Policy-Makers, the International Community and People Living with HIV: The Need for New Commitment Mechanisms+

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Abstract

In order to determine the optimal allocation of responsibilities in disease interventions, and in designing commitment mechanisms, the paper develops a three-period game comprising policy-makers, the international community providing financial aid, and individuals. A policy-maker chooses, in period 1, a fraction of funds to be allocated to disease prevention, and the remaining fraction is allocated to disease treatment. The policy-maker chooses additional funds provision in period 2 for disease treatment. The international community chooses funding in period 3 for disease treatment. Persons engage in risky versus safe behavior which may or may not cause disease contraction. When the international community funds, the policy-maker free rides by not funding additionally. We determine which factors impact how the policy-maker allocates funding between disease prevention and treatment. If the policy-maker funds substantially, the international community free rides by funding less. We quantify how more allocation of funds by the policy-maker to disease prevention causes lower disease contraction probability and higher probability that a person remains sick or dies, and how the international community's funding impacts these two probabilities. We derive seven assertions from the properties of the model. The model is also tested against empirical data on Africa. The results show consistency between the theoretical model and empirical estimates.

Keywords: Disease, policy, game, funding, prevention, treatment, resource allocation, free riding, risky behavior.

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1 Introduction

In 2013 more than 35 million people lived with HIV/AIDS worldwide.[1] Since its outbreak, 78 million people have become infected with HIV and 39 million people have died worldwide, of which 1.5 million died in 2013 alone. The situation is particularly acute in Africa where in 2013, 24.7 million were living with HIV, amounting to 78% of worldwide HIV infections. In sub-Saharan Africa 1.1 million people died from HIV-related illnesses in 2013. With more prevention measures, this number would be lower. But, with adequate treatment many can today live successfully with diseases such as HIV/AIDS.

This papers tries to answer a few questions, as a way to get to some policy solutions: Why do HIV-countries free-ride on their responsibility to treat their citizens? Why do some donors prefer to focus on funding preventions when treatment is also a form of prevention? Why do those who potentially could contract HIV behave the way they do in the face of a deadly disease? To what extent is the amount of funding committed consequential in changing individual behavior? Are the current mechanisms in place to make sure both policy-makers in affected-countries and donors commit to funding both treatment and prevention of HIV?

In order to answer these questions and get to some solutions, the paper uses a game-theoretic approach involving policy-makers, international donors, and people living with HIV. The behavior of the parties is driven by reactions to each other's actions, which results in outcomes that the questions above imply. The paper goes to the heart of the need to create more binding commitment-mechanisms to counter free-riding by affected countries and also donors, in the financing of HIV. The paper also fleshes out the behavioral incentives for those living with HIV and those who could contract it, pointing to the need for to meet their side of the bargain.

The paper assumes that policy-makers and donors have accepted the *duty of rescue* and the people also have a *duty to respond* to rescue efforts (see Collier et al. (2015)). Collier et al. (2015) discuss the implications of the duty of the rescue principle on treatment and prevention choices, and the size of lifecycle financial liabilities for disease intervention and its fiscal implications. Other approaches, such as *accountability for reasonableness*, lend support to prioritization decisions by policy-makers and donors in their interventions (see Daniels and Sabin (2014)). Accountability for reasonableness seeks to establish a framework for prioritization and fair allocation of responsibilities in decision-making, unlike considerations of cost-effectiveness and comparative effectiveness research.

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[1] http://www.amfar.org/about-hiv-and-aids/facts-and-stats/statistics--worldwide/, retrieved April 22, 2015.

In this regard, we develop a three-period game-theoretic model under conditions of perfect information. First a policy-maker associated with the government in a country allocates funds between disease prevention and disease treatment. The fraction of funds allocated to disease prevention impacts whether persons contract the disease. Second, the policy-maker allocates the remaining funds, and possibly additional funds, to disease treatment. Third, the international community[2] determines the extent of funding disease treatment. The adverse effects of disease are observed after disease contraction which may cause substantial media attention and raise the interest of the international community. We focus on that part of the international community that steps in with funding of disease treatment in such cases. Funding of disease prevention by external actors is not analyzed in this paper.

From the model we are able to generate seven (7) outcomes/assertions, which are in line with the questions we posed above. *First*, when the policy-maker has limited funding, the international community is more likely to contribute funds, the country has a large population, the disease contraction probability is large, and when the probability that person remains sick or dies is large. *Second*, when the international community chooses to provide resources, then the policy-maker will free ride. *Third*, when a policy-maker is faced with limited resources, the country should focus on disease prevention rather than treatment activities.

Fourth, the policy-maker's resource allocation to disease prevention increases in the utility of getting sick or dying, increases in the utility of not contracting the disease, and increases in the utility difference between no disease and recovery. *Fifth*, if the policy-maker has substantial available resources, then little need exists for the international community to provide funding, and the international community free rides. *Sixth*, the probability of contracting the disease falls to its minimum when the policy-maker allocates all his resources to disease prevention, and reaches a maximum when the policy-maker allocates no resources to disease prevention.

Finally, the probability of a person remaining sick or dying depends on the proportion of resources allocated to prevention versus treatment. More funding by the international community generally results in a lower probability that a person remains sick or dies. These outcomes of the model/assertions imply that the three parties are in reality involved in a game with perfect information, and react to each others actions in a way that results in undesirable outcomes.

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[2] We use the word international community to cover private and public financial donors, which also can come from within the given country from actors not associated with the country's policy maker.

From the assertions above, which are outcomes of the model, the paper argues for the need to create commitment-mechanisms to ensure that free-riding by both countries and donors is avoided. Although a cooperative game with binding agreements between the players would be desirable, such a game is hard to implement and sustain. Hence in this paper we develop the more fundamental non-cooperative game, with no binding agreements between the players, to illustrate the dilemmas. The paper also argues for commitment to funding both prevention and treatment, by policy-makers and donors. Without these mechanisms the game will result in countries with limited resources only focusing on prevention, which is not desirable. The model also shows why more funding is needed, and how that can reduce the probability of disease contraction, and death from the disease.

The paper also goes to the heart of fiscal sustainability and debt sustainability issues. If, for example, donors free-ride, how could a country meet its future unfunded debt from HIV liabilities. The fiscal stability of any country, with or without resources is threatened.

The model is tested using HIV data for 43 African countries. First, we tested the model's prediction capacity. We find that, for example, the correlation between the model-prediction on HIV-related deaths and actual deaths rates due to the HIV, is 66%, and the correlation with funding by the international community is 92%.

With the advent of antiretroviral treatment (ART), HIV is no longer a death sentence but a chronic condition, with almost 13 million people in low and middle income countries now receiving ART. While the trajectory of the HIV epidemic has begun to change with declining number of new infections and mortality levels, the cost trajectory has continued upwards, driven by lifetime treatment needs, longer living cohorts of persons receiving treatment, expanded treatment guidelines, and rising prevention costs in HIV-affected countries which have expanding populations. The global resource need, which was US$ 3.8 billion in 2002, was US$19.1 billion in 2013[1]. Recent estimates in Atun et al. (2015)) point to about US$22-24 billion being needed per annum, to fund HIV intervention programs.

The economic, social and health benefits of HIV investment have been well researched. Increased labor productivity, reduced orphan care costs, deferred treatment and end-of-life care have been estimated to produce substantial economic gains. Similarly, expanded services have been shown to benefit populations by strengthening health systems and releasing system capacity to treat other conditions. Adding to the mix are cross-sectorial benefits and social protection realized with prudent investment in HIV prevention and treatment. Those benefits nevertheless remain tenuous due to a chasm in financing. Prior modeling suggested that annual resource needs could reach US$35 billion by 2030.

While the resource needs are enormous, in many low-income countries, especially in Africa, domestic sources remain very low, with HIV co-financing dependent on external sources. There is also evidence of free-riding by affected countries. However, the donor sources are now being constrained by the fiscal constraints from the global economic crisis. Inefficiencies in channeling and utilization of available funds, also adds to resource constraints. Clearly, there is a need to create mechanisms for commitment to funding by both affected countries and donors in order to avoid 'free-riding' by both parties.

Domestic financing remains constrained in sub-Saharan Africa. In Nigeria, for example, domestic financing accounted for US$123 million in 2014 compared to US$451 million. From external financing for the same year. In Uganda external financing was US$446 million, nine times more than the US$53 million from domestic resources. In Malawi, external financing accounted for 98% of overall resources spent on HIV intervention.

Health policy decisions are usually analyzed "one player at a time", which has various disadvantages associated with sectorial analyses and non-holistic analyses which may not capture phenomena comprehensively. Sectorial analyses may be incorrect when relevant cause-effect relationships are shut out from consideration. In this paper we bring the relevant players together in a game-theoretic approach to account for their different interests in a holistic approach.

Examples of disease prevention are awareness campaigns so that people are knowledgeable and take precautions e.g. by using condoms to avoid HIV contraction, or development of vaccines against disease contraction. Examples of disease treatment are hospital beds and medicines to treat diseases, and ameliorate the adverse effects of diseases, given that disease contraction has occurred.

Resource allocations between prevention and treatment are sometimes seen as being in competition. Literature abounds on decision-making for resource allocation for HIV treatment and prevention, see Paltiel and Stinnett (1998), Marseille et al. (2002), Kumaranayake and Watts (2005), Canning (2006), Alister and Brandeau (2012), Boily et al. (2012), Bärnighausen et al. (2012) and the HIV Modelling Consortium Treatment as Prevention Editorial Writing Group (2012) among others. There is also quite some focus on cost-effectiveness analysis of prevention measures, see Walker (2001), Hogan et al. (2005). Goldie et al. (2006), and Cohen et al. (2005). Coates et al. (2008) focuses on behavioral strategies for reducing transmission of the disease.

Some of the literature has focused on the incentives for resource allocation by corporations to develop drugs for either prevention (vaccinations) and treatment. Private incentives for developing treatment drugs seem far stronger than those for developing vaccines for prevetion, see Thomas (2002), Kremer and Glennerster (2004), Kremer and Snyder(2003), Kremer and Snyder(2015), inter alia. Countries with high disease prevalence may be forced to allocate large budgets to disease treatment which may leave little left for disease prevention. This paper throws light on such resource allocations.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the model. Section 4 compares the model with empirics. Section 5 concludes. The appendix contains some detailed proofs for the model.

2 The model

We consider the three-period complete information game in Fig. 1 with two players and three nodes. The policy-maker in a country has available funds r and allocates in period 1 a fraction p, 0≤p≤1, to prevent disease, where p is a strategic choice variable. Person i in the country acts upon the information generated by the funds pr, in the sense of choosing risky versus safe behavior, and contracts the disease (becomes sick) with probability q, or contracts no disease (remains healthy) with probability 1-q and utility E.

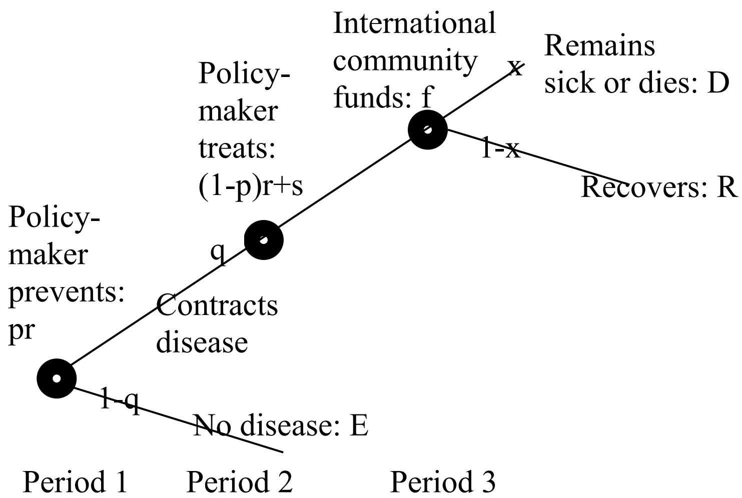

Figure 1: Three-period game for policy-maker, international community, and person i.

We model the disease contraction probability q as probabilistic expressed as

$$q = \frac{\alpha_1}{\beta_1 + pr}, \quad 0 \le \alpha_1 \le \beta_1 > 0 \tag{1}$$

where α_1 and β_1 are country specific parameters dependent on culture, education, employment rate, income, infrastructure, climate, etc. Countries with low α_1 compared with β_1 enjoy low disease contraction probability. In contrast, countries where α_1 approaches β_1 suffer high disease contraction probability. More available funds r and higher allocation fraction p decrease the disease contraction probability, $\partial q / \partial r$<0, $\partial q / \partial p$<0.

In period 2 the policy-maker allocates the remaining resource fraction 1-p, and additional funds s, to treat the disease, where s is a strategic choice variable. With no additional funds, s=0, the policy-maker makes no decision in period 2 since p is determined in period 1. Mathematically, the analysis is the same regardless of whether the decision maker chooses p before s, or r before p, or p and s simultaneously. Observing the policy-maker's decisions in periods 1 and 2, the international community provides additional funding f≥0 in period 3, which is a strategic choice variable. We model the international community in period 3 to cover the common occurrence that the international community postpones acting until funding is potentially imperative. Future research should model cases where the international community acts preventatively. We model the probability x that person i remains sick or dies as

$$x = \frac{\alpha_2}{\beta_2 + (1-p)r + s + f}, \quad 0 \le \alpha_2 \le \beta_2 > 0 \tag{2}$$

where α_2 and β_2 also are country specific parameters. Countries with low α_2 compared with β_2 enjoy low probability x that person i remains sick or dies. Person i receives utility D if he remains sick or dies, and utility R if he recovers, where D<R<E. Summing up, r and the parameters are exogenously given, p, s and f are strategic choices, and q and x are probabilistically determined. Using Fig. 1, person i's utility is

$$U_i = (1-q)E + q\left((1-x)R + xD\right) = E - \frac{\alpha_1}{\beta_1 + pr}\left(E - R + \frac{\alpha_2}{\beta_2 + (1-p)r + s + f}(R-D)\right) \tag{3}$$

Assuming N persons in the country, the policy-maker's utility is

$$u = NU_i - a(r+s) = NE - \frac{\alpha_1}{\beta_1 + pr}N\left(E - R + \frac{\alpha_2}{\beta_2 + (1-p)r + s + f}(R-D)\right) - a(r+s) \tag{4}$$

where a is the unit cost of converting funding into utility for the N persons. Analogously, the international community's utility is

$$v = NU_i - bf = NE - \frac{\alpha_1}{\beta_1 + pr} N\left(E - R + \frac{\alpha_2}{\beta_2 + (1-p)r + s + f}(R-D) \right) - bf \tag{5}$$

where b is the unit cost of converting funding into utility for the N persons. Summing up, the policy-maker chooses one strategy p in period 1, and one strategy s in period 2. The international community chooses one strategy f in period 3. Summing up, the model has 11 parameters $N,r,b,\alpha_1,\beta_1,\alpha_2,\beta_2,a,D,R,E$ which are common knowledge for all players. The disease contraction probability q depends on persons' behaviors which are parameterized to depend on the three parameters r,α_1,β_1, and on the policy-maker's fraction p allocated to disease prevention. The game has three periods and two players. The policy-maker has two free choice variables p and r. That is, he chooses the fraction p of his funds in period 1 to allocate to disease prevention, and the remaining fraction 1-p is allocated to disease treatment. The policy-maker provides additional funds s in period 2 to treat the disease. The international community has one free choice variable which is to choose funding f in period 3.

3 Decisions and strategies by various players

In this section we draw conclusions from the model on how the players in the game prioritize their strategies.

Assertion 1: International community: *The international community is more likely to contribute funds when the policy-maker has limited funds, the country has a large population, the disease contraction probability is large, and the probability that a person remains sick or dies is large (see Property 1).*

Property 1: The international community contributes (f>0) and does not contribute (f=0) when

$$f \begin{cases} > 0 \; if \; \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} > \beta_1 + \beta_2 + r - \dfrac{\alpha_2(R-D)}{2(E-R)} \\ = 0 \; if \; \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} \le \beta_1 + \beta_2 + r - \dfrac{\alpha_2(R-D)}{2(E-R)} \end{cases} \tag{6}$$

Proof: Follows from f>0 in Appendix A.

First, (6) is more likely satisfied when β_1 and r are small and α_1 is large causing the disease contraction probability q in (1) to be large. Second, (6) is more likely satisfied when β_2 is small causing the probability x in (32) that person i remains sick or dies to be large. Third, (6) is more likely satisfied when N is large causing the probability x in (32) that person i remains sick or dies to be large. That is, the international community is more likely to contribute funds when the policy-maker has limited funds r, the country has many persons N, the disease contraction probability q is large, and the probability x that person i remains sick or dies is large.

Assertion 2: Policy-maker: *The policy-maker will free ride on the international community if the international community chooses to provide resources (see Property 2a).*

Property 2a: The policy-maker's disease prevention p and funds provision s are

$$p = \begin{cases} 1 \; if \; \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} > Max\left\{\beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)}, \beta_1+r\right\} \\ \dfrac{1}{r}\left(\dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}-\beta_1\right) if \; Max\left\{\beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)}, \beta_1\right\} < \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} \le \beta_1+r \\ 0 \; if \; \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} > \beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)} \; and \; \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} \le \beta_1 \\ p_t \; if \; \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} \le \beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)} \end{cases}$$

$$s = \begin{cases} 0 \; if \; \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} > \beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)} \\ s_t \; if \; \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} \le \beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)} \end{cases} \tag{7}$$

where p_t and s_t are the fraction of the funds allocated to disease prevention and the additional funds, respectively, provided by the policy-maker when the international community does not fund.

Proof: Appendices A and B.

When the international community provides funds (Property 1 is satisfied), Property 2a states that the policy-maker chooses s=0 which means free riding on the international community's contribution in the choice of f. This result follows for two reasons. First, the policy-maker chooses s in period 2 whereas the international community chooses f thereafter in period 3. Second, the policy-maker and international community have the same benefit NU_i in their utilities in (4) and (5), but different costs, a(r+s) and bf respectively. The costs bf incurred by the international community are irrelevant for the policy-maker regarding choosing s (but not irrelevant regarding choosing p), but the benefits NU_i are not since they coincide with the policy-maker's benefits in (4). Hence the policy-maker can enjoy the same benefits as the international community without incurring the costs of generating the benefits. Consequently the policy-maker chooses no additional funds s, i.e. s=0, knowing that the international community will step up to the plate thereafter and provide funds f.

<u>Assertion 3: Prevention decision</u>: *A policy-maker with limited resources should focus on disease prevention rather than treatment activities (see Property 2a).*

In Property 2a the policy-maker's choice of fraction p to prevent disease is a number between 0 and 1, i.e. $0 \leq p \leq 1$. Nothing goes to disease prevention, p=0, if $\frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} \leq \beta_1$ (third line in (7)), i.e. if the disease contraction probability q is low (consistent with low α_1/β_1), there are few persons in the country (N is low), the probability x that person i remains sick or dies is high (consistent with high α_2), the international community's unit cost b of funds provision is high (causing low f and high x), the utility E of no disease marginally outweighs the utility R of recovery (E-R is low), or the utility R of recovery substantially outweighs the utility D of remaining sick or dying (R-D is large). Conversely, the policy-maker allocates all funds, p=1, to disease prevention if $\frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} > \beta_1 + r$ (first line in (7)), i.e. if all the above relationships are reversed and, additionally, the policy-maker has limited funds (r is low). This last result is remarkable. A policy-maker with limited funds should focus on disease prevention rather than treatment.

When the international community does not provide funds (Property 1 is not satisfied), so that period 3 is eliminated, Property 2a states that the policy-maker chooses fraction p_t and additional funds s_t. Appendix B shows the first order conditions which give a third order equation in p. The analytical expressions for p_t and s_t are omitted due to space considerations, since they are analytically cumbersome to interpret, and since the two-period game is not particularly interesting. That is, the fraction p allocated to disease prevention in the three-period model is the interesting result, and additional funds are what the international community provides in period 3 expressed as f, with utility v in (5) equivalent to u in (4) aside from cost bf instead of a(r+s).

Assertion 4: Prevention resources: *The policy-maker's resource allocation to disease prevention increases in the utility of getting sick or dying, increases in the utility of not contracting the disease, and increases in the utility difference between no disease and recovery (see Property 2b).*

Property 2b. For the interior solution in (7), if $Max\left\{\beta_1+\beta_2+r-\frac{\alpha_2(R-D)}{2(E-R)},\beta_1\right\}<\frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}\le\beta_1+r$, $\partial p/\partial D>0$, $\partial p/\partial E>0$, $\partial p/\partial(E-R)>0$, $\partial p/\partial(R-D)<0$, $\partial p/\partial N>0$, $\partial p/\partial r<0$, $\partial p/\partial b<0$, $\partial p/\partial\alpha_1>0$, $\partial p/\partial\alpha_2<0$, $\partial p/\partial\beta_1<0$.

Proof. Follows from (7).

Property 2b states that the policy-maker's resource fraction p allocated to disease prevention (the remaining fraction is allocated to disease treatment) increases in the utility D of getting sick or dying, increases in the utility E of not contracting the disease, and increases in the utility difference E-R between no disease and recovery, where $(E\text{-}R)^2$ is squared in (7) acknowledging the benefit from a disease prevention viewpoint that persons do not contract to disease. In contrast, the fraction p allocated to disease prevention decreases in the utility difference R-D between recovery on the one hand and getting sick or dying on the other hand. That is, from a disease prevention viewpoint, it is not preferable that the recovery utility R exceeds the getting-sick-or-dying utility D which in contrast is preferable from a disease treatment viewpoint. Furthermore, the fraction p increases in the number N of persons, decreases in the available funds r and the international community's unit cost b of converting funding into utility, increases in the disease contraction probability q (increasing α_1 and decreasing β_1 in (1)), and decreases in α_2 which increases the probability x in (1) that person i remains sick or dies.

<u>Assertion 5: International community vs policy-maker</u>: *If the policy-maker has substantial available resources, then little need exists for the international community to provide funding, and the international community free rides (see Properties 3a and 3b).*

Property 3a. The international community's contribution is

$$f=\begin{cases}\frac{\sqrt{\alpha_1\alpha_2 N(R-D)}}{\sqrt{b(\beta_1+r)}}-\beta_2 \text{ if } \frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}>Max\left\{\beta_1+\beta_2+r-\frac{\alpha_2(R-D)}{2(E-R)},\beta_1+r\right\}\\ \frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}+\frac{\alpha_2(R-D)}{2(E-R)}-\beta_1-\beta_2-r \text{ if } Max\left\{\beta_1+\beta_2+r-\frac{\alpha_2(R-D)}{2(E-R)},\beta_1\right\}<\frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}\le\beta_1+r\\ \frac{\sqrt{\alpha_1\alpha_2 N(R-D)}}{\sqrt{b\beta_1}}-\beta_2-r \text{ if } \frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}>\beta_1+\beta_2+r-\frac{\alpha_2(R-D)}{2(E-R)} \text{ and } \frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}\le\beta_1\\ 0 \text{ if } \frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}\le\beta_1+\beta_2+r-\frac{\alpha_2(R-D)}{2(E-R)}\end{cases} \quad (8)$$

Proof: Appendix A.

Property 3a gives four possible solutions for the international community's choice of funding f. First, no funding f=0 is provided if Property 1 is not satisfied. Second and third, two corner solutions exist, corresponding to p=0 and p=1 in (7). The two corner solutions for f have similar analytical expressions except for r which can enable both corner solutions to be larger than the other. Fourth, an interior solution for f exists corresponding to the interior solution for p, considered in Property 3b.

Property 3b. For the interior solution in (8), if $Max\left\{\beta_1+\beta_2+r-\frac{\alpha_2(R-D)}{2(E-R)},\beta_1\right\}<\frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}\le\beta_1+r$, $\partial f/\partial N>0$, $\partial f/\partial r<0$, $\partial f/\partial b<0$, $\partial f/\partial\alpha_1>0$, $\partial f/\partial\beta_1<0$, $\partial f/\partial\beta_2<0$. For both corner solutions in (8), when p=1 (first line) and p=0 (third line), $\partial f/\partial\alpha_2>0$ and $\partial f/\partial(R-D)>0$.

Proof. Follows from (8).

The derivatives of f in Property 3b have the same signs as the derivatives of p in Property 2b with respect to N,r,b,α_1,β_1, though differentiation of f with respect to the utilities D, R, E, and α_2 gives unclear signs and are omitted, and $\partial f / \partial \beta_2 < 0$. The substantial alignment of f and p follows from the common terms $\frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} - \beta_1$ for the interior solutions in (7) and (8), which means that $f = \frac{\alpha_2(R-D)}{2(E-R)} - \beta_2 - (1-p)r$ as in (A3), which again follows from the aligned interests of the policy-maker and the international community as expressed in the utilities u in (4) and v in (5). The result $\partial f / \partial r < 0$ means that the international community free rides on the policy-maker. That is, if the policy-maker has substantial available funds r, then limited need exists for the international community to provide funding f. Although the signs of the derivatives of f and p are aligned for N,r,b,α_1,β_1, the derivatives $\partial f / \partial \alpha_2 > 0$ and $\partial f / \partial (R-D) > 0$ for the two corner solutions (when p=1 and p=0) in Property 3b are positive while $\partial p / \partial \alpha_2 < 0$ and $\partial p / \partial (R-D) < 0$ are negative for the interior solution in Property 2b. The utility difference R-D expresses the utility of recovery relative to the utility D of remaining sick or dying. These opposite results follow from the denominator in (2) where f and p have opposite impact on the probability x that person i remains sick or dies. That is, higher fraction p on disease prevention gives lower fraction 1-p on disease treatment in (2) where also funding f by the international community contributes disease treatment to decrease the probability x that person i remains sick or dies.

<u>Assertion 6: Disease contraction probability</u>: *The probability of contracting the disease falls to its minimum when the policy-maker allocates all his resources to disease prevention, and reaches a maximum when the policy-maker allocates no resources to disease prevention (see Properties 4a and 4b).*

Property 4a. The disease contraction probability q is

$$q=\begin{cases}\dfrac{\alpha_1}{\beta_1+r} \; if \; \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} > Max\left\{\beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)}, \beta_1+r\right\} \\ \dfrac{b\alpha_2(R-D)}{4N(E-R)^2} \; if \; Max\left\{\beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)}, \beta_1\right\} < \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} \le \beta_1+r \\ \dfrac{\alpha_1}{\beta_1} \; if \; \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} > \beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)} \; and \; \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} \le \beta_1 \\ \dfrac{\alpha_1}{\beta_1+p_t r} \; if \; \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} \le \beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)} \end{cases} \quad (9)$$

Proof: Follows from inserting (7) into (1).

The disease contraction probability reaches its minimum when the policy-maker allocates all his funds r to disease prevention (p=1), and is maximum when the policy-maker allocates no funds r to disease prevention (p=0). In Property 4a the first line in (9) corresponds to maximum fraction p=1 allocated to disease prevention which gives minimum disease contraction probability $q=\alpha_1/(\beta_1+r)$ where all available funds r are allocated to disease prevention. In contrast, the third line in (9) corresponds to minimum fraction p=0 allocated to disease prevention which gives maximum disease contraction probability $q=\alpha_1/\beta_1$ where no funds r are allocated to disease prevention, and all funds instead are allocated to disease prevention. The ratio of these two probabilities is $\beta_1/(\beta_1+r)$.

Property 4b. For the interior solution in (9), if $Max\left\{\beta_1+\beta_2+r-\frac{\alpha_2(R-D)}{2(E-R)}, \beta_1\right\} < \frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)} \le \beta_1+r$, $\partial q/\partial D<0$, $\partial q/\partial E<0$, $\partial q/\partial(E-R)<0$, $\partial q/\partial(R-D)>0$, $\partial q/\partial N<0$, $\partial q/\partial b>0$, $\partial q/\partial \alpha_2>0$.

Proof. Follows from (9).

The simple inverse dependence of the disease contraction probability q in (1) on the fraction p of funds allocated by the policy-maker to disease prevention implies that the signs of all the derivatives in Property 2b for p gets reversed for q, and the derivatives with respect to α_1, β_1 and r are omitted since α_1, β_1 and r are not present in (9), though they are present in (1). The intuition is that a larger fraction p allocated to disease prevention gives lower disease contraction probability q.

<u>Assertion 7: Probability of remaining sick or dying:</u> *The probability of a person remaining sick or dying depends on the fractions of resources allocated to prevention versus treatment. More funding by the international community gives lower probability that a person remains sick or dies (see Properties 5a and 5b).*

Property 5a. The probability x that person i remains sick or dies is

$$x=\begin{cases}\dfrac{\sqrt{b\alpha_2(\beta_1+r)}}{\sqrt{\alpha_1 N(R-D)}}\ \text{if}\ \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}>Max\left\{\beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)},\beta_1+r\right\}\\ \dfrac{2(E-R)}{(R-D)}\ \text{if}\ Max\left\{\beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)},\beta_1\right\}<\dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}\le\beta_1+r\\ \dfrac{\sqrt{b\alpha_2\beta_1}}{\sqrt{\alpha_1 N(R-D)}}\ \text{if}\ \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}>\beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)}\ \text{and}\ \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}\le\beta_1\\ \dfrac{\alpha_2}{\beta_2+r-pr}\ \text{if}\ \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}\le\beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)}\end{cases}\tag{10}$$

Proof: Follows from inserting (7) and (8) into (2).

In Property 5a the first line in (10) corresponds to maximum fraction p=1 allocated to disease prevention which gives maximum probability x that person i remains sick or dies. In contrast, the third line in (9) corresponds to minimum fraction p=0 which gives minimum probability x that person i remains sick or dies since all funds r are allocated to disease prevention. The ratio of these two probabilities is $\sqrt{(\beta_1+r)/\beta_1}$.

Property 5b. For the interior solution in (10), if $Max\left\{\beta_1+\beta_2+r-\dfrac{\alpha_2(R-D)}{2(E-R)},\beta_1\right\}<\dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}\le\beta_1+r$, $\partial x/\partial D>0$, $\partial x/\partial E>0$, $\partial x/\partial(E-R)>0$, $\partial x/\partial(R-D)<0$. For both corner solutions in (10), when p=1 (first line) and p=0 (third line), $\partial x/\partial N<0$, $\partial x/\partial b>0$, $\partial x/\partial\alpha_1<0$, $\partial x/\partial\beta_1>0$, $\partial x/\partial\beta_2>0$. When p=1 (first line in (10)), $\partial x/\partial r>0$.

Proof. Follows from (10).

The signs of the derivatives in Property 5b for x with respect to the utilities D, E, E-R, and R-D are the same as for p in Property 2b. That is, larger fraction p to disease prevention gives smaller fraction 1-p to disease treatment which gives larger probability x that person i remains sick or dies. For both corner solutions in (10), the signs of the derivatives in Property 5b for x with respect to N , b , α_1 , β_1, β_2 are the opposite of the signs of the derivatives in Property 3b for f with respect to the same derivatives. Also, when p=1, $\partial x / \partial r > 0$ in contrast to $\partial f / \partial r < 0$. These results follow since f appears in the denominator in the expression for x in (2). That is, more funding f by the international community gives lower probability x that person i remains sick or dies. In contrast, for the corner solution when p=1, more funds r provided to disease prevention gives higher probability x that person i remains sick or dies.

4 Comparing the model with empirics

In this section we test aspects of the model against data from Africa, the region that is most affected by HIV and accounts for 78% of all people living with HIV. First we classify countries according to various characteristics. Second we run regressions in order to determine which characteristics matter in disease contraction.

4.1 Country classification

Table 1 classifies countries according to the characteristics in (1) that determine the empirically estimated disease contraction probability q_e, i.e. country characteristics α_1 and β_1 (high α_1/β_1 expresses high disease contraction probability), the policy-maker's funds r, and the policy-maker's empirically estimated fraction p_e allocated to disease prevention. Furthermore, the rightmost three columns in Table 1 classifies countries according to the characteristics in (2) that determine the empirically estimated probability x_e that person i remains sick or dies, i.e. country characteristics α_2 and β_2 (high α_2/β_2 expresses high probability that a person remains sick or dies), the policy-maker's funds r, the policy-maker's empirically estimated fraction p allocated to disease prevention, and the international community's empirically estimated funds provision f. Data for the policy-maker's additional funds provision s is not available, and has been omitted, i.e. s_e=0.

Table 1. Characteristics N,r,α_1/β_1 of 43 countries and policy-makers, and policy-makers' empirically estimated strategic choice p_e that impact the empirically estimated disease contraction probabilities q_e, where Int means Intermediate and Lo/I means Low/Int. Additionally, α_2/β_2 and the empirically estimated strategic choice f_e that impact the empirically estimated probabilities x_e that person i remains sick or dies.

	N	r	α_1/β_1	p_e	q_e		α_2/β_2	f_e $ mill(%) 2009-2011	x_e (%)
Angola	21256000	High(18.8%)	Low	0.342’	Low		Low	20.45(0%)	0.061
Benin	9742000	Int(15.4%)	Low	0.134’’	Low		Low	27.80(1%)	0.031
Botswana	2096000	High(35.2%)	High	0.096’	High		High	123.14(3%)	0.286
Burkina Faso	17323000	Int(11.5%)	Low	-	Low		Low	35.63(1%)	0.035
Burundi	9023000	Int(17.4%)	Low	0.203’’	Low		Low	26.79(1%)	0.055
Cameroon	20930000	Int(18.3%)	Low	-	Low		Int	22.00(1%)	0.167
Chad	12948000	Int	Low	0.294’’	Low		Int	15.12(0%)	0.108
Congo, Dem Rep	74618000	Int(13.2%)	Low	-	Low		Low	56.44(0%)	0.043
Cote d'Ivoire	23919000	Int(15.2%)	Low	-	Low		Int	80.54(2%)	0.130
Egypt	84605000	Int(15.8%)	Low	-	Low		-	-	-
Equatorial Guinea	1837000	Low(1.7%)	Int	-	Int		Low	1.06(1%)	0.054
Eritrea	4980000		Low	-	Low		Low	15.53(0%)	0.02
Ethiopia	86614000	Int(11.6%)	Low	-	Low		Low	367.59(8%)	0.054
Gabon	2204000	Int(10.3%+Oil)	Lo/I	0.167’’	Low		Low	2.94(0%)	0.091
Gambia	1794000	Int(18.9%)	Low	-	Low		-	6.76(0%)	-
Ghana	26441000	High(20.8%)	Low	0.281’	Low		Low	51.80(0%)	0.045
Guinea	11861000	Low(8.2%)	Low	0.135’’	Low		Low	8.49(0%)	0.042
Guinea-Bissau	1699000	Int(11.5%)	Lo/I	-	Lo/I		Int	6.24(0%)	0.118
Kenya	43291000	Int(18.4%)	Int	0.270’	Int		Int	425.86(10%)	0.132
Lesotho	1887000	High(15%)	High	-	High		High	52.70(1%)	0.795
Liberia	3881000	Int(13%)	Low	0.313’	Low		Low	12.90(0%)	0.052
Madagascar	21852000	Int(10.7%)	Low	0.515’’	Low		Low	10.15(0%)	0.027
Malawi	15316000	High(20.7%)	High	0.113’	High		High	146.23(3%)	0.300
Mali	16678000	Int(15.3%)	Low	-	Low		Low	22.04(1%)	0.030
Mauritania	3461000	Int(12.9%)	Low	0.144’’	Low		-	0.61(0%)	-
Mauritius	1273000	Int(19%)	Low	-	Low		-	1.58(0%)	-
Morocco	32950000	Int(13.4%)	Low	-	Low		Low	-	0.003
Mozambique	24491000	High(22.3%)	High	0.422’’	High		High	240.32(5%)	0.314
Namibia	2170000	High(28.8%)	High	-	High		High	114.22(3%)	0.230
Niger	17493000	Int(11%)	Low	0.421’’	Low		Low	11.52(0%)	0.017
Nigeria	177096000	Low(6.1%)	Low	-	Low		Int	401.22(9%)	0.136
Rwanda	10780000	Int(14.1%)	Low	-	Low		Low	187.99(4%)	0.056
São Tomé and Príncipe	194000	Int(17.4%)	Low	0.046’’	Low		-	0.30(0%)	-
Senegal	13567000	Int(19.2%)	Low	0.383’	Low		Low	25.34(1%)	0.015
Sierra Leone	5823000	Lo/I(10.5%)	Low	-	Low		Low	17.83(0%)	0.052
South Africa	52982000	High(26.9%)	High	-	High		High	595.11(14%)	0.453
Swaziland	1077000	High(39.8%)	High	-	High		High	50.58(1%)	0.557
Tanzania	45950000	Int(12%)	Int	-	Int		Int	341.80(8%)	0.174

Togo	6675000	Int(15.5%)	Low	0.257’’	Low		Int	14.20(0%)	0.105
Tunisia	10889000	Int(14.9%)	-	-	-		-	-	-
Uganda	35363000	Int(16.1%)	High	-	High		Int	284.60(7%)	0.178
Zambia	14129000	Int(16.1%)	High	-	High		High	255.15(6%)	0.212
Zimbabwe	13098000	High(49.3%)	High	0.152’’	High		High	98.95(2%)	0.298

Notes: r is tax revenues as % of GDP. Low is 0-10%, Intermediate is 10.1-20%, and High is over 20%; ’ and ’’ denote 2011 and 2012 figures, respectively. - means data is not available. Figures for donor funding f are in US$ mill and percentage of total donor funding is in parenthesis. Figures for probability of remaining sick or dying x are in % probability; The ranges for α_2/β_2 are Low(less than 0.1%), Intermediate(between 0.0%-2%) and High(above 2%).

From Table 1 we see that countries with high resource mobilization, as measured by tax revenues to GDP ratio, r, are Angola, Botswana, Ghana, Lesotho, Mozambique, Namibia, South Africa, Swaziland and Zimbabwe. The bulk of the countries are in the intermediate stage of tax resource mobilization capacity. The countries with higher levels of tax mobilization capacity also have a higher HIV prevalence rate. These are also the countries with the highest levels of HIV contraction probability q_e. This perhaps means that those countries with no extractive resource endowment, such as Swaziland and to some extent Lesotho, have no more room to raise taxes and require innovative finance solutions or indeed further external aid.

The probability of remaining sick or dying from HIV-related illness is highest (above 0.2%) in Botswana, Lesotho, South Africa, Swaziland, Namibia, Mozambique, Malawi, Zambia and Zimbabwe. This pattern corresponds to the probability of contracting the disease. The rest of the countries have moderate probability of death from the disease.

Looking at resource allocation between prevention and treatment, countries that are allocating more to prevention, i.e. high p_e , are Angola, Chad, Ghana, Kenya, Liberia, Madagascar, Togo, Senegal, Mozambique, and Niger. Data on the split in resources between treatment and prevention of HIV is scarce.

Regarding external support from donors, Table 1 shows how they are contributing in each country. Contributions would be driven by many factors but one of them would be disease prevalence and capacity to manage the use of the resources. Countries with high and intermediate disease contracting probability receive a higher than average quantity of external aid.

4.2 Model predictions using calibration

Table 1 has estimated $N,r,\alpha_1/\beta_1,\alpha_2/\beta_2$, which is a first step towards getting an empirical grip on the theoretical model. We additionally need $\alpha_1,\beta_1,\alpha_2,\beta_2$ separately. Since α_1/β_1 and α_2/β_2 are only ordinally available, and given the scarcity of data, we proceed heuristically, thus illustrating how the model can be used also when data is scarce. From the denominator in (1) we assume that β_1 and $p_e r$ are of the same order, and set $\beta_1 \approx p_e r$ as determined by Table 1. Further, we estimate α_1/β_1=Low=1/3, α_1/β_1=Int=2/3, α_1/β_1=High=1. Analogously, from the denominator in (2) we assume that β_2 and $(1-p)r+s_e+f_e$ are of the same order, and set $\beta_2 \approx (1-p)r+s+f_e$ where we set $s_e=0$ due to lack of data, and f_e is given by Table 1, and α_2/β_2=Low=1/3, α_2/β_2=Int=2/3, α_2/β_2=High=1.

The international community's unit cost of converting funding into utility for the N persons, was estimated using the method of least squares and found to be b=182.[3] We do not estimate the policy-maker's unit cost *a* of converting funding into utility for the N persons since it plays no role in the analytical results for the strategic choice variables. The parameter *a* is only present in the policy-maker's utility in (4). We estimate the utility D=0 for remaining sick or dying, utility R=1 for recovery, and utility E=3 for no disease (remaining healthy). Table 2 follows from inserting these 11 empirically estimated parameters $N,r,b,\alpha_1,\beta_1,\alpha_2,\beta_2,a,D,R,E$ into (7)-(10) to predict theoretically the strategic choices p, s and f of the policy-maker and international community, and how they impact the disease contraction probability q and the probability x that person i remains sick or dies.

Table 2 shows the predicted values for f, x, q, and p using (7)-(10). The value for p is 1, except for Botswana where it is p_t=0.1 as specified in (7). Hence all countries except Botswana are predicted to allocate all their funding to prevention activities. The value for s is 0, except for Botswana where it is s_t= \$40 million (10% of all HIV funding) as specified in (7). Hence all countries except Botswana are predicted to free-ride in the funding of treatment (s=0).

Table 2. Using the 11 empirically estimated parameters $N,r,b,\alpha_1,\beta_1,\alpha_2,\beta_2,a,D,R,E$ to predict theoretically the strategic choices p, s and f impacting q and x.

Country	f	p	s	q	X
Angola	26.432	1	0	0.0849	0.0292
Benin	11.381	1	0	0.0394	0.0677
Botswana	23.146	0.1	40	0.0876	0.3567
Burkina Faso	-	-	-	-	-
Burundi	13.182	1	0	0.0562	0.057

[3] We estimate using the 18 rows in Table 1 where the empirics is complete for p_e,q_e,f_e,x_e, inserted into (5) to give 18 empirical utilities $v_{e1},v_{e2},\ldots,v_{e18}$ as functions of b for the international community. Inserting our model predictions of p,q,f,x in (7)-(10) into (5) gives 18 theoretical utilities $v_1, v_2,\ldots,v_{18}$ based on the model for the international community's utility, which also are functions of b. The parameter b is estimated to minimize the sum $(v_{e1}-v_1)^2+(v_{e2}-v_2)^2+\ldots+(v_{e18}-v_{18})^2$.

Cameroon	-	-	-	-	-
Chad	-	-	-	-	-
DRC	-	-	-	-	-
CIV	-	-	-	-	-
Egypt	-	-	-	-	-
EQ Guinea	-	-	-	-	-
Eritrea	-	-	-	-	-
Ethiopia	-	-	-	-	-
Gabon	2.541	1	0	0.0477	0.0522
Gambia	-	-	-	-	-
Ghana	39.257	1	0	0.0731	0.0423
Guinea	7.893	1	0	0.0396	0.034
Guinea-Bissau	-	-	-	-	-
Kenya	268.992	1	0	0.1417	0.0927
Lesotho	-	-	-	-	-
Liberia	7.288	1	0	0.0795	0.0507
Madagascar	20.987	1	0	0.1133	0.0162
Malawi	97.38	1	0	0.1015	0.1312
Mali	-	-	-	-	-
Mauritania	1.589	1	0	0.0420	0.0213
Mauritius	-	-	-	-	-
Morocco	-	-	-	-	-
Mozambique	286.921	1	0	0.2967	0.0779
Namibia	-	-	-	-	-
Niger	18.32	1	0	0.0988	0.0206
Nigeria	-	-	-	-	-
Rwanda	-	-	-	-	-
São Tomé and Principe	0.101	1	0	0.0147	0.0876
Senegal	22.266	1	0	0.0923	0.0363
Sierra Leone	-	-	-	-	-
South Africa	-	-	-	-	-
Swaziland	-	-	-	-	-
Tanzania	-	-	-	-	-
Togo	14.206	1	0	0.0682	0.0627
Tunisia	-	-	-	-	-
Uganda	-	-	-	-	-
Zambia	-	-	-	-	-
Zimbabwe	89.237	1	0	0.1319	0.105

Notes: f and s are in US$ mill.

The model was tested for its predictive capacity, at least in predicting the trend. Correlations were calculated between the predicted variables f, x, and p in Table 2, and the empirical values

f_e, x_e, and p_e in Table 1. The correlations are shown in Table 3. The model shows especially strong predictive power for funding allocation f from the international community, and also strong predictive power for the probability q of contracting the disease and the probability x of remaining sick or dying from the disease.

Table 3. Correlations between predictions of f, x, and q, and the empirical f_e, x_e, and q_e.

	f	x	q
f_e	0.92		
x_e		0.657	
q_e			0.661

4.3 Testing model outcomes (assertions) using regression analysis

In order to support and test the outcomes and assertions of the model with respect to expressions for f, p, s, x, q in equations (7)-(10), we try to determine using econometric analysis, as to whether the determinants of the equations make sense.

Equation (9) gives the expression for q, the probability of contracting the disease (incidence). We wish to determine which country characteristics matter, and we estimated a regression equation of HIV incidence (probability of contracting HIV), against country characteristics variables such as GDP per capita, literacy, Voice & accountability indicators, government effectiveness, inequality, and poverty levels. GDP per capita is expected to have a negative relationship with HIV incidence in the sense that poorer countries with low GDP per capita are likely to have weaker health delivery systems and therefore higher levels of HIV incidence. Literacy levels, particularly higher education, is also expected to have a negative relationship with HIV incidence due to the fact that education and campaigns on prevention measures is likely to be more effective in countries with higher literacy levels. Voice & accountability indicators are expected to have a negative relationship with prevalence due to the fact that a higher level of freedom of expression is a good medium for prevention campaigns. On accountability, the higher this indicator the more likely a country will improve service delivery. Higher general governance effectiveness is expected to have a negative relationship with incidence due to higher quality of service delivery to the population. Inequality levels are also expected to impact prevalence levels. The more unequal the society, the higher the prevalence level. Likewise, higher poverty levels are likely to be associated with a higher prevalence level, as the poor have lower access to medical care and are generally more vulnerable to disease. This is for 43 countries in Africa.[4] The results for a multiple regression are as in Table 4.

Table 4. Testing determinants of HIV incidence q in equation (9)

Variable	Coefficient	Std Error	t-Value	Significance level
Constant (Intercept)	1.012	0.558	1.814**	0.087
GDP per capita(current US$)	0.000	0.000	-0.603	0.555
Voice & Accountability	0.001	0.149	-0.009	0.993
Government Effectiveness	0.082	0.222	0.369	0.716
Adult Literacy Rate (%)	0.008	0.005	1.656**	0.116
International Funding (US$1000)	0.022	0.006	3.656*	0.002
Poverty Head Count at US$1(%)	0.002	0.004	-0.598	0.558
GINI (Inequality)	0.023	0.014	1.698**	0.108

* and ** mean significance at the 5% and 10% levels, respectively.

From Table 4, only Inequality, International spending on HIV, Adult literacy, and Constant are significant. The most significant variable is international funding on HIV by financial donors. The Adult literacy rate is also quite significant and has a positive coefficient. However, this is the opposite of what is expected, but perhaps implies that middle-income countries, typically with a higher adult literacy levels, are exhibiting higher incidence levels due to other factors other than adult literacy. The countries just happen to be middle-income in classification. The inequality measure, i.e. the Gini coefficient, has a positive coefficient which is quite significant. This means that the more unequal the society, the higher the prevalence rate, as there is a large percentage of the population with poor access to health. Furthermore, inequality is quite high in some middle-income countries, which are showing higher levels of HIV incidence.

The coefficient for GDP per capita is not significant, which means that the level of income for the country is no predictor for its level of HIV incidence. Some poor countries have low incidence rates, while some middle-income countries have some of the highest incidence rates in the world, such as Botswana, South Africa and Swaziland. Voice & accountability is not a driver of HIV incidence, as some of the countries with a relatively free environment for public expression exhibit high incidence rates, such as South Africa. Again, government effectiveness, which is a proxy for the quality if heath systems, is shown not to be a factor in driving the

[4] Angola, Benin, Botswana, Burkina Faso, Burundi, Cameroon, Chad, Congo, Democratic Rep., Cote d'Ivoire, Egypt, Equatorial Guinea, Eritrea, Ethiopia, Gabon, Gambia, Ghana, Guinea, Guinea-Bissau, Kenya, Lesotho, Liberia, Madagascar, Malawi, Mali, Mauritania, Mauritius, Morocco, Mozambique, Namibia, Niger, Nigeria, Rwanda, São Tomé and Príncipe, Senegal, Sierra Leone, South Africa, Swaziland, Tanzania, Togo, Tunisia, Uganda, Zambia, Zimbabwe.

incidence rate. Equally, the level of poverty, as measured by headcount ratio, does not explain differentials in the incidence rate.

Next, we consider the expression in equation (10), for the probability of dying, x. Again we ran a regression to determine if indeed variables like the level of prevalence, population size, incidence rate, level of domestic funding and level of international funding were significant drivers of x. The results are in Table 5.

Table 5. Testing determinants of probability of dying from HIV, x in equation (10)

Variable	Coefficient	Std Error	t-Value	Significance level
Constant	0.024	0.12	2.094*	0.048
Prevalence Level	-0.003	0.04	-0.871	0.393
Population Size (N)	3.629E-10	0.000	1.262	0.220
Incidence Rate	0.354	0.046	7.773*	0.000
International Funding	-5.001E-11	0.000	-0.644	0.526
Domestic Funding	-2.921E -10	0.000	-2.038*	0.053

R-Squared= 0.978, Adjusted R-Squared= 0.947; * means significance at the 5% level.

From Table 5 the incidence rate is the most significant determinant of likelihood of dying from HIV. Indeed counties with a high incident rate exhibit a high probability of HIV death. The level of domestic funding also is significant. Here the negative sign shows that countries with low domestic funding seem to have a higher HIV death rate. This may imply that they need to commit more resources to HIV. Population size and the prevalence level do not seem to as highly correlated with HV death rates.

Table 6. Determinants of international funding, f in equation (8)

Variable	coefficient	Std Error	t-value	Significance level
Constant	6822084.868	27527999.14	0.248	0.806
Prevalence(No)	6562456.103	10037639.62	0.654	0.519
Population Size(N)	2.643	0.503	5.244*	0.000
Incidence Rate(No)	21186583.86	115023421.114	-0.184	0.855
Domestic Funding	0.451	0.352	1.282	0.211

R-Squared=0.757; Adjusted R-Squared= 0.572; * means significant at the 5% level.

Next we consider equation (8), for international funding f. From Table 6, the most significant driver of f, is population size (N). Table 5 shows that the higher the population size, the higher the aid flows. This may perhaps mean that the formula for allocating funds by donors relies too heavily on size of the country than the level of disease burden. If this is the case, this may need to be corrected.

Table 7. Determinants of domestic funding, p in equation (7)

Variable	coefficient	Std Error	t-value	Significance level
Constant	7408274.859	14835660.7	-0.499	0.622
Prevalence(No)	11851648.1	4955483.549	2.392*	0.024
Population Size(N)	0.252	0.388	-0.650	0.522
Incidence Rate(No)	97496309.5	59244152.819	-1.646	0.112
International Funding	0.132	0.103	1.282	0.211

R-Squared = 0.642; Adjusted R-Squared= 0.412; * means significant at the 5% level.

In terms of drivers of domestic funding (p), the prevalence rate, followed by the incidence rate, are major factors. This seems to be in line with equation 7 for variable p. This makes sense, as one expects countries with high prevalence and incidence rate that feeds prevalence, to allocate more funding to HIV. The econometric results show that the drivers of the theoretic model do determine the outcomes.

4.4 Policy implications of model outcomes (assertions) and predictions

What are the policy implication of the seven (7) outcomes or assertions of the model and its predictions? This section looks into that.

First, when the policy-maker has limited funding, the international community is more likely to contribute funds, if the country has a large population, the disease contraction probability is large, and when the probability that person remains sick or dies is large. This has largely been the case in countries such as Kenya or Mozambique or Uganda with low resources but high disease burden. However, countries with small populations should not be penalized and receive meager donor resources. For example, Swaziland, which has a high disease burden and incidence rate but has a small population and is resource constrained.

Second, if the international community chooses to provide resources, then the policy-maker will free ride. Free-riding by policy-makers should not be allowed. A few countries in Africa have abdicated on their duty of rescuing their citizens, and delegated HIV intervention to international donors. Country ownership of programs is crucial and also helps build domestic systems for service delivery. Also a need exists to broaden and deepen tax-bases and other revenue resources from affected countries. Therefore, there should be stronger commitment mechanisms to stop free-riding practices by affected countries.

Third, when a policy-maker is faced with limited resources, the country should focus on disease prevention rather than treatment activities. Treatment activities using ARTs are also preventative measures, as a person who is well is not only alive and productive, but will also not transmit the disease. Policy-makers should also spend on treatment activities so as to boost productivity and reduce the future disease burden.

Fourth, the policy-makers' resource allocation to disease prevention increases in the utility of getting sick or dying, increases in the utility of not contracting the disease, and increases in the utility difference between no disease and recovery. Again, policy-makers should view treatment as part of disease prevention, and their utility should benefit from sustained productivity of people who are kept alive through treatment. Besides, prevention measures may involve programs aimed at changing people's behavior or changing cultural norms. This is not easy to achieve. It is also not clear what works well and which prevention measures are cost-effective.

Fifth, if the policy-maker has substantial available resources, then little need exists for the international community to provide funding, and the international community free-rides. The international community should not free-ride even if a country is like Botswana with large revenues from natural resources. The international community should still provide funding whose benefit could be higher imposed-standards in the use of financial resources. The efficiency in converting funds into outcomes and utility is something that the international community has been emphasizing through pronouncements in the Paris Declaration and Aid-effectiveness pronouncements. Besides, even the countries with large resources are carrying the future debt from funding the liability of HIV. This debt needs to be financed, and even large domestic resources may not be enough.

Sixth, the probability of contracting the disease falls to its minimum when the policy-maker allocates all his resources to disease prevention, and reaches a maximum when the policy-maker allocates no resources to disease prevention. Obviously, when disease prevention works, it seems to make sense to allocate more resource towards it. However, the marginal cost of increasing coverage of prevention programs may start to rise as the policy-maker seeks to add more people to the programs. It could, for instance, if more people live in rural areas with poor road access, the cost of reaching them and general service delivery costs increase.

Finally, the probability of a person remaining sick or dying depends on the proportion of resources allocated to prevention versus treatment. More funding by the international community generally results in a lower probability that a person remains sick or dies. Here again, both treatment and prevention should receive funding. The tipping point should be driven by the relative marginal costs of prevention versus that of treatment. This happens when the two marginal costs are equal. Therefore allocation for either activity should take into account the relative marginal costs of provision.

5 Conclusion

We have developed a three-period game between a policy-maker and the international community on how to fund prevention and treatment of diseases. We account for the behavior of persons who engage in risky versus safe behavior which may or may not cause disease contraction. The policy-maker chooses in period 1 which fraction of his funds to allocate to disease prevention, and the remaining fraction is allocated to disease treatment. The policy-maker provides additional funds for disease treatment in period 2. The international community provides funding in period 3.

We find that when the international community provides funding, the policy-maker free rides by not providing additional funding. The policy-maker allocates a large fraction of his funds to disease prevention when the utility of getting sick or dying is large, the utility of not contracting the disease is large, the utility difference between no disease and recovery is large, the number of persons is large, and the disease contraction probability is large. Conversely, the fraction allocated to disease prevention is low when the utility difference between recovery on the one hand and getting sick or dying on the other hand is large, the available funds are substantial, the international community's unit cost of converting funding into utility is large, and the parameter proportional to the probability that a person remains sick or dies is large.

The international community's funding is aligned with the policy-maker's allocation of the fraction to prevent disease when five of the parameters vary, i.e. the number of persons, the policy-maker's funding, the international community's unit cost of converting funding into utility, and the two parameters that impact the disease contraction probability. This follows since the two players have utilities which are equivalent regarding benefits and different regarding costs of funding. But, the parameter that is proportional to the probability that a person remains sick or dies, and the utility difference between recovery and remaining sick or dying, impact differently. That is, the probability that a person remains sick or dies benefits from large international community funding, and low fraction which gives large fraction 1-p allocated to disease treatment. Furthermore, if the policy-maker has substantial available funds, the international community free rides on the policy-maker by providing less funding.

The disease contraction probability intuitively is minimal when the policy-maker allocates all his funds to disease prevention, and maximal when no funds are allocated to disease prevention. Thus, for example, a large utility difference between no disease and recovery causes the policy-maker to allocate a large fraction of his funds to disease contraction causing a low disease contraction probability. Furthermore, a large unit cost for the international community in converting funding into utility causes large disease contraction probability.

If the policy-maker allocates all funds to disease prevention and no funds to disease treatment, the probability that a person remains sick or dies is intuitively larger than if all funds are allocated to disease treatment. Furthermore, more funding f by the international community gives lower probability x that a person remains sick or dies. Especially, if the utility of recovery is substantially higher than the utility of getting sick or dying, then the probability that a person remains sick or dies is low caused by funding.

From outcomes of the model, the paper argues for the need to create commitment-mechanisms to ensure that free-riding by both countries and the international community is avoided. Basically, moving from a non-cooperative game for a cooperative game being more desirable. Such commitment mechanisms, such in the Global Fund for HIV/AIDS, TB, and Malaria, which require countries to co-finance, may be effective, as countries never fully co-finance. This needs to be strengthened. It also seeks to argue for commitment to funding both prevention and treatment, by policy-makers and the international community. Without these mechanisms the game will result in countries with limited resources only focusing on prevention, which is not desirable. The model also shows why more funding is needed, and how that can reduce the probability of disease contraction, and death from the disease.

We classify countries according to characteristics and strategic choices that determine the empirically estimated disease contraction probability. We also tested for the drivers for expressions for the fraction of resources allocated to disease prevention, the disease contraction probability, the probability that a person remains sick or dies, and additional funding by the international community provides. We estimate the regression equations. The results largely confirm the various theoretical relationships. We discuss the policy implications of the outcomes (assertions) of the model. Future research should look more thoroughly into a "commitment technology" in the form of a global governance mechanism that forces policy-makers and donors to both commit to funding prevention and treatment, and not to free-ride.

Appendix A Solving the three-period game when f>0

Definition 1. A strategy pair (s, p, f) is a subgame-perfect *Nash Equilibrium* if and only if

$$f = f(s,p) = \arg\max_{f\geq 0} v(s,p,f), \quad (s,p) = \arg\max_{s\geq 0, p\geq 0} u(s,p,f(s,p)) \tag{A1}$$

Starting with period 3, differentiating the international community's utility v in (5) with respect to f, and equating with 0, give the first order conditions

$$\begin{cases} f > 0 \Leftrightarrow \dfrac{\partial v}{\partial f} = -\dfrac{\alpha_1\alpha_2 N(R-D)}{[\beta_1 + pr][\beta_2 + (1-p)r + s + f]^2} - f = 0 \\ f = 0 \Leftrightarrow \dfrac{\partial v}{\partial f} = -\dfrac{\alpha_1\alpha_2 N(R-D)}{[\beta_1 + pr][\beta_2 + (1-p)r + s + f]^2} - f < 0 \end{cases} \tag{A2}$$

which is solved to yield

$$f = \begin{cases} \dfrac{\sqrt{\alpha_1\alpha_2 N(R-D)}}{\sqrt{b(\beta_1 + pr)}} - \beta_2 - (1-p)r - s \; if \; \dfrac{\sqrt{\alpha_1\alpha_2 N(R-D)}}{\sqrt{b(\beta_1 + pr)}} > \beta_2 + (1-p)r + s \\ 0 \; otherwise \end{cases} \tag{A3}$$

The second order condition

$$\frac{\partial^2 v}{\partial f^2} = -\frac{2\alpha_1\alpha_2 N(R-D)}{[\beta_1 + pr][\beta_2 + (1-p)r + s + f]^3} \tag{A4}$$

is always satisfied as negative. Inserting (A3) into the policy-maker's utility in (4) gives

$$u = \begin{cases} NE - a(r+s) - \dfrac{\alpha_1 N(E-R)}{\beta_1 + pr} - \dfrac{\sqrt{b\alpha_1\alpha_2 N(R-D)}}{\sqrt{\beta_1 + pr}} \; if \; \dfrac{\sqrt{\alpha_1\alpha_2 N(R-D)}}{\sqrt{b(\beta_1 + pr)}} > \beta_2 + (1-p)r + s \\ NE - a(r+s) - \dfrac{\alpha_1 N(E-R)}{\beta_1 + pr} - \dfrac{\alpha_1\alpha_2 N(R-D)}{(\beta_1 + pr)[\beta_2 + (1-p)r + s]} \; otherwise \end{cases} \tag{A5}$$

The second line in (A5) is analyzed in Appendix B. Proceeding with periods 2 and 1 (analyzed simultaneously since the sequence is mathematically irrelevant), differentiating the policy-maker's utility u in the first line in (A5) with respect to p and s, and equating with 0, give

$$\begin{aligned} &\frac{\partial u}{\partial p} = \frac{\alpha_1 Nr(E-R)}{(\beta_1 + pr)^2} + \frac{r\sqrt{b\alpha_1\alpha_2 N(R-D)}}{2(\beta_1 + pr)^{3/2}} = 0 \; if \; \frac{\sqrt{\alpha_1\alpha_2 N(R-D)}}{\sqrt{b(\beta_1 + pr)}} > \beta_2 + (1-p)r + s \\ &\frac{\partial u}{\partial s} = -a \; if \; \frac{\sqrt{\alpha_1\alpha_2 N(R-D)}}{\sqrt{b(\beta_1 + pr)}} > \beta_2 + (1-p)r + s \end{aligned} \tag{A6}$$

which is solved to yield

$$p=\begin{cases}1\,if\ \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}+\dfrac{\alpha_2(R-D)}{2(E-R)}>\beta_1+\beta_2+r\ and\ \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}\geq\beta_1+r\\ \dfrac{1}{r}\left(\dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}-\beta_1\right)if\ \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}+\dfrac{\alpha_2(R-D)}{2(E-R)}>\beta_1+\beta_2+r\ and\ \beta_1<\dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}<\beta_1+r\\ 0\,if\ \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}+\dfrac{\alpha_2(R-D)}{2(E-R)}>\beta_1+\beta_2+r\ and\ \dfrac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}\leq\beta_1\end{cases}$$

$$s=0\ if\ \frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}+\frac{\alpha_2(R-D)}{2(E-R)}>\beta_1+\beta_2+r \tag{A7}$$

which is rewritten as (7). The second order condition for p is negative, i.e.

$$\frac{\partial^2 u}{\partial p^2}=\frac{-2\alpha_1 Nr^2(E-R)}{(\beta_1+pr)^3}-\frac{3r^2\sqrt{b\alpha_1\alpha_2 N(R-D)}}{4(\beta_1+pr)^{5/2}}<0\ if\ \frac{\sqrt{\alpha_1\alpha_2 N(R-D)}}{\sqrt{b(\beta_1+pr)}}>\beta_2+(1-p)r+s \tag{A8}$$

Inserting (A7) into (A3) gives (8).

Appendix B Solving the two-period game when f=0

No funding by the international community, f=0, means eliminating period 3. We solve periods 2 and 1 simultaneously since the sequence is mathematically irrelevant. Differentiating the policy-maker's utility u in (5) with respect to p and s, and equating with 0, gives the first order conditions

$$\frac{\partial u}{\partial p}=\frac{\alpha_1 Nr}{(\beta_1+pr)^2}\left(E-R+\frac{\alpha_2(R-D)}{\beta_2+(1-p)r+s+f}-\frac{\alpha_2(R-D)(\beta_1+pr)}{(\beta_2+(1-p)r+s+f)^2}\right)=0$$
$$if\ \frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}\leq\beta_1+\beta_2+r-\frac{\alpha_2(R-D)}{2(E-R)} \tag{B1}$$
$$\frac{\partial u}{\partial s}=\frac{\alpha_1\alpha_2 N(R-D)}{(\beta_1+pr)(\beta_2+(1-p)r+s+f)^2}-a=0\ if\ \frac{4\alpha_1 N(E-R)^2}{b\alpha_2(R-D)}\leq\beta_1+\beta_2+r-\frac{\alpha_2(R-D)}{2(E-R)}$$

which gives a third order equation in p. We refer to the solution as $p=p_t$ and $s=s_t$.

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