection. In principle then, for any given fine there exists an optimal mix of the crime level and the avoidance activity. Given the endogenous avoidance activity, we consider how an increased fine will affect the crime level. An increased fine will lead to a higher avoidance activity and thereby a lower probability of detection. This may, in turn, make it profitable to commit a more severe crime.

We show that whether an increased fine leads to a more severe crime depends on whether the marginal fine or the fine level in general is increased. Not surprisingly, if the marginal punishment is increased, so that the fine increases more rapidly with the severeness of the crime, then the increased fine has the intended effect on the criminal activity. The agents avoid paying the higher fine by reducing the severeness of the crime. However, if the fine is increased uniformly for any given crime level, so that the marginal punishment is not affected, the agents may react differently. The higher fine will certainly reduce the expected gain from committing the crime, and to sustain a collusive equilibrium where no one informs the police, the agents will have to invest to reduce the detection probability. The crime level is then no longer at the optimal level, and will thus be adjusted to take into account the now lower probability of detection. In a setting where the criminals have to give up all the criminal surplus if detected, the optimal response to the lower detection probability is clearly to increase the crime level. This is so because the deviation profits, and hence the temptation to deviate, are then not affected by the criminal surplus.

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However, if the criminals can retain the illegal surplus even if the crime is detected, they also have to take into account that a higher crime level will make it more tempting to inform the police. The latter effect makes it less profitable to increase the crime level, and thus indicates that the criminals' response to a higher fine will depend on whether or not they can retain the illegal surplus after detection. The analysis confirms this intuition.

We apply our model to antitrust enforcement. It is a violation of competition law if competing firms fix prices, and the firms will typically be given fines if they are detected and convicted. The crime is the overcharge (price above the competitive price level), and we interpret a higher overcharge as a more severe crime. The main difference between the basic crime model and the cartel model, is that there are now two ways in which an agent (or firm) can deviate from a collusive equilibrium: either report to the antitrust authorities and thereby apply for leniency, or deviate by setting a lower price and thus appropriate the cartel profit. If the option to apply for leniency is the binding constraint for the cartel's choice of overcharge and avoidance activity, there is no difference between this antitrust model and the general setting outlined above. Higher fine levels for detected cartels can in such a case lead to more avoidance activity which, in turn, leads to a larger cartel overcharge. In particular, a general increase in the fine level (rather than an increase in the marginal fine) may lead to more avoidance activity and a higher overcharge.

Without a leniency program in place, however, a higher overcharge will always increase the temptation to appropriate the cartel profit by slashing the price. An increase in the fine will in this case always lead to a lower overcharge in order to discipline the incentive to deviate.

We also find that even though there is a leniency program in place, the binding constraint for the cartel can be that firms are tempted to slash prices. However, the existence of a leniency program broadens the range of collusive strategies. This occurs when the option to apply for leniency is used as a credible threat to punish a firm who deviates from the cartel agreement regarding pricing (and hence makes the agreement more sustainable). The

deviating firm will then be fined and may also have to give up the illegal gains. Hence, the presence of a leniency program may allow the cartel to sustain higher overcharges, and an increase in the fine may lead to even higher overcharges by the cartel. Thus, our model shows that introducing a leniency program may (a) help to sustain the cartel and (b) also affect – in some cases negatively – the way the cartel reacts to higher fines.

The main mechanism behind these negative effects is that leniency programs enable firms to coordinate on equilibria in which all apply for leniency once someone deviates. As a result, the overcharge level may not affect the temptation to deviate from the cartel. One way to mitigate the negative effects is to let the firms retain cartel surplus after detection, so that a higher overcharge also increases the temptation to deviate. We show that if the cartel surplus is retained, the leniency program can be designed such that higher fines reduce the overcharge. This requires a sufficiently small difference between the leniency granted when a single firm applies and the leniency granted when all firms apply.

The article is organized as follows. In the next section we relate our approach and results to the existing literature. In Section 3 we present our model and the main result concerning the relationship between punishment and crime. In Section 4 we apply our model to antitrust violations, focusing on how higher fines can affect the overcharge. Some concluding remarks are offered in Section 5, where we also contrast our results with the tougher fine policy in many jurisdictions during the last decade.

## 2    Relation to the literature

The idea that fines should be set at a maximum, *i.e.,* setting fines equal to an individual's wealth, has been criticized by many.[2] Malik (1990) was the

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[2]For example, Stigler (1970) argued that it ignores the need to maintain marginal deterrence since the fine for an offense does not increase with its seriousness. Polinsky and Shavell (1979) show that when persons are risk averse, the optimal fine is lower than the offender's wealth.

first to model an offender's avoidance activity, and he found that if such an activity is costly for society, it is an argument for setting fines lower than at an individual's wealth.[3] The reason is that a fine is no longer a costless transfer. More recently, it has been shown that with the presence of avoidance activities an increase in the punishment may have counterintuitive results. In particular, both Langlais (2008) and Nussim and Tabbach (2009) have shown that a tougher punishment may lead to more crime being committed. Although the direct effects of a tougher punishment is less crime and more avoidance activity, they show that the interplay between crime and avoidance may lead to such a counterintuitive result. Our modeling is in the same spirit as theirs, with some important distinctions. First, in contrast to them we endogenize the severeness of the crime, and show that more punishment can lead to a more severe crime being committed. Second, we study a repeated game model of organized crime, and tailor-make our model to antitrust violations, showing that higher fines for price fixing can lead to even higher overcharges.

There are studies that endogenize firms' avoidance activity concerning antitrust violations. For example, Aubert, Rey and Kovacic (2006) investigate the incentive to destroy hard evidence of price fixing. Their main issue, though, is how whistleblowing programs may affect the incentives to keep hard evidence, and they do not discuss the relationship between the fine size and the overcharge. Avramovich (2010) takes into account that avoidance activities are costly.[4] Her main concern is how each firm allocates resources between avoidance activities and traditional cost reducing activities. In contrast, our main focus is on how a tougher punishment (which in this case

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[3]Although not modelled, Friedman (1981) did argue that avoidance activities might be relevant for the question of optimum enforcement. See also Skogh (1973), who discussed how the costs of planning and carrying out offences should affect the optimal punishment. It has also been argued that victim precaution, effort by the victim that lowers the probability of being injured by an offender, will also lead to a fine that is lower then the maximum one (see Hylton, 1996).

[4]See also Jellal and Souam (2004), who also consider a setting with an endogenous detection probability. Both firms and inspectors make costly effort to hide and discover collusion, respectively. Their main issue is the design of the payment schemes for inspectors.

means a higher fine) will affect the severeness of the crime (in this case the overcharge).

The ambiguous effects of the introduction of leniency programs on cartel stability are also studied elsewhere in the literature.[5] On one hand, leniency programs can undermine the collusive outcome since a firm has the option to report and thereby reduce, or even eliminate, any fine that it might otherwise have to pay if the cartel is detected. On the other hand, the prospects for a less severe punishment can make it more profitable to form a cartel. Yet another mechanism for how leniency programs affect cartel stability is that applying for leniency can be used as a threat against defecting firms and thus also help to sustain the cartel. The latter approach is more in line with our model.[6] In contrast to the existing literature on leniency programs, we investigate in detail how the presence of a leniency program will influence the effect of higher fines on cartels' avoidance activities and, in turn, on the overcharge.

In the existing literature it is seemingly a widely recognized result that leniency should be restricted to the first informant only, as implemented in the US leniency program. This is for instance found in the works of Harrington (2008), Spagnolo (2008) as well as in Chen and Rey (2012). They all show that letting leniency be limited to the first informant maximizes the program's impact when it comes to destabilizing the cartel. Our results suggest that it might be desirable to modify the design of the leniency program to take into account the possible countervailing effect of the first-informant rule on cartel stability. This suggests that the type of leniency program adopted by EU might be more efficient in this respect.

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[5]This is shown in Motta and Polo (2003), and further analyzed in, among others, Aubert *et al.* (2006) and Harrington (2008). For a survey of the literature, see Spagnolo (2008).

[6]This effect was first discussed in Spagnolo (2000) and Ellis and Wilson (2001), and is further elaborated in Chen and Harrington (2007).

# 3　The model

We consider first a basic model of crime and punishment, and then extend and apply this model to antitrust enforcement in the next section of the paper.

Consider a group of $n$ agents who each period can cooperate on committing a crime of seriousness $\alpha$, where $\alpha$ is determined by the group. The net payoff from committing the crime equals $\alpha\pi$ for each agent, where $\alpha > 1$. If one agent chooses not to cooperate, all agents earn $\pi$ each. Let $p$ denote the probability that the crime is detected a given period. Assume that the agents can reduce $p$, but that this is costly: Each agent has avoidance costs $c(p)$ where $c_p < 0$. If the crime is detected, then each agent gets a punishment $F(\alpha)$, where $F_\alpha > 0$, earning $\pi - F(\alpha)$.[7] We also assume that detection prevents the agents from cooperating in the future. After the crime has been committed, but before it is potentially detected, agents can inform the police and attain leniency, giving a utility $u = \pi - L + v$, where $L < F$ is the (reduced) punishment and $v$ is a non-pecuniary utility from informing the police.

We analyze a repeated relationship where the following stage game is played each period:

1. The agents (simultaneously) choose whether or not to cooperate on a crime $\alpha$ with avoidance costs $c(p)$. If they cooperate the game proceeds to stage 2. If they do not cooperate, the game ends.

2. The agents simultaneously choose whether or not to inform the police and apply for leniency.

3. If no-one informs the police, the crime is detected with probability $p$.

We consider trigger strategies where the agents cooperate if all agents have

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[7]For simplicity we assume that $F(\alpha)$ captures any kind of punishment, including imprisonment.

cooperated in the past, while they choose not to cooperate forever after if at least one agent have defected in the past.[8]

The present value of committing the crime each period is then, for each agent:

$$V(\alpha, p) = (1 - p)\left[\alpha\pi + \delta V(\alpha, p)\right] + p\left[\frac{\pi}{1 - \delta} - F(\alpha)\right] - c(p) \qquad (1)$$

Solving for $V$ yields

$$V(\alpha, p) = \frac{(1 - p)\alpha\pi - c(p) + p(\frac{\pi}{1-\delta} - F(\alpha))}{1 - \delta(1 - p)}$$

The present value of informing the authorities is $u - c(p) + \frac{\delta\pi}{1-\delta}$. An agent will thus not inform the police if[9]

$$V(\alpha, p) \geq u - c(p) + \frac{\delta\pi}{1 - \delta}$$

Our main question is now as follows: Will the agents commit a less severe crime (here a lower $\alpha$) if the punishment becomes stronger? To analyze this, let $\phi$ be a shift parameter with $F_\phi > 0$. The constraint to sustain cooperation is now

$$V(\alpha, p; \phi, \delta) = \frac{(1 - p)\alpha\pi - c(p) + p(\frac{\pi}{1-\delta} - F(\alpha, \phi))}{1 - \delta(1 - p)} \geq u + \frac{\delta}{1 - \delta}\pi - c(p) \quad (\text{L})$$

Examining how the crime level ($\alpha$) reacts to variations in the fine, we find the following:

**Proposition 1** *The relationship between crime $\alpha$ and punishment $F$ is am-*

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[8]Not cooperating is the optimal punishment.

[9]Note that a general attribute with all leniency programs is that for the leniency constraint to be binding (and not the participation constraint), agents must earn more from committing a crime and then report, than from not committing a crime at all. Since $V(\alpha, p) \geq \frac{\pi}{1-\delta}$, we must have $u - c(p) > \pi$ for the constraint to be binding. Hence, the constraint binds if $v - L - c(p) > 0$ , implying either a high $v$ or a sufficiently low $L$ (where $L < 0$ is also possible).

*biguous. In particular, a tougher punishment leads to a more severe crime*
*($\alpha'(\phi) > 0$) if and only if:*

$$\frac{\pi + F_\alpha}{1 - \delta(1-p)} F_\phi + F_{\alpha\phi} \frac{c_p}{1 + \lambda} > 0,$$

*where $\lambda > 0$ is the shadow cost associated with the constraint (L). Thus, the crime becomes more severe ($\alpha'(\phi) > 0$) if the level of the fine increases ($F_\phi > 0$) while the marginal fine remains constant ($F_{\alpha\phi} = 0$). And conversely, the crime becomes less severe if the marginal fine increases ($F_{\alpha\phi} > 0$) while the level of the fine stays constant ($F_\phi = 0$).*

We see from this that a shift which increases the level of the fine without affecting its slope ($F_{\alpha\phi} = 0$ and $F_\phi > 0$) will lead to a more severe crime. The reason why this type of tougher punishment leads to a more severe crime can be seen more directly as follows. Note that the RHS of the constraint (L) is independent of $\alpha$ and $F$. It is thus not directly affected by changes in the fine. Define

$$\tilde{V}(p; \phi, \delta) = \max_\alpha \frac{(1-p)\alpha\pi - c(p) + p(\frac{\pi}{1-\delta} - F(\alpha, \phi))}{1 - \delta(1-p)}$$

i.e. the optimal value (wrt to crime level $\alpha$) for given $p$.

The problem of maximizing $V$ subject to the constraint is then equivalent to maximizing $\tilde{V}(p)$ subject to the constraint (i.e. subject to $\tilde{V}(p) \geq u + \frac{\delta}{1-\delta}\pi - c(p)$).

Consider now a shift that increases the level of the fine without affecting its slope ($F_{\alpha\phi} = 0$ and $F_\phi > 0$). This will shift down the value; we have

$$\tilde{V}_\phi = V_\phi = \frac{-pF_\phi}{1 - \delta(1-p)}$$

This downward shift will reduce $p$; the higher $F$ is accommodated with increased investments so as to reduce the detection probability. Note that the

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marginal value of $\alpha$ is

$$V_\alpha = \frac{(1-p)\pi - pF_\alpha}{1 - \delta(1-p)}$$

When $F_\alpha$ is unaffected, the lower $p$ will increase this marginal value and hence make a heavier crime profitable. The result is thus that $\alpha$ increases and $p$ is reduced. The higher fine/penalty level leads to a lower detection probability and, in turn, to a more severe crime.

This illustrates that the fine policy as such, and in particular whether the marginal punishment changes or not, is crucial for whether tougher punishment has an unintended affect shown in Proposition 1.

## 3.1   Retaining surplus after detection

In the basic model, we have made the assumption that the agents cannot retain any surplus from the criminal activity if they are being detected. Let us now assume that they can "consume" the criminal surplus before they are detected so that the first period payoff after detection is $\alpha\pi - F(\alpha)$ , which also implies that the first period payoff after being granted leniency is $u = \alpha\pi - L + v$. As we shall see, this implies a less ambiguous relationship between crime and punishment, since it turns out that $\alpha'(\phi) < 0$ is possible also for $F_{\phi\alpha} = 0$.[10]

The present value of committing the crime is now, for each agent:

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\begin{aligned}
V(\alpha, p) &= (1-p)\left[\alpha\pi + \delta V(\alpha, p)\right] + p\left[\alpha\pi + \frac{\delta\pi}{1-\delta} - F(\alpha)\right] - c(p) \\
&= \alpha\pi + (1-p)\delta V(\alpha, p) + p\left[\frac{\delta\pi}{1-\delta} - F(\alpha)\right] - c(p)
\end{aligned}
$$

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[10]It is true for this case as well that $\alpha'(\phi) < 0$ if $F_{\phi\alpha} > 0$ and $F_\phi = 0$, i.e. that a higher marginal fine yields a reduced crime level. Formally this can be seen from the formulas (7) and (8) in the proof of Proposition 2 below.

Solving for $V$ yields

$$V(\alpha, p) = \frac{\alpha\pi - c(p) + p(\frac{\delta\pi}{1-\delta} - F(\alpha))}{1 - \delta(1 - p)}$$

An agent will not inform the police if $V(\alpha, p) \geq \alpha\pi - L - c(p) + \frac{\delta\pi}{1-\delta}$. So the problem here is to maximize $V(\alpha, p)$ subject to

$$V(\alpha, p) = \frac{\alpha\pi - c(p) + p(\frac{\delta\pi}{1-\delta} - F(\alpha))}{1 - \delta(1 - p)} \geq \alpha\pi - L - c(p) + \frac{\delta\pi}{1 - \delta} \qquad \text{(M)}$$

Let now $\mu = \frac{\lambda}{\lambda+1} \in [0, 1)$ be a transformation of the Lagrange multiplier $\lambda$ for the constraint (M), and let $\varepsilon(p) = -\frac{c_{pp}}{c_p}(1 - p) > 0$ denote the elasticity of the marginal cost function. Here we get the following.

**Proposition 2** *Given that $F_{\phi\alpha} = 0$ (the marginal punishment remains constant), for the model with constraint (M) we have that a higher punishment $(F_\phi > 0)$ leads to a a more severe crime $(\alpha'(\phi) > 0)$ if:*

$$\varepsilon(p) \left[\frac{\mu - 1}{1 - p} - \delta\mu\right] p + [(1 - \mu)(2 - \delta) + \mu\delta(1 - 2p)] > 0 \qquad (2)$$

*A tougher punishment thus leads to a more severe crime $(\alpha'(\phi) > 0)$ if:*

$$
\begin{aligned}
1 - (\varepsilon(p) + 2)p &> 0 & \text{for } \mu \to 1 & \qquad (3)\\
1 - \delta + \frac{1 - (\varepsilon(p) + 1)p}{1 - p} &> 0 & \text{for } \mu \to 0 & \qquad (4)
\end{aligned}
$$

The elasticity of the marginal cost function affects the agents' responsiveness regarding $p$, and this in turn affects their response regarding $\alpha$. Given that the elasticity is bounded, we see that for the limiting cases $(\mu \to 1, \mu \to 0)$

the sign of $\alpha'(\phi)$ is positive only if the (equilibrium) probability of detection ($p$) is 'small'. Hence, higher punishment will more often lead to less severe crime in this case where the agents can consume the criminal value before they are detected. The reason is that it is tempting to deviate to reap the benefit from the crime, and the severeness of the crime must be reduced to avoid such a deviation.

This does not mean that $\alpha'(\phi) > 0$ is not possible. For quadratic costs $c(p) = k(1-p)^2$ the elasticity is $\varepsilon(p) = -\frac{c_{pp}}{c_p}(1-p) = 1$, so the expression in (3) is positive for $p < \frac{1}{3}$, while the expression in (4) is (for $\mu \to 0$) certainly positive for $p \leq \frac{1}{2}$.

We see that the scope for an unintended effect of a higher punishment is more limited when the agents can retain the surplus after detection. The reason is that in such a case the agents can find it tempting to inform the police, since they even then can retain the criminal surplus. To avoid such an outcome, it is less likely that the agents set a higher crime level when the probability of detection is reduced.

Formally this is captured by constraint (M) being more difficult to satisfy for higher $\alpha$ than constraint (L). First, the deviation profit is larger; there is an additional term $(\alpha - 1)\pi$ on the RHS of (M) compared to (L). Second, although the collusive profit value is also larger – the difference in values amounts to $\frac{p(\alpha-1)\pi}{1-\delta(1-p)}$ – this higher value does not make up for the higher deviation profit. (The net difference is $(\frac{p}{1-\delta(1-p)} - 1)(\alpha - 1)\pi$, which is decreasing in $\alpha$.) Hence the retained surplus makes constraint (M) more difficult to satisfy for higher $\alpha$ than constraint (L), and the cartel therefore more reluctant to increase $\alpha$ in this case.

# 4    Antitrust enforcement

We now extend and apply the basic model in Section 3 to study antitrust enforcement. Forming a cartel is then the criminal activity, and the profit

from overcharge is the crime level. Let $k = (p^C - p^N)/p^N$ denote cartel overcharge where $p^C$ is cartel price and $p^N$ is the non-collusive price. Each firm's (per period) profit from collusion is $\alpha(k)\pi$, $\alpha > 1$. If the firms do not collude, they earn $\pi$ each. If all other firms are colluding, a firm's profit from deviation is $\beta\alpha\pi$, where $\beta > 1$. As before, $p$ denotes the probability that the cartel is detected within a given period. If a firm is detected, it gets a fine $F(\alpha)$ in addition to a restitution fine that permits to seize back the illegal profits realized by the cartel (in the given period). The payoff after detection is then $\pi - F(\alpha)$.

The cartel faces avoidance costs $c(p)$ per firm per period. Since the overcharge $k$ only works via $\alpha$, we will (mostly) omit the variable $k$ in what follows. The present value of repeated cartel activity is then given by

$$V(\alpha, p) = (1 - p)\left[\alpha\pi + \delta V(\alpha, p)\right] + p\left[\frac{\pi}{1 - \delta} - F(\alpha)\right] - c(p) \qquad (5)$$

which is exactly the same as in the basic model in the previous section.

Consider now the following stage game:

1. The cartel agrees on overcharge $k$ (and thus $\alpha$) and hiding costs $c(p)$.

2. Firms set prices. They can honor the cartel agreement or deviate by lowering the price.

3. Firms can report to the competition authority (CA) about the cartel and apply for leniency. If leniency is admitted, the expected fine is $L(m) < F$, where $m \geq 1$ is the number of firms applying for leniency. Any non-applicant is then fined $F$.

4. If no-one applies for leniency, CA detects the cartel with probability $p$.

When the stage game is played repeatedly, firms play trigger strategies: If at least one firm deviates by lowering the price or applies for leniency, the game reverts to non-cooperative Nash strategy forever after, earning $\pi$ each.

Now, the major difference between the basic crime model, and the cartel model, is that the agents (labeled firms in this section) can deviate by charging a lower price and thus appropriate the cartel profit. Hence there are two ways in which a firm can in principle deviate: Report to CA or just deviate from the cartel agreement without reporting.

First note that a strategy profile where all firms apply for leniency in Stage 3 (and then revert to Nash play) is a continuation equilibrium in that stage, irrespective of the outcome in stage 2. As long as at least one firm applies, it is a best response for another firm to also apply, because of the milder punishment under leniency. This is in particular an equilibrium after a deviation in Stage 2, and we will assume that the firms play this strategy profile after such a deviation.[11] A firm will then not deviate from the collusive agreement in Stage 2 if

$$V(\alpha, p) \geq \pi - L + \upsilon + \frac{\delta}{1 - \delta}\pi - c(p) \tag{L}$$

where now $L = L(n)$.

Given no deviation in Stage 2, it is a continuation equilibrium that no firm applies for leniency in Stage 3 if condition (L) holds for $L = L(1)$. Since it is reasonable to assume $-L(1) \geq -L(n)$, the requirement in condition (L) is stronger for $L = L(1)$ than for $L = L(n)$. Hence, if the requirement to deter a first deviation in Stage 3 (after no previous deviations) is fullfilled, then the requirement to deter a first deviation in Stage 2 is also fullfilled. This equilibrium will thus be sustained if condition (L) holds for $L = L(1)$, and in equilibrium no-one will then deviate from the collusive agreement. To simplify notation, we will in the following take $L$ to mean $L(1)$.

There are of course other equilibria. In particular, a strategy profile where no firm applies for leniency in stage 3 after a deviation in stage 2 is a continuation equilibrium if $pF \leq L$. If firms play according to this strategy, then a unilateral deviation in stage 2 is not profitable (for $pF \leq L$) if

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[11]If $pF \leq L$, it is also a continuation equilibrium that no firm applies for leniency. This is discussed below in the text.

$$V(\alpha, p) \geq (1 - p)\beta\alpha\pi + p(\pi - F) + \frac{\delta}{1 - \delta}\pi - c(p) \qquad \text{(D)}$$

Given this continuation equilibrium, it is an overall equilibrium that no-one ever deviates if condition (L) holds (to deter a first deviation in stage 3) *and in addition* that condition (D) holds if $pF \leq L$. The requirement that both these conditions hold is of course a (weakly) stronger requirement than requiring only condition (L) to be fulfilled. The attainable value $V$ in this equilibrium must therefore be (weakly) lower than the attainable value in the equilibrium considered above (where only condition (L) is required). We therefore assume that the cartel coordinates on the former equilibrium.

To attain a maximal value, the cartel thus maximizes $V(\alpha, p)$ subject to (L) which is the same as the problem in the previous section.[12] The cartel chooses the crime level $\alpha$ by choosing the cartel price $p^C$ and thus overcharge $k = (p^C - p^N)/p^N$, where $\alpha'(k) > 0$ as long as $p^C$ does not exceed the monopoly price $p^M$. Then the condition in Proposition 1 applies, and we have the following result:

**Proposition 3** *When the leniency program binds, a higher fine leads to a higher overcharge $(\alpha'(\phi) > 0)$ if the condition in Proposition 1 is met. In particular, the overcharge increases if the level of the fine increases $(F_\phi > 0)$ while the marginal fine remains constant $(F_{\alpha\phi} = 0)$.*

The results shown earlier thus carry over to the case of antitrust enforcement with a binding leniency program. As shown above, higher fines may well lead to higher overcharges.

If there were no leniency program present, i.e. if it was not possible to apply for leniency in Stage 3 of the game above, condition (D) would be the (single) relevant condition for sustaining the cartel.[13] It is convenient to rewrite this

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[12] Similarly to the previous section, for the leniency constraint to be binding (and not the participation constraint $V(\alpha, p) \geq \frac{\pi}{1-\delta}$), we must here have $-L + v - c(p) > 0$, implying either a high $v$ or a sufficiently low $L$ (where $L < 0$ is also possible).

[13] The participation constraint is not relevant in this case, since as we show below, (D) is equivalent to condition (DV), and is hence a stricter condition.

condition somewhat. Using the expression for $V(\alpha, p)$ given in (5) above, we see that (D) can be written as

$$(1-p)\left[\alpha\pi + \delta V(\alpha, p)\right] + p\left[\frac{\pi}{1-\delta} - \pi\right] \geq (1-p)\beta\alpha\pi + \frac{\delta}{1-\delta}\pi$$

By collecting terms and dividing by $(1-p)\delta$, we see that this condition is equivalent to

$$V(\alpha, p) \geq \frac{1}{\delta}(\beta - 1)\alpha\pi + \frac{1}{1-\delta}\pi \qquad \text{(DV)}$$

We now obtain the following comparative statics result:

**Proposition 4** *When there is no leniency program, and the relevant constraint (D) to sustain the cartel binds, a higher fine F will always lead to lower cartel overcharge ($\alpha'(F) < 0$).*

When the constraint (D) – or equivalently (DV) – binds, the elasticity of $c_p$ does not matter because an increase in $\alpha$ would also increase the temptation to deviate. An increase in the fine will in this case always lead to a lower overcharge in order to discipline the incentive to deviate. The higher fine will lead to larger investment in avoidance activity and to a reduction in the detection probability. In isolation, this would make it tempting to commit a more severe crime. This is not optimal in this case since it would increase the incentive to deviate.

The results in the last two propositions show that the cartel may react quite differently to a higher fine, depending on whether a leniency program is in place or not. When such a program is present, higher fines may well lead to higher overcharges by the cartel, while if it is not in place, a higher fine will always result in a lower overcharge.[14]

As the analysis here also has pointed out, the mere sustainability of the cartel may depend on whether a leniency program is in place or not. In particular, the relevant constraint (D) – or equivalently (DV) – to sustain

---

[14]It is straightforward to show that a lower overcharge will result also if the constraint (D) is not binding, which will be the case if, say, $\delta$ is high.

the cartel in the absence of a leniency program, may be stricter than the relevant constraint (L) when a program is present. Comparing constraints (L) and (DV), we see that this is the case when

$$\frac{1}{\delta}(\beta - 1)\alpha\pi > -L + \upsilon - c(p) \tag{6}$$

In such cases constraint (L) may hold, but not constraint (D). This will imply that, whenever the cartel is viable with no leniency program in place, it will also be viable when such a program is present, but not the other way around.[15] The intuition for this is that applying for leniency can be used as a credible threat to punish a firm who deviates from the cartel agreement regarding pricing, and hence make this agreement more sustainable.

Summing up, we have seen that introducing a leniency program may (a) help to sustain the cartel, but (b) also affect – and in some cases negatively – the way the cartel reacts to higher fines.

## 4.1 Retaining surplus after detection

Assume, as in Section 3.1 that the agents (now firms in the cartel) can retain the surplus after detection. This means that we no longer impose a restitution fine, as we allowed for in the previous discussion in this Section. If no-one deviates, the cartel value is then, as in Section 3.1, given by

$$\begin{aligned} V(\alpha, p) &= (1 - p)\left[\alpha\pi + \delta V(\alpha, p)\right] + p\left[\alpha\pi + \frac{\delta\pi}{1 - \delta} - F(\alpha)\right] - c(p) \\ &= \alpha\pi + (1 - p)\delta V(\alpha, p) + p\left[\frac{\delta\pi}{1 - \delta} - F(\alpha)\right] - c(p) \end{aligned}$$

---

[15]Condition (6) wil definitely hold if $-L + \upsilon - c(p) \leq 0$, but then the relevant constraint under leniency is the participation constraint; see fn. 11 above. The leniency program will then still ease cartel viability, since the participation constraint is always weaker than (DV).

Absent a leniency program, the condition to sustain the cartel is then

$$V(\alpha, p) \geq \beta\alpha\pi - pF + \frac{\delta\pi}{1-\delta} - c(p) \qquad \text{(E)}$$

When a leniency program is present, the condition to deter a first deviation in stage 3 (given no previous deviations) is

$$V(\alpha, p) \geq \alpha\pi - L + v + \frac{\delta\pi}{1-\delta} - c(p) \qquad \text{(M)}$$

where $L = L(1)$. This is just as in Section 3.1.

As before, it is an equilibrium that all firms apply for leniency in Stage 3. Assuming this equilibrium is played after a deviation in Stage 2, the condition to deter such a deviation in Stage 2 is then

$$V(\alpha, p) \geq \beta\alpha\pi - L(n) + v + \frac{\delta\pi}{1-\delta} - c(p) \qquad \text{(N)}$$

It is not obvious whether this condition is stronger or weaker than (M). It is stronger if $(\beta - 1)\alpha\pi > L(n) - L$, i.e. if the profits gain from the price deviation exceeds the loss from a less favorable leniency treatment. Denoting the latter by $\Delta L$, we can write the conditions combined as

$$V(\alpha, p) \geq \alpha\pi - L + v + \max\{(\beta - 1)\alpha\pi - \Delta L, 0\} + \frac{\delta\pi}{1-\delta} - c(p) \quad \text{(MN)}$$

where $\Delta L = L(n) - L$.

Comparing leniency and no leniency, ie comparing constraints (MN) and (E), wee see that leniency entails a stricter constraint if

$$pF - L + v + \max\{(\beta - 1)\alpha\pi - \Delta L, 0\} > (\beta - 1)\alpha\pi$$

If $(\beta - 1)\alpha\pi \leq \Delta L$, then the inequality above holds if $pF - L + v > (\beta - 1)\alpha\pi$, i.e if the leniency program is sufficiently favorable (for a single applicant)

and/or the fine is sufficiently large.

If $(\beta-1)\alpha\pi > \Delta L$, then the inequality holds if $pF - L(n) + v > 0$. Thus, if a price deviation is very profitable $((\beta-1)\alpha\pi > \Delta L)$, the leniency constraint will entail a stricter constraint for the cartel merely if $pF - L(n) + v > 0$. In this case the (credible) threat to punish a price deviation by all applying for leniency is not at all effective in sustaining the cartel. On the contrary, the leniency program (at least in this equilibrium) will make it more difficult to sustain the cartel.

The last considerations above have some implications for the design of a leniency program. For given deviation profitability $(\beta-1)\alpha\pi$, one may design the system with $\Delta L \geq (\beta-1)\alpha\pi$, or with $\Delta L < (\beta-1)\alpha\pi$. In the former case (a 'big' difference between the leniency treatments when all report and when only a single firm reports), the leniency treatment of the single firm must be very favorable in order for the program to negatively affect the viability of the cartel $(L < pF + v - (\beta-1)\alpha\pi)$ This may not even be feasible, e.g. if it requires $L < 0$.

An alternative is to design a program with a smaller difference in the leniency treatments between a single firm and all firms reporting $(\Delta L < (\beta-1)\alpha\pi)$. In this case the viability of the cartel is negatively affected if just $L(n) < pF + v$, which may well be feasible. This indicates that a program with small differences in leniency treatments may be more effective in deterring the cartel.

Regarding the way the cartel reacts to a higher fine, we obtain here the following result:

**Proposition 5** *Assume the surplus is retained after detection.*
*(i) If $\Delta L > (\beta-1)\alpha\pi$ (so that (M) is binding), the effect of a higher fine on the cartel's overcharge is given as in Proposition 2.*
*(ii) If $\Delta L < (\beta-1)\alpha\pi$ (so that (N) is binding), the effect of a higher fine tends to be smaller than in case (i): the marginal effects in the two cases*

$(\alpha'_M(\phi)$ and $\alpha'_N(\phi)$, respectively); satisfy, all else equal; $\alpha'_N(\phi) < \alpha'_M(\phi)$ when $F_{\alpha\phi} = 0$ and $F_\phi > 0$

As we can see, this result shares some similarities with the results shown in Proposition 2 for the general case of crime activity. When the agents can retain surplus when deviating, there is less scope for an unintended effect of a higher fine on the overcharge. Again, the reason is that it becomes more tempting to deviate since the agent can capture the short term gain. A higher overcharge would make it even more profitable to deviate, and there is therefore less scope for the cartel members to respond by setting a higher overcharge when the detection probability is reduced.

# 5   Concluding remarks

In this paper we have analyzed a repeated game model of organized crime in which criminals i) can spend resources in order to reduce the probability of being detected and ii) are admitted reduced punishment if they inform the police about a committed crime. Our analysis shows that a higher general punishment level can increase the crime level. The reason is that higher punishment-levels lead criminals to spend much more resources on hiding their criminal activity, which in turn leads to lower probability of detection, and thus weaker law enforcement, in equilibrium.

We then apply the model to antitrust enforcement. The main difference between the basic crime model, and the cartel model, is that the agents (or firms) now can deviate by charging a lower price and thus appropriate the cartel profit. Hence there are in principle two ways in which a firm can deviate: report to the competition authority or just deviate from the cartel agreement regarding pricing without reporting. As we have seen, the latter behavior can be credibly deterred by the other firms when there is a leniency program in place, and thus ease the sustainability of the cartel. And as we also have shown, with a generous (binding) leniency program in place, a

higher general fine level may lead to higher cartel overcharges. As this will not occur in the absence of a leniency option, the analysis has pointed out that introducing a leniency program may, under some conditions (notably when surplus cannot be retained by the firms after detection) both help to sustain the cartel, and also affect the way the cartel reacts to higher fines. We have also seen that leniency may help to deter the cartel (when surplus can be retained), and that this may also affect the way the cartel reacts to higher fines.

We have found that it matters significantly how the fine is increased. While an increase in the marginal fine - the additional fine for an additional overcharge - will always in itself lead to a lower overcharge, we find that an increase in the fine level as such may lead to more avoidance activity and, in turn, a higher overcharge. During the last decade we have in many jurisdictions seen a substantial increase in the fines for price fixing. Unfortunately, it seems as if the tougher sanctions are primarily implemented as an increase in the fine level as such rather than higher fines for those cartels with the highest overcharges. For example, according to the US guidelines the base fine level should set at 20% of annual affected commerce.[16] It is argued that *'the purpose of specifying a percent of the volume of commerce is to avoid the time and expense that would be required for the court to determine actual gain or loss'*.[17] In EU, the fine is limited to 10% of the overall annual turnover of the company. Although there is a scope for higher fines with more severe offences, it is not obvious that higher fines will be set in EU when cartels have set higher overcharges.[18] These observations, coupled with the observation

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[16]Such a fine setting procedure may in itself produce some unintended effects. As shown in Bageri et al. (2012), a fine that is calculated as a percentage of the affected commerce will give the cartel incentive to set a price above the monopoly price and thereby to lower the revenues.

[17]See U.S. Sentencing Guidelines Manual § 2R1.1 (2005). The base fine level should be adjusted by a number of factors, such as adjusted upwards if bid rigging or other aggravating factors are involved or downward if the firm cooperates with antitrust authority. However, it is hard to see that such adjustments introduce anything that would imply that the fine should depend on the actual damage.

[18]See Guidelines on the method of setting fines imposed pursuant to Article 23(2)(a) of Regulation No 1/2003. The fines are increased in line with a set of "aggravating circumstances" which include recidivism, leading role, retaliatory measures against other

that cartels with serious harm to society are still detected, indicates that we should be concerned that the present fine policy can have unintended effects on cartels' overcharges.

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undertakings, refusal to co-operate with or attempts to obstruct the European Commission in carrying out its investigation.

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# Appendix

**Proof of Propositions 1 and 2.**

To accomodate both models (M) and (L), and in addition a case to be considered in a later section, introduce the index $m \in \{0,1\}$, the parameter $\beta > 1$ and consider the following optimization problem

$$\max_{\alpha,p} V(\alpha, p; \phi, \delta, m) \quad \text{s.t.} \quad G(p, \alpha, \phi, \delta, m) \geq 0,$$

where

$$V(\alpha, p; \phi, \delta, m) = \frac{\alpha\pi - c(p) + p(\frac{\delta\pi}{1-\delta} - F(\alpha, \phi)) - mp(\alpha - 1)\pi}{1 - \delta(1 - p)}$$

and

$$G = V - H, \quad \text{with} \quad H = mu + \frac{\delta}{1 - \delta}\pi - c(p) + (1 - m)(\beta\alpha\pi - L)$$

Then model (L) is obtained for $m = 1$, and model (M) for $m = 0, \beta = 1$. (The function $H()$ equals the right hand sides of constraints (L) and (M), respectively, for $m = 1$ and $m = 0, \beta = 1$.)

Forming the Lagrangian $L = V + \lambda G$, we have the following standard comparative statics result:

$$\alpha'(\phi) = \frac{1}{D} \left( [L_{pp}G_\alpha - L_{\alpha p}G_p] G_\phi + [L_{\alpha\phi}G_p - L_{p\phi}G_\alpha] G_p \right),$$

where $D > 0$ is the determinant of the bordered Hessian.

Note that from $L = V + \lambda G$, $G = V - H$ and the FOCs $V_k = -\lambda G_k$, $k = \alpha, p$, we have

$$G_k L_{ij} = G_k V_{ij} + G_k \lambda G_{ij} = (V_k - H_k)V_{ij} - V_k(V_{ij} - H_{ij}) = V_k H_{ij} - H_k V_{ij}$$

Substituting this in the formula for $\alpha'(\phi)$ yields

$$\begin{aligned}
\alpha'(\phi)D &= \left[ (V_\alpha H_{pp} - H_\alpha V_{pp}) - (V_p H_{\alpha p} - H_p V_{\alpha p}) \right] G_\phi \\
&\quad + \left[ (V_p H_{\alpha\phi} - H_p V_{\alpha\phi}) - (V_\alpha H_{p\phi} - H_\alpha V_{p\phi}) \right] G_p
\end{aligned}$$

Note that $H_\phi = 0$ and hence $G_\phi = V_\phi$. From FOC we have $0 = V_p + \lambda G_p = (1+\lambda)V_p - \lambda H_p$ and thus $G_p = V_p - H_p = -H_p/(1+\lambda) = c_p/(1+\lambda) < 0$. So we have

$$\begin{aligned}
\alpha'(\phi)D &= \left[ (V_\alpha H_{pp} - H_\alpha V_{pp}) - (V_p H_{\alpha p} - H_p V_{\alpha p}) \right] V_\phi \\
&\quad + \left[ -H_p V_{\alpha\phi} + H_\alpha V_{p\phi} \right] (c_p)/(1+\lambda)
\end{aligned}$$

We have $H_\alpha = (1-m)\beta\pi$. We further have from FOC $V_p = \frac{\lambda}{1+\lambda}H_p = -\mu c_p > 0$, where $\mu = \frac{\lambda}{1+\lambda}$, and $V_\alpha = \frac{\lambda}{1+\lambda}H_\alpha = \mu(1-m)\beta\pi$

So we have, since $H_{\alpha p} = 0$

$$\begin{aligned}
\alpha'(\phi)D &= \left[ (\mu(1-m)\beta\pi H_{pp} - (1-m)\beta\pi V_{pp}) - (0 + c_p V_{\alpha p}) \right] V_\phi \quad (7) \\
&\quad + \left[ c_p V_{\alpha\phi} + (1-m)\beta\pi V_{p\phi} \right] (c_p)/(1+\lambda) \\
&= \left[ (-\mu c_{pp} - V_{pp}) (1-m)\beta\pi - c_p V_{\alpha p} \right] V_\phi \\
&\quad + \left[ c_p V_{\alpha\phi} + (1-m)\beta\pi V_{p\phi} \right] (c_p)/(1+\lambda)
\end{aligned}$$

Consider

$$V_\phi = \frac{-pF_\phi}{1-\delta(1-p)}, \qquad V_{\phi\alpha} = \frac{-pF_{\alpha\phi}}{1-\delta(1-p)}, \qquad V_{p\phi} = \frac{-(1-\delta)F_\phi}{(1-\delta(1-p))^2} \quad (8)$$

$$
\begin{aligned}
V_{\alpha p} &= \frac{\partial}{\partial p} \frac{\pi - pF_\alpha - mp\pi}{1 - \delta(1-p)} \tag{9}\\
&= \frac{(-m\pi - F_\alpha)(1 - \delta(1-p)) - [(1-mp)\pi - pF_\alpha]\delta}{(1 - \delta(1-p))^2}\\
&= \frac{(-m\pi - F_\alpha)}{1 - \delta(1-p)} - V_a \frac{\delta}{1 - \delta(1-p)}\\
&= \frac{(-m\pi - F_\alpha)}{1 - \delta(1-p)} - \frac{\mu(1-m)\beta\pi\delta}{1 - \delta(1-p)} \qquad \text{(by FOC } V_\alpha = \mu(1-m)\beta\pi)
\end{aligned}
$$

(I) For model L, where $m = 1$, we thus have

$$
\begin{aligned}
\alpha'(\phi)D &= [-V_{\alpha p}V_\phi + V_{\alpha\phi}c_p/(1+\lambda)]\, c_p\\
&= \left[ -\frac{(-\pi - F_\alpha)}{1 - \delta(1-p)} \frac{-pF_\phi}{1 - \delta(1-p)} + \frac{-pF_{\alpha\phi}}{1 - \delta(1-p)} \frac{c_p}{1+\lambda} \right] c_p\\
&= \left[ \frac{\pi + F_\alpha}{1 - \delta(1-p)} F_\phi + F_{\alpha\phi} \frac{c_p}{1+\lambda} \right] \frac{-pc_p}{1 - \delta(1-p)}
\end{aligned}
$$

This proves the formula in Proposition 1.

(II) Consider now the case $m = 0$. (Model M is obtained for $m = 0, \beta = 1$.) Suppose further that $F_{\alpha\phi} = 0$, and hence that $V_{\phi\alpha} = 0$. Then we have, from the formulas (7), (8) and (9) above:

$$
\begin{aligned}
\alpha'(\phi)D &= [(-\mu c_{pp} - V_{pp})\beta\pi - c_p V_{\alpha p}]\, V_\phi + [\pi\beta V_{p\phi}]\,(c_p)/(1+\lambda)\\
&= \left[ (-\mu c_{pp} - V_{pp})\beta\pi - c_p \frac{-F_\alpha - \mu\beta\pi\delta}{1 - \delta(1-p)} \right] \frac{-pF_\phi}{1 - \delta(1-p)} + \frac{\pi\beta c_p}{1+\lambda} \frac{-(1-\delta)F_\phi}{(1 - \delta(1-p))^2}
\end{aligned}
$$

We show below that we have

$$
V_{pp} = \frac{-c_{pp} + 2\delta\mu c_p}{1 - \delta(1-p)} < 0 \tag{10}
$$

Hence, defining $D_1 = (1 - \delta(1-p))^2 D$, and noting that $\frac{1}{1+\lambda} = 1 - \mu$ we then

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obtain

$$\begin{aligned}
\alpha'(\phi)D_1 &= [-\mu c_{pp}(1-\delta(1-p))\beta\pi - (-c_{pp}+2\delta\mu c_p)\beta\pi + c_p(F_\alpha + \mu\beta\pi\delta)](-pF_\phi) \\
&\quad - \beta\pi c_p(1-\mu)(1-\delta)F_\phi \\
&= \beta\pi\{c_{pp}[\mu - 1 - \delta\mu(1-p)]p + [-\delta\mu p + pF_\alpha/\beta\pi + (1-\mu)(1-\delta)](-c_p)\}F_\phi
\end{aligned}$$

From FOC $V_\alpha = \mu H_\alpha = \mu\beta\pi$ we get

$$V_\alpha = \frac{\pi - pF_\alpha}{1 - \delta(1-p)} = \mu\beta\pi$$

and hence $pF_\alpha/\pi\beta = \frac{1}{\beta} - \mu(1 - \delta(1-p)) = \frac{1}{\beta} - \mu + \mu\delta(1-p)$.

This yields

$$\begin{aligned}
\alpha'(\phi)D_2 &\hspace{4cm} (11)\\
&= \beta\pi\{c_{pp}[\mu - 1 - \delta\mu(1-p)]p + [1/\beta - \mu + \mu\delta(1-2p) + (1-\mu)(1-\delta)](-c_p)\}F_\phi \\
&= \beta\pi\{c_{pp}[\mu - 1 - \delta\mu(1-p)]p + [(1/\beta - 1) + (1-\mu)(2-\delta) + \mu\delta(1-2p)](-c_p)\}F_\phi
\end{aligned}$$

Setting $\beta = 1$ (as required in model M), we see that $\alpha'(\phi)$ has the same sign as.

$$\{c_{pp}[\mu - 1 - \delta\mu(1-p)]p + [(1-\mu)(2-\delta) + \mu\delta(1-2p)](-c_p)\}F_\phi$$

Taking account of $\varepsilon(p) = \frac{c_{pp}}{-c_p}(1-p)$, we then obtain the formula (2) in Proposition 2..

It remains to verify the formula (10) above. Consider, for $m = 0$:

$$\begin{aligned}
V_p &= \frac{\left(-c_p + (\frac{\delta\pi}{1-\delta} - F)\right)(1 - \delta(1-p)) - \delta\left(\alpha\pi - c(p) + p(\frac{\delta\pi}{1-\delta} - F)\right)}{(1 - \delta(1-p))^2} \\
&= \frac{-c_p + (\frac{\delta\pi}{1-\delta} - F)}{1 - \delta(1-p)} - \frac{\delta}{1 - \delta(1-p)}V
\end{aligned}$$

$$
\begin{aligned}
V_{pp} &= \frac{-c_{pp}\left(1-\delta(1-p)\right)-\delta\left(-c_p+\left(\frac{\delta\pi}{1-\delta}-F\right)\right)}{\left(1-\delta(1-p)\right)^2}+\frac{\delta^2}{\left(1-\delta(1-p)\right)^2}V-\frac{\delta}{1-\delta(1-p)}V_p \\
&= \frac{-c_{pp}}{1-\delta(1-p)}-\frac{\delta}{1-\delta(1-p)}\frac{-c_p+\left(\frac{\delta\pi}{1-\delta}-F\right)}{1-\delta(1-p)}+\frac{\delta^2 V}{\left(1-\delta(1-p)\right)^2}-\frac{\delta V_p}{1-\delta(1-p)} \\
&= \frac{-c_{pp}}{1-\delta(1-p)}-\frac{\delta}{1-\delta(1-p)}\left(V_p+\frac{\delta}{1-\delta(1-p)}V\right)+\frac{\delta^2 V}{\left(1-\delta(1-p)\right)^2}-\frac{\delta V_p}{1-\delta(1-p)} \\
&= \frac{-c_{pp}}{1-\delta(1-p)}-\frac{2\delta}{1-\delta(1-p)}V_p
\end{aligned}
$$

Then (10) follows from FOC $V_p = -\mu c_p$. This completes the proof.

### Proof of Proposition 4

Taking the constraint (DV) into account, the optimization problem is now of the form

$$
\max_{\alpha,p} V(\alpha,p;F,\delta,\beta) \quad \text{s.t.} \quad G(p,\alpha,F,\delta,\beta) \geq 0,
$$

where $V(\alpha,p;F,\delta,\beta) = \frac{(1-p)\alpha\pi-c(\alpha,p)+p\left(\frac{\pi}{1-\delta}-F\right)}{1-\delta(1-p)}$ and

$$
G = V - H, \quad \text{with} \quad H(\alpha;\beta,\delta) = \frac{\beta-1}{\delta}\alpha\pi+\frac{1}{1-\delta}\pi.
$$

Let $L = V + \lambda G$ be the Lagrangian. Given sufficient second order conditions (SOC), standard comparative statics yield

$$
\alpha'(F) = \frac{1}{D}\left(\left[L_{pp}G_\alpha - L_{\alpha p}G_p\right]G_F + \left[L_{\alpha F}G_p - L_{pF}G_\alpha\right]G_p\right),
$$

where $D > 0$ is the determinant of the bordered Hessian of $L$.

Note that $H()$ doesn't depend on $p$, nor on $F$, hence $G_F = V_F$ and $G_p = V_p$. From $L = V + \lambda G$, $G = V - H$ and the FOCs $0 = L_k = V_k + \lambda G_k =$

$V_k(1+\lambda) - \lambda H_k$, $k = \alpha, p$, we then have $V_p = 0 = G_p$, and hence

$$
\begin{aligned}
\alpha'(F)D &= [L_{pp}G_\alpha - L_{\alpha p}0]\, V_F + [L_{\alpha F}G_p - L_{pF}G_\alpha]\, 0 \\
&= L_{pp}G_\alpha V_F
\end{aligned}
$$

Computing $D$ from the bordered Hessian of $L$ yields

$$
\begin{aligned}
D &= -L_{pp}G_\alpha^2 + 2L_{\alpha p}G_\alpha G_p - L_{\alpha\alpha}G_p^2 \\
&= -L_{pp}G_\alpha^2
\end{aligned}
$$

where the last equality follows from FOC; $G_p = V_p = 0$. Hence we now have

$$
\alpha'(F) = -\frac{G_\alpha V_F}{G_\alpha^2} = -\frac{V_F}{G_\alpha}
$$

From FOC $0 = V_\alpha + \lambda G_\alpha = V_\alpha(1+\lambda) - \lambda H_\alpha$ we have $\lambda H_\alpha = V_\alpha(1+\lambda) = -\lambda G_\alpha(1+\lambda)$. Hence

$$
\alpha'(F) = -\frac{V_F}{G_\alpha} = -\frac{V_F}{-H_\alpha/(1+\lambda)} = (1+\lambda)\frac{V_F}{H_\alpha} < 0
$$

where the inequality follows from $H_\alpha > 0$ and $V_F < 0$. This completes the proof.

**Proof of Proposition 5**

From the formula (11) in the proof of Propositions 1-2 we obtain the derivative $\alpha'_M(\phi)$ for $\beta = 1$, and $\alpha'_N(\phi)$ for $\beta > 1$. Using that formula we can write

$$
\begin{aligned}
&\alpha'_N(\phi)D_2 \\
={}& \beta\pi\left\{c_{pp}\left[\mu - 1 - \delta\mu(1-p)\right]p + \left[(1/\beta - 1) + (1-\mu)(2-\delta) + \mu\delta(1-2p)\right](-c_p)\right\}F_\phi \\
={}& \beta\pi\left\{c_{pp}\left[\mu - 1 - \delta\mu(1-p)\right]p + \left[(1-\mu)(2-\delta) + \mu\delta(1-2p)\right](-c_p)\right\}F_\phi \\
&+ \beta\pi(1/\beta - 1)(-c_p)F_\phi \\
={}& \beta\alpha'_M(\phi)D_2 + \pi(1-\beta)(-c_p)F_\phi
\end{aligned}
$$

Hence

$$\alpha'_N(\phi) = \alpha'_M(\phi) - \frac{\pi}{\beta D_2}(\beta - 1)(-c_p)F_\phi \qquad \text{(CMN)}$$

This shows that, all else equal, $\alpha'_N(\phi) < \alpha'_M(\phi)$, and completes the proof.