A Game-Theoretic Model with Empirics of Economic Crises

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Abstract

A game theoretic model of six kinds of players, i.e. countries, central banks, banks, firms, households, and financial inter-governmental organizations. Each player has a strategy set, with strategies such as setting interest rates, lending, borrowing, producing, consuming, investing, importing, exporting, defaulting, and penalizing default. Markets for goods, debt, and capital are modeled endogenously. This rich conceptualization of strategic opportunities for as many as six types of players is richer than anything that has been attempted earlier. The 2005-2011 empirical data for Greece is used to analyze how utility is impacted by public consumption and lump sum transfers, assuming negative productivity shocks, analyzing several time periods, with and without the possibility of default. The 2007-2008 empirical data for Greece and Germany is used to determine how the two countries' utilities depend on Greece's public 2007 consumption, with and without negative productivity shocks. Greece's

high debt burden is shown to make default optimal with high shock magnitude is low default penalty. Germany has limited ability, through its available strategies, to prevent the default, and may resort to unconventional tools such as debt forgiveness and changing the default penalty.

1. Introduction

1.1 Background and Motivation

A game-theoretic model is developed comprising six types of players, i.e. countries, central banks, banks, firms, households, and financial intergovernmental organizations (FIGOs). Countries produce, consume, trade, invest, borrow, and lend. Central banks lend, borrow, set interest rates, may default, penalize default, set inflation target. Banks borrow, lend, set interest rates, default, and penalize default. Firms produce, demand labor, invest, import, export, borrow, lend, and default. Households consume, invest, borrow, lend, and default. FIGOs borrow, lend, set interest rates, default, and penalize default.

In this work, we present a general model, characterize the players, and define Nash equilibria. The players' strategy sets are large, making the model extensively endogenous. However, the numerous strategic-choice variables in the model make solution of the full model extremely complicated. The model can be used with all strategic variables as strategic variables, or used interpreting some of the variables as exogenous parameters. We consider several simplifications of the model. For example, single-country numerical analyses and sensitivity analysis show how a country can respond to a temporary negative shock with increased borrowing to finance public consumption.

We present a dynamic model that guides readers to understand the strategies of all stakeholders in international crises. We present findings from relevant numerical examples. Our conclusions explain how crises can plausibly spread and how each player's strategy can contribute to the spread. We also put forth ideas for how players may adjust their strategy to stem the spread of contagion. Strategic interaction implies contagion. For example, we illustrate in this paper that a shock to one country may impact another country through contagion. The literature on contagion builds on the literature of sovereign default. We highlight several methods for modeling sovereign default including financial models and sovereign default models.

Two-country numerical analyses demonstrate contagion through the credit channel since a shock that causes default in a borrower country can propagate to a lending country. The trade channel is similar, as a negative shock in one country can diminish demand for imported goods, transmitting the adverse shock to another country through reduced trade. Finally, two-country numerical analyses are used to demonstrate how common macroeconomic conditions can create adverse shocks without contagion, since a joint interest-rate shock can negatively affect both countries. Our results demonstrate that each of the three causes discussed above (contagion through credit channels, contagion through trade channels, or common macroeconomic conditions with no contagion) can lead to crises even if all players in the model behave rationally.

1.2 The Use of Game Theory in Modeling Economic Crises

Most of the literature applying game-theoretic approaches to economic contagion is limited in either its use of game-theoretic and decision-theoretic tools, or its ability to address the multiple-channels problems. Morris and Shin (2012, 2000) model contagion in a coordination game of investors to currency crises, bank runs, and debt pricing to assess adverse selection. The importance of behavior is demonstrated by Chari and Kehoe (2004) who argue that models of herd behavior can explain financial crises. Broner (2008) argues that the private information in models of currency crises can create multiple equilibria and unpredictable currency devaluations. Furthermore, many have argued that crises can be self-fulfilling. Obstfeld (1984) argues that speculative attacks can make balance-of-payments crises self-fulfilling. Lorenzoni and Werning (2013) create a model where self-fulfilling debt crises follow from investor expectations on default. They argue that a crisis follows from a shift from a good equilibrium to a bad one. Hausken and Plumper (2002) propose a contagion game for how a crisis spreads and can be contained through intervention by the IMF and collective action. Baral (2013) proposes a network formulation game between United States borrower banks, lender banks, and the Federal Reserve to model contagion. Similarly, Acemoglu et al. (2013) compute Nash equilibria in a network formulation of the interbank market to describe contagion and counterparty risk. Most similar in spirit to this paper, Welburn and Hausken (2015) present a model consisting of many of the players in this paper except FIGOs. Whereas they present results from a numerical analysis using data for the United States, we present results from numerical analyses using data for Greece and Germany in order to give insight into the Eurozone debt crisis.

1.3 The Sovereign Debt Literature

The discussion of contagion builds on the sovereign debt literature which seeks to explain debt and default for small open economies through single country models. Reinhart and Rogoff (2009) consider debt crises through history. Eaton and Gersovitz (1981) and Eaton et al. (1986), describe the unique problems of sovereign lending presented by a lack of enforcement. While domestic lending can be legally enforced in default through seizure, foreclosure, as well as other methods, foreign lending is more challenging to enforce. Eaton et al. (1986) further elaborate that given the lack of enforcement, payment on sovereign debt is mostly voluntary.

Eaton and Gersovitz (1981) explain that the potential for repudiation drives repayment. They explain that default has a cost by harming the reputation of the borrower. In their framework, the decision to default follows the path that gives the highest utility; default or no default. Bulow and Rogoff (1988) continue the sovereign debt discussion. They argue that reputation alone is not enough to ensure repayment. Bulow and Rogoff (1988) develop a model to explain that sanctions are required for payment. While the type of penalty varies, the literature on sovereign default often uses some penalty (e.g., reduced output, loss in utility, autarky) to enforce repayment.

Default risk can, in fact, be handled in several ways. It is popular in the finance literature to model default exogenously. Duffie and Singleton (1999) develop a reduced-form model to price bonds with the risk of default. In this approach, default follows an exogenous stochastic process. Andritzky (2006) presents an extensive assessment of sovereign lending and associated risks within the framework of financial models such as the Duffie and Singleton (1999) model. He presents a detailed history of sovereign lending and the numerous default crises that have occurred in recent history. Andritzky (2006) study of debt crises focuses on what the results of past crises has been, what their impact to investors has been, and how default risk can be modeled. He highlights how default risk can be handled by financial models (e.g., structural vs. reduced-form models). Additionally, sovereign debt risk can be evaluated using credit ratings from major ratings agencies. Gaillard (2011) discusses credit ratings for sovereign debt. In light of criticisms of credit ratings following the 1997 East Asian Crisis and 2010 Greek Debt Crisis, Gaillard (2011) explains how ratings are made & assigned and the methodology used to rate debt. Ratings are demonstrated to be procyclical and offer limited value. Furthermore, Gaillard (2014) demonstrates flaws in ratings following the Eurozone crisis where downgrades occurred too slowly. As a result, Gaillard (2014) argues that policymakers and investors should decrease their reliance on credit ratings and improve in-house modeling of default risk. Finally, Das et al. (2007) seek to understand common cause failures as correlated corporate defaults.

We address the need for improved modeling of default risk by modeling default endogenously and through time. In their seminal default model, Eaton and Gersovitz (1981) consider debt with repudiation. Aguiar and Gopinath (2006) explain how defaults can occur in a quantitative model of small open economies. Arellano (2008) estimates default risk using a stochastic recursive equilibrium model applied to emerging economies. Mendoza and Yue (2011) explain defaults in the context of business cycle dynamics using a general equilibrium model.

Default is often handled as a one-time event. Those following the Eaton and Gersovitz (1981) setting often model default as a single decision. In contrast, those following the Andritzky (2006) settings model default as a single random event. Yue (2010) and Arellano and Bai (2013) offer another approach by including Nash bargaining for debt renegotiation in a model of linkages in sovereign debt markets accounting for default. The outcome of a failed negotiation is autarky for the borrower and a total loss for the lender.

1.4 The Contagion Literature

The sovereign debt literature (in the previous subsection) discusses crises in a single country. Crises, however, are often not isolated to a single country. In many cases crises spread across countries and regions through a contagion process. In 1989, a Brazilian crisis contributed to a Latin American crisis. In 1994, a Mexican crisis contributed to another Latin American crisis. The 1990s led to further notable examples. In 1994, a crisis in Thailand spread across Southeastern Asia driving the Asian financial crisis. This crisis, in turn, contributed to a Russian area crisis in 1998. The most recent crisis, the 2009-2012 Eurozone crisis which began in Greece, serves as a primary motivation for this paper. We seek to quantitatively explain key elements of the Eurozone crisis, specifically the crisis in Greece, applying our game-theoretic model.

Contagion follows from globalization, trade, and interconnectedness. This problem is not modern; Reinhart and Rogoff (2009) document crises dating back to the fourteenth century. The consensus for modelling contagion risks is incomplete. Forbes and Rigobon (2002, 2001) argue that so-called contagion episodes actually follow from increased interdependence following an adverse shock and that the effect can be viewed through increased co-movements. Hernández and Valdés (2001) and Kaminsky et al. (2003) clarify the possible mechanisms of contagion, that it can be driven by a debt channel, a trade channel, or that the process that appears to be contagion may be driven by a common macroeconomic cause.

Much of the contagion literature focuses on the debt channel which has many ties to the interbank market. Kiyotaki and Moore (1995) present a model of cross-sector contagion following a vicious cycle of declining asset prices and investment. Allen and Gale (2000) address the ability for shocks to spread across regions of the economy due to network structure. Furthermore, Giesecke and Weber (2006) extend the reduced form models of financial default risk to assess contagion between firms. Acharya and Yorulmazer (2008) add to the literature on contagion specifying the importance of information on contagion through the banking sector. They find that information (e.g. bad news) can lead banks to act as a group driving contagion. These sources provide insight into contagion, however do not specifically address the risk of sovereigns with contagion.

Many others extend the sovereign debt literature to discuss contagion. Arellano and Bai (2013) extend Yue (2010) to present a multi-country model with debt renegotiation where all defaulting countries renegotiate simultaneously. They calibrate their model to the Eurozone crisis predicting commoving interest rates and finding that a default in one country increases the probability of default in another. Park (2012) presents a sequential equilibrium model of contagion where the mechanism of contagion is the investor (i.e. lender) in sovereign debt.

1.5 Financial Regulation: Austerity, Capital Controls and Financial Repression

Following the several notable debt crises including the recent Eurozone crisis, much attention in the sovereign debt literature has been given to developing macroprudential policies. Galati and Moessner (2013) describe that the recent development of literature towards macroprudential policy shares the objective of shaping macro-focused policy that improves financial stability. Of course, we argue that our approach using game theory offers a strong framework for which macro policy can be developed. Here, we acknowledge that several approaches can be used for managing crises which we either do not model, or model indirectly.

Austerity is often presented as a solution for managing crises. Austerity describes the sharp reduction in spending used by government to reduce deficits avoiding further crisis. An ardent critic of austerity is Krugman (2013), see also Gauti B. Eggertsson and Krugman (2012). While the austerity approach offers certain advantages in winding down budget deficits, it sharply reduces spending, consumption, and investment in a potentially harmful manner. We explore and endogenize these effects in this paper.

An additional approach for managing crises is the use of capital controls. Capital controls refer to policies enforced by the government which restrict capital flows out of the country. This approach has mostly been used by developing countries to boost domestic assets. Edwards (1999) argues that the use of capital controls is often ineffective and causes distortions and corruption.

Financial repression represents another form of governmental policy used to stem debt crises. The term financial repression refers to the governmental policy of setting interest rates on government debt at or below the inflation rate. Financial repression, thereby, works as a tax of the saving of households who purchases government bonds providing governments with a cash flow to repay debts. This approach has some similarities to austerity and the imposition of capital controls in both its desired effect and its implementation. Reinhart et al. (2011) argue that financial repression can be used as an alternative to debt restructuring and default. That is, a government could generate income through financial repression to pay down debts avoiding debt crises. Others argue that this approach comes with significant drawbacks that reduce the policy's ability to be an effective tool for managing debt crises. Roubini and Sala-i-Martin (1992) argue that the use of financial repression inhibits growth making it less desirable. Financial repression, consequently suffers from a similar argument as do capital controls in their limited ability to serve as macroprudential policy.

In this paper we focus on strategic behavior in a highly interconnected system. We provide a framework for discussing how crises can be managed through strategic interaction without the use of capital controls or financial repression. We account for degrees of austerity which are relevant to discussions of policy. Our paper advocates the use of game theory in constructing macroprudential policy.

1.6 Paper Organization

Section 2 presents a model with six types of players; households, firms, banks, central banks, countries and FIGOs. We characterize the strategy sets of each player and present a description of variables used in the model. In this section we present a model with similarities to a small open economy framework with endogenous default. We describe markets for debt and markets for goods. We explain the basis for endogenous default and define the penalty for defaulting. We conclude this section with a definition of the Nash Equilibrium. In section 3, we numerically analyze a single country with households and firms in equilibrium. We illustrate the effect of public consumption, lump sum transfers, austerity, a positive interest rate shock, and a negative productivity shock in a ten period analysis. In section 4, we numerically analyze two countries, determining the effect of a negative productivity shock and default. In section 5, we use 2005-2011 empirical data for Greece and numerically analyze the effect of public consumption, negative productivity shocks, austerity, public investment, and default, assuming one period, two periods, and seven periods. In section 6, we use 2007-2008 empirical data for Greece and Germany and numerically analyze the effect of public consumption, negative productivity shocks, public investment, and default, assuming two periods. Section 7 presents policy recommendations from the findings and intuition of our framework. Section 8 concludes.

2. The Model

2.1 Players, Variables, and Parameters

A non-cooperative, simultaneous-move, repeated game is developed. Complete information is assumed making all parameters common knowledge for all players. This assumption can be extended to incomplete information in future research. There are six types of players interacting as shown in Figure 1[1] and defined in Table 1, where red arrows represent the market

[1] Figure 1 expands upon a figure presented by Welburn and Hausken (2015) by incorporating FIGOs.

for goods and services and black arrows represent the debt market. Table 2 shows the six players' strategic choice variables. Table 3 provides the guide to setting interest rates. Table 4 defines the parameters. Table 5 delineates the dependent variables.

The collectively exhaustive set of players consists of countries, central banks, banks, firms, households, and FIGOs. All players have the ability to borrow, lend, and default while their incentives for each vary significantly. On top of this, households have the strategic choice set of consumption, investment, and supplying labor. Firms demand labor, invest, trade, and produce goods. Countries consume, invest, and tax. Central banks set inflation targets, while banks and FIGOs simply borrow and lend.

Central banks set the key interest rates in the game. Banks use this rate to determine the interest rates on loans to others. Countries can set rates, but only on loans to households, firms, and banks. These loans occur only in special circumstances such as bailouts. Players who borrow cannot set rates on borrowing. Countries are not permitted to practice financial repression. We now proceed to elaborate more thoroughly.

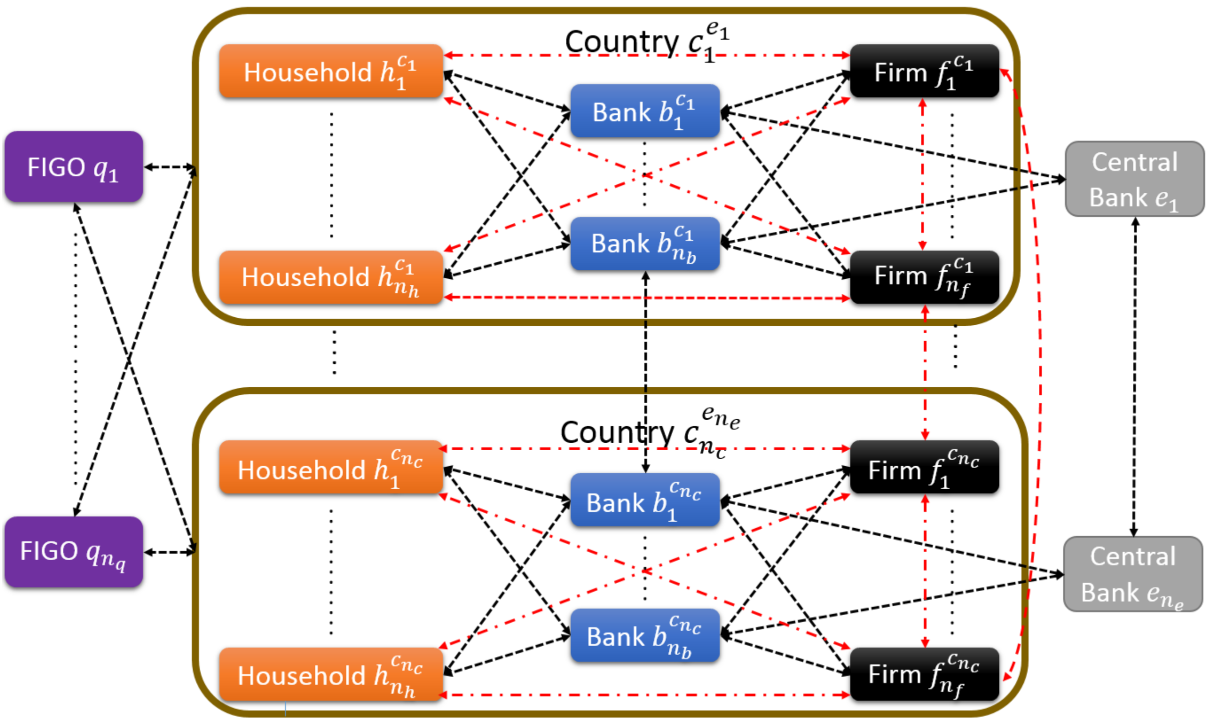

Figure 1: The international market for goods and services across six types of players.

Table 1: Player descriptions and strategy sets.

Name	Definition	Players $p = \{c, e, b, f, h, q\}$	Strategy Set, $\{S_i\}_{i \subseteq p}$
Countries, c	n_c countries borrow B, lend (negative borrowing, B), set interest rates r on lending and bailouts, default D, penalize default P, consume C, invest I, tax N.	$c = \{c^{e_1}, \dots, c^{e_{n_e}}\}$: $c^{e_j} = \{c_1^{e_j}, \dots, c_n^{e_j}\}$	$S_c = \{B, r, D, P, C, I, N\}$
Central banks, e	n_e central banks (lenders of last resort) borrow, lend, set interest rates, default, penalize default, set inflation target π^*.	$e = \{e_1, \dots, e_{n_e}\}$	$S_e = \{B, r, D, P, \pi^*\}$
Banks, b	n_b banking institutions borrow, lend, set interest rates, default, penalize default.	$b = \{b^{c_1}, \dots, b^{c_{n_c}}\}$: $b^{c_j} = \{b_1^{c_j}, \dots, b_n^{c_j}\}$	$S_b = \{B, r, D, P\}$
Firms, f	n_f firms produce, demand labor L, invest, import M, export, borrow B, lend, default.	$f = \{f^{c_1}, \dots, f^{c_{n_c}}\}$: $f^{c_j} = \{f_1^{c_j}, \dots, f_n^{c_j}\}$	$S_f = \{L, I, M, B, D\}$
Households, h	n_h households labor, consume, invest, borrow, lend, default.	$h = \{h^{c_1}, \dots, h^{c_{n_c}}\}$: $h^{c_j} = \{h_1^{c_j}, \dots, h_n^{c_j}\}$	$S_h = \{L, C, I, B, D\}$
FIGOs, q	n_q financial inter-governmental organizations (FIGOs) borrow, lend, set interest rates, default, penalize default.	$q = \{q_1, \dots, q_{n_q}\}$	$S_q = \{B, r, D, P\}$

Table 2: Strategic choice variables.

Variable	Definition	Player Domain $n \in \{1, \dots, n_i\}$, $m \in \{1, \dots, n_j\}$	Variable Domain
$B_{i_n j_m t}$	Amount player i_n borrows from player j_m at time t	$(i_n, j_m) \in p$	$B_{i_n j_m t} \in \mathbb{R}^2$
$D_{i_n j_m t}$	Binary indicator for whether player i_n defaults on positive debt to player j_m at time t (1=yes, 0=no)	$(i_n, j_m) \in p$	$D_{i_n j_m t} \in \{0,1\}$
$r_{i_n j_m t}$	Interest rate at which player i_n borrows $B_{i_n j_m t}$ from player j_m at	$(i_n, j_m) \in p$	$r_{i_n j_m t} \in \mathbb{R}^{2+}$

	time t. (Rate setting player shown in Table 3)[2]		
$E_{i_n j_m t} = -E_{j_m i_n t}$	Amount player i_n exchanges of its home currency with player j_m for j_m's home currency at time t	$(i_n, j_m) \in p$	$E_{i_n j_m t} \in [E_{i_n j_m t}^{min}, E_{i_n j_m t}^{max}]$
$P_{i_n j_m t}$	Amount player i_n pays to player j_m as a penalty for defaulting on debt at time t	$(i_n, j_m) \in p$	$P_{i_n j_m t} \in (0, \infty)^2$
$\Psi_{i_n t}$	Fraction of player i_n's utility penalized for defaulting on debt at time t	$i_n \in p$	$\Psi_{i_n t} \in [\vec{\mathbf{0}}, \vec{\mathbf{1}}]$
$C_{c_n t}^P$	Public consumption for country c_n at time t	$c_n \in c$	$C_{c_n t}^P \in \mathbb{R}^+$
$I_{c_n t}^P$	Public capital investment for country c_n at time t	$c_n \in c$	$I_{c_n t}^P \in \mathbb{R}^+$
$\pi_{e_n t}^*$	Target inflation rate of central bank e_n at time t	$e_n \in e$	$\pi_{e_n t}^* \in [0,1]$
$N_{i_n c_m t}$	Net lump sum transfer payment from player i_n to the country c_m at time t	$i_n \in (b, h, f)$, $c_m \in c$	$N_{i_n c_m t} \in \mathbb{R}$
$I_{f_n t}$	Investment of firm f_n at time t	$f_n \in f$	$I_{f_n t} \in \mathbb{R}^+$
$L_{h_m f_n t}$	Labor demanded from household h_m by firm f_n at time t	$h_m \in h$, $f_n \in f$	$L_{h_m f_n t} \in \mathbb{R}^+$
$I_{h_n t}$	Investment of household h_n at time t	$h_n \in h$	$I_{h_n t} \in \mathbb{R}^+$
$M_{f_n f_m t}$	Imports of firm f_n in country $c_i$from firm f_m in country c_j at time t	$f_n \in f^{c_i}, f_m \in f^{c_j}$ $(c_i, c_j) \in c$	$M_{f_n f_m t} \in \mathbb{R}^+$
$C_{h_n t}$	Consumption of household h_n at time t	$h_n \in h$	$C_{h_n t} \in \mathbb{R}^+$
$L_{f_m f_n t}$	Labor supplied from household h_m by firm f_n at time t	$h_m \in h$, $f_n \in f$	$L_{h_m f_n t} \in \mathbb{R}^+$

Table 3: Interest rate setting guide.

Rate setting player	Positive borrowing rate	Equivalent negative borrowing rate
Country, c_m	$r_{i_n c_m t}^+ \; \forall \; c_m \in c, i_n \in \{c, e, b, f, h\}$	$r_{c_m j_n t}^- \; \forall \; c_m \in c, j_n \in \{c, e, b, f, h\}$
Country, c_n	$r_{c_n j_m t}^+ \; \forall \; c_n \in c, j_m \in \{c, e, b, f, h\}$	$r_{i_m c_n t}^- \; \forall \; c_n \in c, i_m \in \{c, e, b, f, h\}$
Central bank, e_m	$r_{i_n e_m t}^+ \; \forall \; e_m \in e, i_n \in \{e, b, f, h\}$	$r_{e_m j_n t}^- \; \forall \; e_m \in e, j_n \in \{e, b, f, h\}$

[2] Interest rates are allowed to range from 1 to -1. Although negative interest rates are not typical, a negative interest rate of -1 is exemplified by a full bailout. In this case there would be no repayment of debts.

Central bank, e_n	$r^{+}_{e_n j_m t}$ $\forall\ e_n \in e, j_m \in \{e,b,f,h\}$	$r^{-}_{i_m e_n t}$ $\forall\ e_n \in e, i_m \in \{e,b,f,h\}$
Bank, b_m	$r^{+}_{i_n b_m t}$ $\forall\ b_m \in b, i_n \in \{b,f,h\}$	$r^{-}_{b_m j_n t}$ $\forall\ b_m \in b, j_n \in \{b,f,h\}$
Bank, b_n	$r^{+}_{b_n j_m t}$ $\forall\ b_n \in b, j_m \in \{b,f,h\}$	$r^{-}_{i_m b_n t}$ $\forall\ b_n \in b, i_m \in \{b,f,h\}$
FIGO, q_m	$r^{+}_{i_n q_m t}$ $\forall\ q_m \in q, i_n \in p$	$r^{-}_{q_m j_n t}$ $\forall\ q_m \in q, j_n \in p$
FIGO, q_n	$r^{+}_{q_n j_m t}$ $\forall\ q_n \in q, j_m \in p$	$r^{-}_{i_m q_n t}$ $\forall\ q_n \in q, i_m \in p$

Table 4: Parameters.

Parameter	Definition	Player Domain	Parameter Domain
$\beta_{i_n t}$	Utility discount factor for player i_n at time t	$i_n \in p$	$\beta_{i_n t} \in (0,1)$
$\bar{Y}_{i_n t}$	GDP potential of player i_n at time *t*	$i_n \in (c_n, e_n)$	$\bar{Y}_{c_n t} \in \mathbb{R}^+$, $\bar{Y}_{e_n t} = \sum_{c_m^{e_n} \in c^{e_n}} \bar{Y}_{c_n t}$
$\epsilon_{i_n q_m t}$	Amount player i_n endows FIGO q_m at a given time *t*	$i_n \in p, q_m \in q$	$\epsilon_{i_n q_m t} \in \mathbb{R}^+$
$\pi_{i_n t}$	Inflation rate observed by player i_n at time *t*	$i_n \in p$	$\pi_{i_n t} \in \mathbb{R}$
$\pi_{e_n t}$	Inflation rate of central bank e_n's monetary union c_n^u at time *t*	$e_n \in e$	$\pi_{e_n t} \in \mathbb{R}$
$r^*_{e_n t}$	Assumed equilibrium real interest rate of central bank e_n at time *t*	$e_n \in e$	$r^*_{e_n t} \in [0,1]$
μ_t	Market risk (systematic) at time *t*		$\mu_t \in \mathbb{R}$
$A_{c_n t}$	Productivity (technology) factor for production in Country c_n at time t	$c_n \in c$	$A_{c_n t} \in \mathbb{R}^+$
α_{c_n}	Elasticity parameter for country c_n's utility	$c_n \in c$	$\alpha_{c_n} \in [0,1]$
ρ_{h_n}	Household h_n's elasticity of intertemporal substitution	$h_n \in h$	$\rho_{h_n} \in [0,1]$
ς_{c_n}	Endogenous growth parameter (capital-output ratio) for production in country c_n	$c_n \in c$	$\varsigma_{c_n} \in [0,1]$
α_{c_n}	Elasticity parameter for country c_n's utility	$c_n \in c$	$\alpha_{c_n} \in [0,1]$
γ_{c_n}	Debt seeking parameter of country c_n	$c_n \in c$	$\gamma_{c_n} \in [0,1]$
ν_{c_n}	Z-factor seeking parameter of country c_n	$c_n \in c$	$\nu_{c_n} \in [0,1]$
α_{c_n}	Austerity parameter of country c_n where increasing α_{c_n} increases the austerity level	$c_n \in c$	$\alpha_{c_n} \in \mathbb{R}^+$

a_π	Taylor rule weight on inflation		$a_\pi > 0$
a_Y	Taylor rule weight on GDP		$a_Y > 0$
ω_{e_n}	Relative weight of central bank $e_n's$ monetary union's utilities against profit and loss from banking activities	$e_n \in e$	$\omega_{e_n} \in [0,1]$
ω_{q_n}	Weight given to the collective benefit of the countries to which FIGO q_n lends against its own costs of borrowing	$q_n \in q$	$\omega_{q_n} \in [0,1]$
$w_{h_n t}$	Wage for household h_n at time t as determined exclusively by the households exogenous ability	$h_n \in h$	$w_{h_n} \in \mathbb{R}^+$
$\xi_{h_n f_m}$	Fraction of firm f_m owned by household h_n	$h_n \in h, f_m \in f$	$\xi_{h_n f_m} \in [0,1]$
$\delta_{i_n t}$	Capital depreciation of player i_n at time t (negative depreciation means appreciation)	$i_n \in p$	$\delta_{i_n t} \in \mathbb{R}$
η	Set of all currencies		$\eta = (\eta_1, \dots, \eta_{n_e})$
κ	GDP-to-debt parameter		$\kappa \in (0, \infty)$
T	Number of time periods	$t \in T$	$T \in [1, \infty)$
$R_{c_n t}$	Rent (relevant e.g. for dictatorships)	$c_n \in c$	$R_{c_n t} \in \mathbb{R}$
$Z_{c_n t}$	Unspecified input (relevant e.g. for military powers)	$c_n \in c$	$Z_{c_n t} \in \mathbb{R}$

Table 5: Dependent variables (depend on strategic choice variables and parameters).

Variable	Definition	Player Domain	Variable Domain
$d_{i_n t}$	Debt of player i_n to all players at time t	$i_n \in p$	$d_{i_n t} = \sum_{j_m \in p} d_{i_n j_m t}$
$\phi_{i_n j_m t}$	The percentage loss (haircut) on debt of player i_n to player $j_m$given default at time t	$(i_n, j_m) \in p$	$\phi_{i_n j_m t} = 1 - \frac{\psi_{i_n t} U_{i_n t}(x_{i_n t}, x_{-i_n t})}{r^+_{i_n j_m t} d^+_{i_n j_m t}}$
$\psi_{j_m i_n t}$	Percentage of utility penalized for default imposed by player j_m on player i_n at time t	$(i_n, j_m) \in p$	$\psi_{j_m i_n t} \stackrel{\text{def}}{=} \begin{bmatrix} \psi^+_{j_m i_n t} \\ -\psi^-_{j_m i_n t} \end{bmatrix}$

$\Delta_{i_n j_m t}$	Debt of player i_n to player j_m at time t conditional on default	$(i_n, j_m) \in p$	$\Delta_{i_n j_m t} = \begin{bmatrix} \Delta^{+}_{i_n j_m t} \\ \Delta^{-}_{i_n j_m t} \end{bmatrix} = \begin{bmatrix} (1-\phi_{i_n j_m t} D_{i_n j_m t}) d^{+}_{i_n j_m t} \\ (1-\phi_{j_m i_n t} D_{j_m i_n t}) d^{-}_{i_n j_m t} \end{bmatrix}$
$\Delta_{i_n t}$	Debt of player i_n to all players at time t conditional on default	$i_n \in p$	$\Delta_{i_n t} = \sum_{j_m \in p} \Delta_{i_n j_m t}$
$\lambda_{j_m t}$	Idiosyncratic risk (unsystematic) on negative borrowing to player j_m at time t	$j_m \in p$	$\lambda_{j_m t}$=$\phi_{j_m i_n t} \Pr(D_{j_m i_n t} = 1)$
$x_{i_n t}$	Strategy profile of player i_n at time t	$i_n \in p$	$x_{i_n t} \in S_{i_n}$
$x_{-i_n t}$	Strategy profile of all players not including player i_n at time t	$-i_n \in p$	$x_{-i_n t} \in S_{i_n}$
$U(x_{i_n t}, x_{-i_n t})$	Utility of player i_n at time t	$i_n \in p$	$U(x_{i_n t}, x_{-i_n t}) \in \mathbb{R}$
$\Pi_{i_n t}$	Profit of player i_n at time t	$i_n \in p$	$\Pi_{i_n t}(x_{i_n t}, x_{-i_n t}) \in \mathbb{R}$
$C^{H}_{c_n t}$	Aggregate household consumption in country c_n at time t	$c_n \in c$	$C^{H}_{c_n t} = \sum_{h^{c_n}_m \in h^{c_n}} C_{h^{c_n}_m t}$
$I^{H}_{c_n t}$	Aggregate household investment in country c_n at time t	$c_n \in c$	$I^{H}_{c_n t} = \sum_{h^{c_n}_m \in h^{c_n}} I_{h^{c_n}_m t}$
$I^{F}_{c_n t}$	Aggregate firm investment in country c_n at time t	$c_n \in c$	$I^{F}_{c_n t} = \sum_{f^{c_n}_m \in f^{c_n}} I_{f^{c_n}_m t}$
$C_{c_n t}$	Aggregate consumption of country c_n at time t	$c_n \in c$	$C_{c_n t} = C^{P}_{c_n t} + C^{H}_{c_n t}$
$I_{c_n t}$	Aggregate investment of country c_n at time t	$c_n \in c$, $h^{c_n}_m \in h$, $f^{c_n}_l \in f$	$I_{c_n t} = I^{P}_{c_n t} + I^{H}_{c_n t} + I^{F}_{c_n t}$

$M_{f_n t}$	Firm $f_n's$ imports at time t	$f_n \in f$	$M_{f_n t} = \sum_{c_j \in c, f_m \in f^{c_j}} s_{c_j f_n t} M_{f_n f_m t}$
$M_{c_n t}$	Country c_n's imports from all other countries at time t	$(c_n, c_m) \in c$, $c_n \in c$	$M_{c_n t} = \sum_{f_i \in f^{c_j}} M_{f_i t}$
$X_{f_n f_m t}$	Firm $f_n's$ exports to f_m at time t	$f_n \in f^{c_i}$, $f_m \in f^{c_j}$, $(c_i, c_j) \in c, i \neq j$	$X_{f_n f_m t} = M_{f_m f_n t}$
$X_{f_n t}$	Exports from firm f_n to all other countries at time t	$f_n \in f$	$X_{f_n t} = \sum_{f_n \in f} X_{f_n f_m t}$
$X_{c_n t}$	Exports from country c_n to all other countries at time t	$(c_n, c_m) \in c$	$X_{c_n t} = \sum_{f_n \in f^{c_n}} X_{f_n t}$
$TB_{c_n t}$	Trade Balance for country c_n at time t	$c_n \in c$	$TB_{c_n t} = Y_{c_n t} - C_{c_n t} - I_{c_n t} = (X_{c_n t} - M_{c_n t})$
$CA_{c_n t}$	Current account for country c_n at time t	$c_n \in c$	$CA_{c_n t} = TB_{c_n t} - \sum_{j_m \in p} (r_{c_n j_m t} \times \Delta_{c_n j_m t})$
$Y_{i_n t}$	GDP/production of player i_n at time t	$i_n \in (f, c, e)$	$Y_{c_m t} = A_{c_m t} K_{c_m t}^{\varsigma_{c_m}} L_{c_m t}^{1-\varsigma_{c_m}} = \sum_{(f_j \in f^{c_m})} Y_{f_j t}$ $Y_{e_n t} = \sum_{c_m^{e_n} \in c^{e_n}} Y_{c_m^{e_n} t}$
$K_{i_n t}$	Total capital for player i_n at time t	$i_n \in (f, c)$	$K_{c_n t} = \sum_{f_l \in c_n} K_{f_l t}$, $K_{i_n t} = \sum_{\tau=1}^{t} (1-\delta)^{\tau} I_{i_n \tau}$
$L_{i_n t}$	Labor supplied by player i_n at time t	$i_n \in (h, c)$	$L_{c_n t} = \sum_{h_i \in h^{c_n}} L_{h_i t}$, $L_{h_i t} = \sum_{f_j \in f^{c_n}} L_{h_i f_j t}$
$s_{i_n j_m t}$	Exchange rate; the price of player j_m's home currency in terms of player i_n's home currency at time t where i_n is in the monetary	$(i_n, j_m) \in p$	$s_{i_n j_m t} = s_{i_n j_m t-1} \exp\left(r_{e_i b_k t-1}^{-} - r_{e_j b_k t-1}^{-}\right)$

	union of e_i and j_m of e_j		

Countries, central banks, and FIGOs make up a set of players (c, e, q) which is mutually exclusive and collectively exhaustive. We set $c = \{c^{e_1}, \dots, c^{e_{n_e}}\} : c^{e_j} = \{c_1^{e_j}, \dots, c_n^{e_j}\} \forall\, j \in (1, \dots, n_e)$ where each of the n_c countries is indexed by its respective central bank e_j. Thus each country has one central bank, though several countries may share a central bank, which is the case e.g. for the European Central Bank. The set of players (b, f, h) consisting of banks, firms, and households is also mutually exclusive. However, these players are members of a given country, as shown in Figure 1. The reason for this conceptualization is that the production and consumption of banks, firms, and households add up to constitute the aggregate production and consumption of the country to which these three kinds of players belong. Consequently, we define $b = \{b^{c_1}, \dots, b^{c_{n_c}}\} : b^{c_j} = \{b_1^{c_j}, \dots, b_n^{c_j}\}, f = \{f^{c_1}, \dots, f^{c_{n_c}}\} : b^{c_j} = \{f_1^{c_j}, \dots, f_n^{c_j}\}, h = \{h^{c_1}, \dots, h^{c_{n_c}}\} : h^{c_j} = \{h_1^{c_j}, \dots, h_n^{c_j}\},$ where$c_j = c_j^{e_k}\ \forall\, j \in (1, \dots, n_c), k \in (1, .., n_e)$. This means that a country's strategic choices depend on the choices of its banks, firms, and households. However, each country makes many additional strategic choices, e.g. public sector consumption.

<u>Assumption 1. Home currency</u>

Central bank e_n has its own currency which is the home currency of country $c_j^{e_n}$ and all players within country $c_j^{e_n}$, i.e. households h^{c_j}, firms, f^{c_j}, and banks b^{c_j}. In some instances, (e.g. the European Union) multiple countries can have the same home currency.

Table 1 shows that countries have seven strategic choice variables, while the other five types of players have four of five strategic choice variables. Several of the strategic choice variables are vectors, which means that the strategic choices to be made by each player can be gigantic. Borrowing is a fundamental strategic choice available to all players. We distinguish between positive borrowing and negative borrowing. Positive borrowing is how we usually perceive borrowing, i.e. incurring debt from another player (receiving a loan). In contrast, negative borrowing is equivalent to lending which means providing credit to another player (giving a loan). The reason for this definition is that we cannot allow lending to be a strategic choice which would generate an over-determined system. Allowing one player to borrow presupposes the availability of the loan, which in reality is provided by the lender. But the lender cannot choose the size of the loan which the borrower chooses. Table 3 shows that countries, central banks, banks, and FIGOs determine interest rates, which impacts players' willingness to borrow negatively.

Borrowing is defined as $B_{i_n j_m t} \stackrel{\text{def}}{=} \begin{bmatrix} B^+_{i_n j_m t} \\ -B^-_{i_n j_m t} \end{bmatrix}$, which is a 2x1 vector where $B^+_{i_n j_m t}$ is the amount player i_n borrows positively from player j_m at time t and $B^-_{i_n j_m t}$ is the amount player i_n borrows negatively from player j_m at time t. Assuming $B^{min,+}_{i_n j_m t} \in \mathbb{R}^+$ and $B^{max,-}_{i_n j_m t} \in \mathbb{R}^+$, we define $B^{min}_{i_n j_m t} \stackrel{\text{def}}{=} \begin{bmatrix} 0 \\ -B^{min,-}_{i_n j_m t} \end{bmatrix}$ as the minimum amount of borrowing, and $B^{max}_{i_n j_m t} \stackrel{\text{def}}{=} \begin{bmatrix} B^{max,+}_{i_n j_m t} \\ 0 \end{bmatrix}$ as the maximum amount of borrowing.

The cost of borrowing is defined as $r_{i_n j_m t} \stackrel{\text{def}}{=} \begin{bmatrix} r^+_{i_n j_m t} & r^-_{i_n j_m t} \end{bmatrix}$, where $r^+_{i_n j_m t}$ is the interest rate associated with borrowing $B^+_{i_n j_m t}$, and $r^-_{i_n j_m t}$ is the interest rate associated with negative borrowing $B^-_{i_n j_m t}$. A player's debt accumulates from borrowing over time. Without loss of generality, for technical reasons we assume that debt accumulated from borrowing is repaid in each time period. Mathematically, this means that if player i_n positively borrows $B^+_{i_n j_m t}$ from player j_m at time t, it must repay $B^+_{i_n j_m t}(1 + r^+_{i_n j_m t+1})$ in period $t+1$. In the next time period $t+1$, player i_n makes a new strategic choice about how much to borrow, thus implicitly choosing his accumulated debt in period $t+1$. This means that debt can be enlarged ($B^+_{i_n j_m t+1} > B^+_{i_n j_m t}(1 + r^+_{i_n j_m t+1})$), rolled over in its entirety to the next period ($B^+_{i_n j_m t+1} = B^+_{i_n j_m t}(1 + r^+_{i_n j_m t+1})$), partially rolled over ($0 < B^+_{i_n j_m t+1} < B^+_{i_n j_m t}(1 + r^+_{i_n j_m t+1})$), or repaid in its entirety in period $t+1$, i.e. $B^+_{i_n j_m t+1} = 0$.

Analogously, debt is defined as the 2x1 vector

$$d_{i_n j_m t} \stackrel{\text{def}}{=} \begin{bmatrix} d^+_{i_n j_m t} \\ -d^-_{i_n j_m t} \end{bmatrix} = \begin{bmatrix} B^+_{i_n j_m t-1}(1 + r^+_{i_n j_m t-1}) \\ -B^-_{i_n j_m t-1}(1 + r^-_{i_n j_m t-1}) \end{bmatrix} \tag{1}$$

<u>Assumption 2. No Ponzi schemes</u>
In the final time period T each player cannot hold debt, thus preventing Ponzi schemes, i.e.

$$d_{i_n j_m T} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \ \forall \ (i_n, j_m) \in p \tag{2}$$

Although players can hold massive debt through periods 1,2,…, $T-1$, they cannot keep the debt through the entire game, i.e. for T periods, which would induce everyone to borrow without bounds. Assumption 2 is justified by the illegality of Ponzi schemes, and is also common in the literature (Uribe and Schmitt-Grohé 2013).

Assumption 3. Strategic default
Player i_n can choose to strategically default $\left(D_{i_n j_m t} = 1\right)$ on positive debt, $d^{+}_{i_n j_m t}$.

Assumption 3 enables player i_n to be relieved from its debt obligations, $d^{+}_{i_n j_m t}$, through default, though a penalty is incurred. Default can either be a strategic choice where a player defaults if they are unwilling to repay debts or can be modeled through a default rule. Equivalent to revoked collateral, the default rule requires a default penalty that a lender imposes on a borrower if default occurs. The default penalty is defined as the following 2x1 vector:

$$P_{i_n j_m t} \stackrel{\text{def}}{=} \begin{bmatrix} P^{+}_{i_n j_m t} \\ -P^{-}_{i_n j_m t} \end{bmatrix} \stackrel{\text{def}}{=} \begin{bmatrix} \psi^{+}_{i_n t} U_{i_n t}\left(x_{i_n t}, x_{-i_n t}\right) \\ -\psi^{-}_{i_n t} U_{j_m t}\left(x_{j_m t}, x_{-j_m t}\right) \end{bmatrix} \text{such that } P^{+}_{i_n j_m t} = P^{-}_{j_m i_n t} \tag{3}$$

where $P^{+}_{i_n j_m t}$ is the penalty j_m imposes on i_n, $P^{-}_{i_n j_m t}$ is the penalty i_n imposes on j_m, and $\psi_{i_n t} \stackrel{\text{def}}{=} \begin{bmatrix} \psi^{+}_{i_n t} \\ -\psi^{-}_{i_n t} \end{bmatrix}$ is the fraction of player i_n's utility at time t.

Assumption 4. A default rule
For strategic default the default rule is

$$D_{i_n j_m t} = 1\left(r^{+}_{i_n j_m t-1} B^{+}_{i_n j_m t-1} > P^{+}_{i_n j_m t}\right) \tag{4}$$

where $P^{+}_{i_n j_m t}$ is given by (3). A player defaults if interest payments $r^{+}_{i_n j_m t-1} B^{+}_{i_n j_m t-1}$ on borrowing exceed the default penalty $P^{+}_{i_n j_m t}$.

We assume that players can default even when they can afford to pay debt, cf. Eaton and Gersovitz (1981). The default rule in Assumption 4 expresses the maximum product of the prior interest $r^{+}_{i_n j_m t-1}$ and prior borrowing $B^{+}_{i_n j_m t-1}$ that player i_n is willing to pay. This maximum product equals the penalty imposed when default occurs. That is, player i_n is indifferent regarding defaulting and not defaulting when $r^+_{i_n j_m t-1} B^+_{i_n j_m t-1} = P^+_{j_m i_n t}$, which is player i_n's default threshold towards player j_m at time *t*. This means, first, that for a given interest $r^+_{i_n j_m t-1}$, player i_n is unwilling to borrow $B^+_{i_n j_m t-1}$ more than what is specified by the threshold value $P^+_{j_m i_n t}$. Second, it means that for a given borrowing level $B^+_{i_n j_m t-1}$, player i_n is unwilling to accept higher interest than that specified by $P^+_{j_m i_n t}$. Consequently, the default threshold $\tilde{d}^+_{i_n j_m t}$, which is the is the maximum debt player i_n is willing to incur from player j_m at time *t*, is defined as

$$\tilde{d}^+_{i_n j_m t} \stackrel{\text{def}}{=} \tilde{B}^+_{i_n j_m t-1}\left(1 + r^+_{i_n j_m t-1}\right) = \frac{P^+_{i_n j_m t}\left(1 + r^+_{i_n j_m t-1}\right)}{r^+_{i_n j_m t-1}} \tag{5}$$

<u>Assumption 5. Partial default</u>

Player i_n's fraction of default to player j_m at time *t* equals

$$\phi_{i_n j_m t} \stackrel{\text{def}}{=} 1 - \frac{\tilde{d}^+_{i_n j_m t}}{d^+_{i_n j_m t}} = 1 - \frac{\Psi_{i_n t} U_{i_n t}\left(x_{i_n t}, x_{-i_n t}\right)}{\tilde{B}^+_{i_n j_m t-1} r^+_{i_n j_m t}} \tag{6}$$

which is often referred to as the haircut on debt, see Duffie and Singleton (1999). Assumption 5 expresses that player i_n at default pays player j_m its default threshold of debt divided by its debt. This means that when a player defaults, it pays what it is able to pay. This allows us to reason that player i_n repays its debt in each time period conditional on default and borrowing in the past period, i.e.

$$\Delta^+_{i_n j_m t} = \left(1 - \phi_{i_n j_m t} D_{i_n j_m t}\right) d^+_{i_n j_m t}, \Delta^-_{i_n j_m t} = \left(1 - \phi_{j_m i_n t} D_{j_m i_n t}\right) d^-_{i_n j_m t} \tag{7}$$

where $\Delta^-_{i_n j_m t} = \Delta^+_{j_m i_n t}$.

Assumption 6. Firms are owned by households

We define the fraction of firm f_m owned by household h_n as $\xi_{h_n f_m}$. As a result, $\xi_{h_n f_m} \Pi_{f_m t}$ is household h_n's dividend payment from firm f_m's profit in period t.

In Assumption 6 we state that households are "investors." They are endowed with non-tradable equity in firms. As a result, they share in firm profits in the form of dividend payments each period. Additionally, an important implication of household ownership of domestic firms is that $\xi_{h_n f_m} = 0 \; \forall \; h_n \in h^{c_i}, f_m \in f^{c_j}, i \neq j, \forall f_m \in f^{c_j}$ and $\sum_{h_n \in h^{c_i}} \xi_{h_n f_m} = 1$.

Assumption 7. Firms are the sole importers and exporters

Assumption 7 implies that countries, central banks, banks, households and FIGOs neither import nor export. Thus, consumption of goods by a player occurs after trade from a domestic firm which imports. Therefore, households and countries only trade in their domestic currency, while firms manage currency exchange.

Assumption 8. Domestic ownership for all players

For firm ownership we ~~have~~ assume that $\forall h_n \in h^{c_j}: f_m \notin f^{c_j}, \xi_{h_n f_m} = 0$. Consumption and investment are not defined directionally. Assumption 8 follows the fact that aggregate consumption, for example, is $C^H_{c_n t} = \sum_{h^{c_n}_m \in h^{c_n}} C_{h^{c_n}_m t}$.

All strategic interactions of the household occur within the country; it borrows from domestic banks, owns domestic firms, invests in domestic capital, and consumes domestic goods. Firms borrow from domestic banks, demand labor from domestic households, and invest in domestic capital. This assumption is made for tractability in finding an equilibrium solution.

Assumption 8 is without loss in generality. Firms trade goods between countries. As a result, households may consume goods produced in another country by consuming a good from a domestic firm who has done the importation. Thus, for example, households in the US may consume goods from Microsoft USA who imports goods from Microsoft Germany. Similarly, the restriction that domestic firms borrow domestic implies that Microsoft Norway borrows from Norwegian banks while Microsoft UK borrows from British banks. The firms make investments as such, Microsoft France is allowed to purchase land in France while Microsoft Spain can purchase land in Spain. Consequently, each assumption is easily generalized to the actions of a single multinational (e.g. Microsoft) that interacts with households, banks, and other firms across countries. Furthermore, given our emphasis on modeling international contagion rather than cross-sector contagion, these assumptions clarify the role of the trade channel through firms and the debt channel across countries.

FIGOs, countries, central banks, and banks have the ability to set interest rates. However, on a given borrowed amount, only the most powerful player may set the rates. In the borrowing between banks and firms, for example, banks set interest rates on both deposits and loans. Table 3 presents a guide for when a player is capable of the strategic choice of setting interest rates. FIGOs have the most power and are able to set interest rates in all cases, whereas banks are only able to set interest rates to households and firms as well as other banks.

If player i_n negatively borrows (i.e. lends) $B^-_{i_n j_m t}$ to player j_m at time t, then player i_n chooses strategically the interest rate $r^-_{i_n j_m t}$ on the of debt to maximize its own utility. Following player i_n's strategic choice, which determines the conditions, player j_m chooses strategically the optimal amount, $B^+_{j_m i_n t}$ to borrow from player i_n at time t. Players may alternatively determine interest rates e.g. to optimize the balance between risk and reward. For example, they may set interest rates on negative borrowing above their own positive borrowing in addition to inflation, market, and idiosyncratic risk. A bank, for example may set borrowing rates according to

$$\hat{r}^-_{b_n i_m t} = \hat{r}^-_{e_m b_n t} + \pi_t + \mu_t + \Pr\left(D_{b_n i_m t} = 1\right)\phi_{b_n i_m t} = \hat{r}^-_{e_m b_n t} + \pi_t + \mu_t + \lambda_{b_n i_m t} \quad (8)$$

More generally, a player may set interest rates according to

$$\hat{r}^-_{i_n j_m t} = \inf\left(\hat{r}^-_{k_l i_n t}\right) + \pi_t + \mu_t + \lambda_{i_n j_m t}\ \forall i_n \in (b, e, c, q), (j_m, k_l) \in p \quad (9)$$

The interest rate $r_{i_n j_m t}$ may alternatively be specified parametrically based on various assumptions or empirics.

Assumption 9. No negative investment.

$\xi_{h_n f_m} \in [0,1]$.

Assumption 9 states that the fraction $\xi_{h_n f_m}$ of firm f_m owned by household h_n has to be positive, which means that households cannot short-sell firms. That is, we only model firm ownership rather than asset markets with the key distinction that $\sum_{h_n \in h} \xi_{h_n f_m} = 1$. While asset markets may be related to other topics in the financial risk literature, we model debt crises and account for contagion through debt and trade channels. As a result, Assumption 9 does not inhibit the generality of our results.

2.2 Currency and Covered Interest Parity

Currency plays an important role in international transactions. Everything of value (e.g. production, consumption, and borrowing) is valued in a player's home currency[3]. Thus, exchange rates serve as a good approximation of the terms-of-trade between countries. We determine exchange rates are determined using covered interest rate parity. Although uncovered interest rate parity is commonly used due to its enhancements in the treatment of uncertainty and risk, we present a framework of the world that is deterministic with extensive endogenization. Consequently, covered interest parity will capture elements of risk that are endogenous in this framework.

The condition for covered interest rate parity states that the difference in two countries' interest rates should equal the difference in one period exchange rates[4]. Using the central bank's interest rate for both countries and logarithmic differences for period exchange rates for countries $c_n \in c^{e_i}$ and $c_m \in c^{e_j}$ shows that

$$r^{-}_{e_j b_k t} + \ln\left(s_{c_n c_m t+1}\right) - \ln\left(s_{c_n c_m t}\right) - r^{-}_{e_i b_l t} = 0 \tag{10}$$

where banks $b_k \in b$ and $b_l \in b$ are arbitrary banks. Exchange rates in each period are, thus, able to be found from the previous periods' exchange rate and interest rate difference, i.e.

$$s_{c_n c_m t} = s_{c_n c_m t-1} \exp\left(r^{-}_{e_i b_l t-1} - r^{-}_{e_j b_k t-1}\right) \tag{11}$$

2.3 Equations and Utility Functions for the Players

2.3.1 <u>Households</u>

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[3] Many countries have their own central bank, but not all. Individual so-called central banks of a monetary union's members are not lenders of last resort. They act on behalf of the common central bank, and not as an individual central bank player. Consequently, they are not central banks in our notation. Instead they are individual players as regular banks, though they have various privileged statuses distinguishing them from regular banks. For example, the German Bundesbank acts on behalf of the European Central Bank (ECB) which is the central bank for Germany and the other members of the Eurozone.

[4]The fundamental assumption of covered interest rate parity is that relative changes in interest rates will be reflected in the price of currencies. This, in fact, subsumes a behavior of rational investors in foreign exchange markets. If, for example, country c_1 with currency X increases its interest rate relative to country c_2 with currency Y, covered interest rate parity implies immediate gains for selling currency Y for currency X. Thus, all investors will sell Y for X increasing the relative value of X until the relative change in interest rates is fully priced into the exchange rate satisfying (11). Consequently, without directly modeling currency exchange, (11) encapsulates exchange indirectly. It accounts for interest rate and triangular arbitrage currency exchange possibilities. Through the effect of (35) on interest rates, (11) accounts for the ability of exchange rates to be affected by monetary policy. Furthermore, without explicit modeling of currency exchange, currency exchange is generically allowed by firms importing and exporting unit goods.

We enable households to invest, consume, labor, borrow, pay taxes, and default. Households invest in a single depreciable durable good (e.g. a house), and consume a single nondurable good. Household consumption consists of individual consumption, $C_{h_n t}$, and public consumption divided equally among all households in country c_n, i.e. $C^P_{c_n t}/|h^{c_n}|$, which sums to $C_{h_n t} + C^P_{c_n t}/|h^{c_n}|$. As is common in the literature, we assume that the Arrow-Pratt measure of absolute risk aversion is satisfied, where $1/\rho_{h_n}$ is the intertemporal elasticity of substitution. We further use the preference function suggested by Greenwood et al. (1988). Using these inputs, we assume that household h_n maximizes the isoelastic utility function

$$U_{h_n t} = \frac{\left(C_{h_n t} + C^P_{c_n t}/|h^{c_n}|\right)^{1-\rho_{h_n}}}{1-\rho_{h_n}} \tag{12}$$

Household h_n has time discount factor β_{h_n}, raised to $\tau - t$ so that the current period $\tau = t$ gets no discounting. Multiplying $\beta_{h_n}^{\tau-t}$ with $U_{h_n \tau}$ in (12) and summing from the current period $\tau = t$ to the final period $\tau = T$, gives household h_n's utility discounted to period t,

$$U\left(x_{h_n t}, x_{-h_n t}\right) = \sum_{\tau=t}^{T} \beta_{h_n}^{\tau-t} U_{h_n \tau} \tag{13}$$

which is maximized subject to the budget constraint

$$\begin{aligned} I_{h_n t} + C_{h_n t} + N_{h_n c_m t} + \sum_{j_m \in p} \left[B^-_{h_n j_m t} + \Delta^-_{h_n j_m t} + P^+_{h_n j_m t} D_{h_n j_m t}\right] + \sum_{q_m \in q} \epsilon_{h_n q_m t} \\ = w_{h_n t} L_{h_n t} + \sum_{f_m \in f} \xi_{h_n f_m} \Pi_{f_m t} + \left(1-\delta_{h_n t}\right) I_{h_n t-1} \\ + \sum_{j_m \in p} \left[B^+_{h_n j_m t} + \Delta^+_{h_n j_m t} + P^-_{h_n j_m t} D_{j_m h_n t}\right] \end{aligned} \tag{14}$$

We assume that labor markets clear such that

$$L_{h_n f_m t} = L_{f_m h_n t} \tag{15}$$

Labor markets clear by letting firms have the strategy of demanding labor while households do not have the strategy of supplying labor. Thereby, we assume that households supply whatever firms demand.

2.3.2 Firms

Firm f_n invests $I_{f_n t}$ in a single nondurable good to produce capital according to the firm's capital law of motion

$$K_{f_n t} = \left(1 - \delta_{f_n t}\right) K_{f_n t-1} + I_{f_n t} \tag{16}$$

Assuming that firm f_n at time t has capital $K_{f_n t}$, and also labor $L_{h_i f_n t}$ summed over all households $h_i \in h$, firm f_n's production is defined generically as

$$Y_{f_n t} = f\left(K_{f_n t}, \sum_{h_i \in h} L_{h_i f_n t}\right) \tag{17}$$

Firm f_n's profit is

$$\begin{aligned} \Pi_{f_n t} = Y_{f_n t} - I_{f_n t} - \sum_{h_i \in h} \left(\theta_{h_n f_m t} w_{h_i t} L_{h_i f_n t}\right) + \left(X_{f_n t} - M_{f_n t}\right) - N_{f_n c_m t} \\ + \sum_{j_m \in p} \left(\vec{\mathbf{1}}\left(B_{f_n j_m t} - \Delta_{f_n j_m t}\right) - \begin{bmatrix} D_{f_n j_m t} \\ D_{j_m f_n t} \end{bmatrix}^T P_{f_n j_m t}\right) - \sum_{q_m \in q} \epsilon_{f_n q_m t} \end{aligned} \tag{18}$$

where

$$\theta_{h_n f_m t} = \begin{bmatrix} 1 \text{ if } h_n \text{ works for } f_m \\ 0, \text{otherwise} \end{bmatrix} : \sum_{f_m \in f} \theta_{h_n f_m t} \leq 1 \; \forall \; t \in [1, T] \tag{19}$$

and import costs and export revenues are given by

$$M_{f_i t} = \sum_{c_j \in c} s_{c_j f_i t} M_{f_i c_j t} \; , X_{f_i t} = \sum_{c_j \in c} X_{f_i c_j t} \tag{20}$$

We assume that firms borrow, invest, produce, import, export, pay taxes, pay wages, and can default. Each firm produces a single good and receives income in its home currency.

Multiplying firm f_n's discount factor β_{f_n}, raised to $\tau - t$, with its profit $\Pi_{f_n\tau}$, and summing from period $\tau = t$ to the final period $\tau = T$, gives its utility discounted to period t,

$$U\left(x_{f_nt}, x_{-f_nt}\right) = \sum_{\tau=t}^{T} \beta_{f_n}^{\tau-t} \Pi_{f_n\tau} \tag{21}$$

2.3.3 Banks

Assuming a market risk premium, μ_t, the interbank lending rate is defined as

$$r_{b_nb_mt}^{-} = r_{b_ne_kt}^{+} + \mu_t \;\forall\; (b_n, b_m) \in b, e_k \in e,\; r_{b_nb_mt}^{-} = r_{b_ib_jt}^{-} \;\forall\; \left(b_n, b_m, b_i, b_j\right) \in b \tag{22}$$

Thus, accounting for central bank e_k's lending rate, bank b_n borrows negatively from (i.e. lends to) bank b_m at the strategically chosen interest rate $r_{b_nb_mt}$ at time t (empirically measured by LIBOR). Assuming an idiosyncratic risk premium, λ_{i_nt}, bank b_n also strategically chooses interest rates on borrowing to firms and households, i.e.

$$r_{h_mb_nt}^{-} = r_{b_nb_kt}^{+} + \lambda_{h_mt} \;\forall\; (b_n, b_k) \in b, h_m \in h \tag{23}$$

Accounting for interest payments on positive and negative debt, penalties from default, and subtracting net lump sum transfers $N_{b_nc_mt}$ and endowments to FIGOs, bank b_n's profit is

$$\Pi_{b_nt} = \sum_{j_m \in p} \left(\vec{\mathbf{1}}\left(B_{b_nj_mt} - \Delta_{b_nj_mt}\right) - \begin{bmatrix} D_{b_nj_mt} \\ D_{j_mb_nt} \end{bmatrix}^T P_{b_nj_mt} \right) - N_{b_nc_mt} - \sum_{q_m \in q} \epsilon_{b_nq_mt} \tag{24}$$

Multiplying bank b_n's discount factor β_{b_n}, raised to $\tau - t$, with its profit $\Pi_{b_n\tau}$, and summing from period $\tau = t$ to the final period $\tau = T$, gives its utility discounted to period t,

$$U\left(x_{b_nt}, x_{-b_nt}\right) = \sum_{\tau=t}^{T} \beta_{b_n}^{\tau-t} \Pi_{b_n\tau} \tag{25}$$

2.3.4 Countries

Country c_n's production supplied to the market at time t is

$$Y_{c_nt} = A_{c_nt} K_{c_nt}^{\varsigma_{c_n}} {L_{c_nt}}^{1-\varsigma_{c_n}} \tag{26}$$

assuming labor L_{c_nt}, and capital K_{c_nt} which grows according to the capital law of motion,

$$K_{c_n t} = (1 - \delta_{c_n}) K_{c_n t-1} + I_{c_n t-1}, \tag{27}$$

and productivity $A_{c_n t}$ which grows with growth rate g_{c_n}, i.e.:

$$A_{c_n t+1} = (1 + g_{c_n}) A_{c_n t} \tag{28}$$

The production demanded by the market is the sum of consumption $C_{c_n t}$, investment $I_{c_n t}$, and net exports $(X_{c_n t} - M_{c_n t})$, i.e.

$$\begin{aligned} & C_{c_n t} + I_{c_n t} + (X_{c_n t} - M_{c_n t}) \\ &= C^P_{c_n t} + I^P_{c_n t} + \sum_{h_n \in h^{c_n}} (C_{h_n t} + I_{h_n t}) + \sum_{h_n \in h^{c_n}} (C_{f_n t} + I_{f_n t}) + (X_{c_n t} - M_{c_n t}) \\ &= C^P_{c_n t} + I^P_{c_n t} + C^H_{c_n t} + I^H_{c_n t} + I^F_{c_n t} + (X_{c_n t} - M_{c_n t}) \end{aligned} \tag{29}$$

We assume clearance of the market for goods

$$\sum_{c_n \in c} Y_{c_n t} = \sum_{c_n \in c} \left(C_{c_n t} + I_{c_n t} + (X_{c_n t} - M_{c_n t}) \right) \tag{30}$$

From the definitions of (26)-(30) one can define the trade balance as well as the current account. The trade balance is defined by subtracting consumption and investment from production:

$$TB_{c_n t} \stackrel{\text{def}}{=} Y_{c_n t} - C_{c_n t} - I_{c_n t} = (X_{c_n t} - M_{c_n t}) \tag{31}$$

The current account is found by adding interest payments on external debt:

$$CA_{c_n t} \stackrel{\text{def}}{=} TB_{c_n t} - \sum_{j_m \in p} (r_{c_n j_m t} \times \Delta_{c_n j_m t}) \tag{32}$$

Equations (30), (31), and (32) imply that $\sum_{c_n \in c} TB_{c_n t} = 0$ and $\sum_{c_n \in c} CA_{c_n t} = 0$.

The budget constraint is

$$\sum_{i_m \in c_n} N_{i_m c_n t} + \sum_{j_m \in p} \left(\vec{\mathbf{1}} \left(B_{c_n j_m t} - \Delta_{c_n j_m t} \right) - \begin{bmatrix} D_{c_n j_m t} \\ D_{j_m c_n t} \end{bmatrix}^T P_{c_n j_m t} \right) = C^P_{c_n t} + I^P_{c_n t} + \sum_{q_m \in q} \epsilon_{c_n q_m t} \tag{33}$$

where $\sum N_{i_m c_n t}$ is lump sum transfers between households, firms, and banks on the one hand, and country c_n (the government) on the other hand.

We introduce additional realism and generality beyond the common assumption (e.g. Chari and Kehoe (1999)) that the aggregate utility of the households jointly constitutes country c_n's utility. That is, we assume that each country maximizes four objectives assigned different weights. First, to account for social welfare, we consider aggregate household utility. Second, to account for debt, we consider the GDP-to-debt ratio $Y_{c_n t}/\max\{\Delta^+_{c_n t}, \kappa Y_{c_n t}\}$. This max function equals $Y_{c_n t}/d^+_{c_n t}$ if country c_n has large positive debt, and $1/\kappa$ otherwise. We introduce $1/\kappa$ since country c_n with low debt otherwise gets infinite utility, thus bounding utility as debt approaches zero. The third objective is rent $R_{c_n t}$, which is maximized by various dictatorships. Fourth, to account for all possibilities, we consider various non-economic or alternative motives $Z_{c_n t}$ maximized by some countries (e.g. military powers, North Korea). Assuming Cobb-Douglas preferences with parameters $0 \leq \zeta_{c_n}, \alpha_{c_n}, \gamma_{c_n}, \nu_{c_n} \leq 1$ for these four objectives, multiplied with country c_n's discount factor β_{c_n} raised to $\tau - t$, and summing from period $\tau = t$ to the final period $\tau = T$, gives country c_n's utility discounted to period t,

$$U\left(x_{c_n t}, x_{-c_n t}\right) = \sum_{\tau=t}^{T} \beta_{c_n}^{\tau-t} \left[\sum_{h_m \in c_n} U_{h_m \tau} \right]^{\zeta_{c_n}} \left[\frac{Y_{c_n \tau}}{\max\{\Delta^+_{c_n \tau}, \kappa Y_{c_n \tau}\}} \right]^{\alpha_{c_n}} R_{c_n \tau}^{\gamma_{c_n}} Z_{c_n \tau}^{\nu_{c_n}} \tag{34}$$

2.3.5 <u>Central Banks</u>

Central bank e_n is the lender of last resort for a monetary union and determines the target inflation rate $\pi^*_{e_n t}$ and the interest rate for negative borrowing (lending) to banks, often expressed with the Taylor (1993) rule (Gauti B Eggertsson 2003)

$$r^{-}_{e_n b_m t} = \max\left\{\pi_{e_n t} + r^{*}_{e_n t} + a_{\pi}\left(\pi_{e_n t} - \pi^{*}_{e_n t}\right) + a_{Y}\left(\log\left(\frac{Y_{c^u_n t}}{\bar{Y}_{c^u_n t}}\right)\right), 0\right\} \tag{35}$$

where $a_{\pi} > 0, a_{Y} > 0$.

Central bank e_n's utility $U_{e_n t}$ is a Cobb-Douglas function with two inputs and parameters $0 \leq \alpha_{e_n}, \chi_{e_n} \leq 1$, i.e. aggregate household utility defined in (12) and an unspecified input $\Gamma_{e_n t}$ reflecting various objectives. Multiplying these two inputs with central bank e_n's discount factor β_{e_n}, raised to $\tau - t$, and summing from period $\tau = t$ to the final period $\tau = T$, gives its utility discounted to period t,

$$U\left(x_{e_n t}, x_{-e_n t}\right) = \sum_{\tau=t}^{T} \beta_{e_n}^{\tau-t} \left(\sum_{h_m \in c^u_n} U_{h_m \tau}\right)^{\alpha_{e_n}} \Gamma_{e_n \tau}^{\chi_{e_n}} \tag{36}$$

subject to the budget constraint

$$\sum_{j_m \in p} \left[\left(B^{+}_{e_n j_m t} + \Delta^{-}_{e_n j_m t} + P^{-}_{e_n j_m t} D_{j_m e_n t}\right) - \left(B^{-}_{e_n j_m t} + \Delta^{+}_{e_n j_m t} + P^{+}_{e_n j_m t} D_{e_n j_m t}\right)\right] = \sum_{q_m \in q} \epsilon_{e_n q_m t} \tag{37}$$

2.3.6 Financial Inter-Governmental Organizations

FIGOs receive money in the form of contributions from countries and lend money to selected countries assessing market risk μ_t and idiosyncratic risk $\lambda_{j_m t}$. Households, firms, and banks pay taxes to their country which then pay FIGOs. Although contributions to FIGO q_n, $\epsilon_{i_m q_n t}$, typically come from countries, for generality, we allow for contributions from a generic player $i_m \in p$. We assume that FIGO q_n's negative borrowing rate to country c_m at time t equals the inflation rate $\pi_{c_m t}$, i.e.

$$r^{-}_{q_n c_m t} = \pi_{c_m t} \;\forall\; c_m \in c \tag{38}$$

FIGO q_n maximizes a weighted average of production in each country, i.e.

$$U\left(x_{q_n t}, x_{-q_n t}\right) = \sum_{\tau=t}^{T} \beta_{q_n}^{\tau-t} \sum_{c_m \in c} \omega_{c_m} Y_{c_m \tau} \tag{39}$$

discounting to period t, from period $\tau = t$ to the final period $\tau = T$, where ω_{c_m} is the weight on country c_m's production such that $\omega_{c_m} \in [0,1]$ and $\sum_{c_m \in c} \omega_{c_m} = 1$. FIGO q_n chooses ω_{c_m} to target countries in various manners. For example, in (39), $\omega_{c_m} = \frac{1}{n_c}$ means equal weight to each country, and $\omega_{c_m} = \frac{1/Y_{c_m t}}{\sum_{c_j \in c} 1/Y_{c_m t}}$ means that FIGO q_n allocates to country c_m inversely proportional to country c_m's GDP $Y_{c_m t}$ (where the denominator ensures $\sum_{c_m \in c} \omega_{c_m} = 1$).

FIGO q_n maximizes its utility in (39) subject to the budget constraint

$$\sum_{i_m \in p} \epsilon_{i_m q_n t} + G_{q_n t}$$
$$= \sum_{j_m \in p} \left[\left(B^{+}_{q_n j_m t} + \Delta^{-}_{q_n j_m t} + P^{-}_{q_n j_m t} D_{j_m q_n t}\right) - \left(B^{-}_{q_n j_m t} + \Delta^{+}_{q_n j_m t} + P^{+}_{q_n j_m t} D_{q_n j_m t}\right)\right] \tag{40}$$

The objective of FIGOs is to maximize the benefit of the countries it lends to. Equation (39) however, sums over all countries in the world. This is consistent with the objective for two reasons. First, FIGOs only have a direct impact on the countries they lend to. If there is no indirect impact, then this sum is a monotone transformation of countries they lend to. Moreover, any indirect impacts may be considered externalities, and maximizing these externalities would also be consistent with FIGOs objectives. Consequently, we sum over all countries in the world to maximize FIGO q_n's utility.

2.4 Conditions for Nash Equilibrium

An equilibrium is a set of strategies $\left(x^{*}_{c_n t}, x^{*}_{e_n t}, x^{*}_{b_n t}, x^{*}_{h_n t}, x^{*}_{q_n t}\right)$ such that

$$U\left(x^{*}_{i_n t}, x^{*}_{-i_n t}\right) \geq U\left(x_{i_n t}, x_{-i_n t}\right) \ \forall \ i_n \in p, x_{i_n t} \in S_i, x_{i_n t} \neq x^{*}_{i_n t} \tag{41}$$

3. Numerical Analysis of a Single Country in One Period Games

In sections 3-6 we illustrate the model with specific examples. The many strategic choices quickly complicate so for each country we consider the four strategic choice variables public consumption C^{P}_{ct}, public investment I^{P}_{ct}, borrowing B_{ijt}, and default D_{ijt}, parametrically setting the other choice variables to baseline values. Additionally, in section 6 Germany, acting as a lender, strategically chooses the interest rate $r_{i_n j_m t}$.

Let us consider one country, 10 identical firms, and 1000 identical households. The parameter values for these players are listed in Table 6. Neither households nor firms borrow. Assume that public consumption C^P_{ct}, and investment I^P_{ct}, are funded through lump sum transfers and borrowing. Therefore C^P_{ct}, I^P_{ct}, and B_{ijt} are decision variables in the countries' optimization problem.

Table 6: Baseline parameters values.

Parameter	Value
A_{ct}	1
K_{ct}	41.25
L_{ct}	$1/3$
ς_c	0.8
ρ_h	0.5
$I^H_{ct} = I^F_{ct}$	0
δ	0.05
$\beta_c = \beta_h$	0.95
$\Pi_{f_n t}$	0
$B_{ft} = B_{ht}$	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
$\gamma_c = \nu_c$	0
α_c	0.5
κ	0.2
F_{cqt}	0
$r_{cjt} = r_{ct}$	$\begin{bmatrix} 0.05 \\ 0.05 \end{bmatrix}$
$M_{ct} = X_{ct}$	15
D_{cjt}	0
N^H_{ct}	3
N^F_{ct}	5

The single country chooses strategies to maximize utility given (26)-(30) and (33)-(34). However, we assume the strategies of all other players are fixed. Hence household investment (capital) and labor are constants. More specifically, given the parameter values in Table 6, household before tax income is $(Y_{ct} - N^F_{ct})/n^c_h$, making aggregate household consumption $C^H_{ct} = Y_{ct} - N^F_{ct} - N^H_{ct}$. Household consumption is therefore determined solely by the strategic choices of the single country. Consequently, the single country optimizes the tradeoff of debt for social welfare with utility that it maximizes subject to budget constraints of the country, firm, and household as well as production constraints, laws of motion, and market clearing conditions.

3.1 Sensitivity of Utility in a One Period Game

We start by analyzing the sensitivity of the single country's utility to changes in domestic allocations and parameters.

3.1.1 Sensitivity of Utility to Public Consumption

In Figure 2, household utility increases as public consumption increases. As the country's public consumption increases above the revenue of net lump sum transfers, debt increases leading to an optimal country public consumption of $C_{ct}^{P} = 11$. As consumption increases above $C_{ct}^{P} = N_{ct}^{H} + N_{ct}^{F} = 8$, debt increases according to the budget constraint in (33). However the country utility in (34) decreases after debt increases above $d_{ct}^{+} = \kappa Y_{ct} = 3$, as shown in Figure 2 when C_{ct}^{P} exceeds 8+3=11.

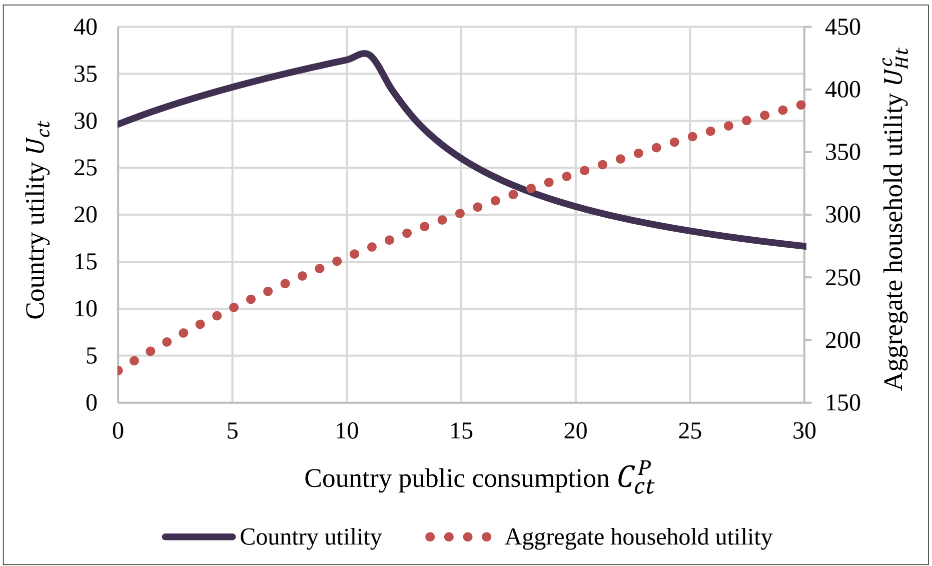

Figure 2: The effect of public consumption C_{ct}^{P} on country utility U_{ct}.

3.1.2 Sensitivity of Utility to Lump Sum Transfers

In Figure 3, country c has the additional strategic choice of selecting a lump sum transfer for households, N_{hct}. Assuming $C_{ct}^{P} = 11$, Figure 3 shows that as N_{ct}^{H} increases, country c's utility increases to a maximum at $N_{ct}^{H} = 3$ and thereafter decreases. However, household utility decreases as N_{ct}^{H} increases.

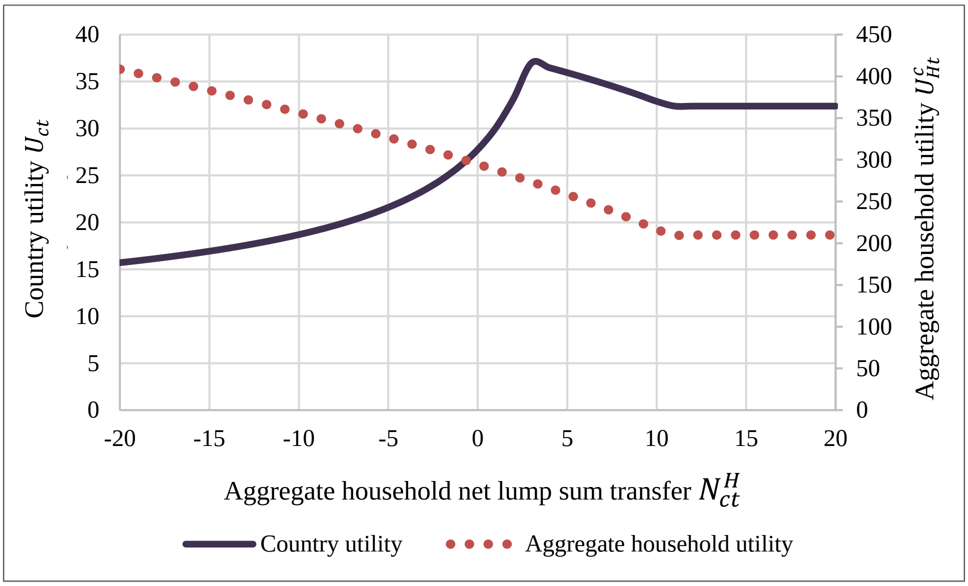

Figure 3: The effect of lump sum transfers N^H_{ct} on country utility U_{ct}.

3.1.3 Sensitivity of Utility to Public Consumption and Austerity

Figure 4 specifies the country's utility as a function of public consumption, C^P_{ct}, assuming two austerity levels. First we consider Consumptionland with low austerity $\alpha_{Consumptionland} = 0.1$. Second we consider Austerityland with high austerity $\alpha_{Austerityland} = 0.9$. Consumptionland's utility increases unboundedly in debt and therefore it chooses maximal debt. Austerityland has maximum utility when its debt is $d^+_{Austerityland,t} = 3$, and thereafter its utility decreases.

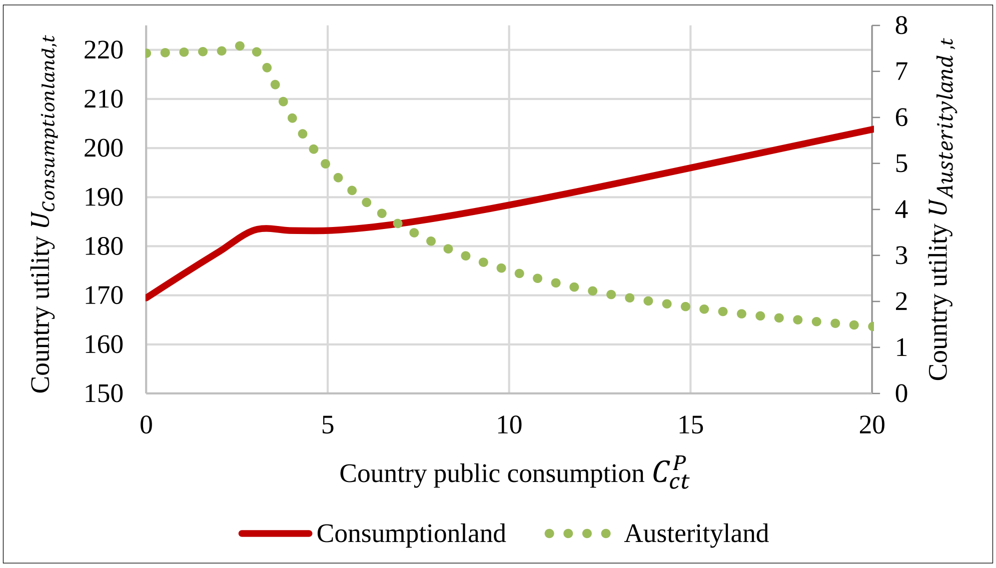

Figure 4: The effect of public consumption C^P_{ct} and austerity α_c on country utility U_{ct}.

3.1.4 Sensitivity of Utility to Interest Rate Shocks

The effects of various shocks pose risks to the country and its firms and households. First consider the effect of a positive interest rate shock. Figure 5 shows the effect of a fivefold increase in interest rate r_{ct}, from 5% to 25%, where a country has debt $d^+_{ct-1} = 6$. The shock decreases the country's utility and decreases optimal public consumption C^P_{ct} leaving household utility unaffected as a function of C^P_{ct}.

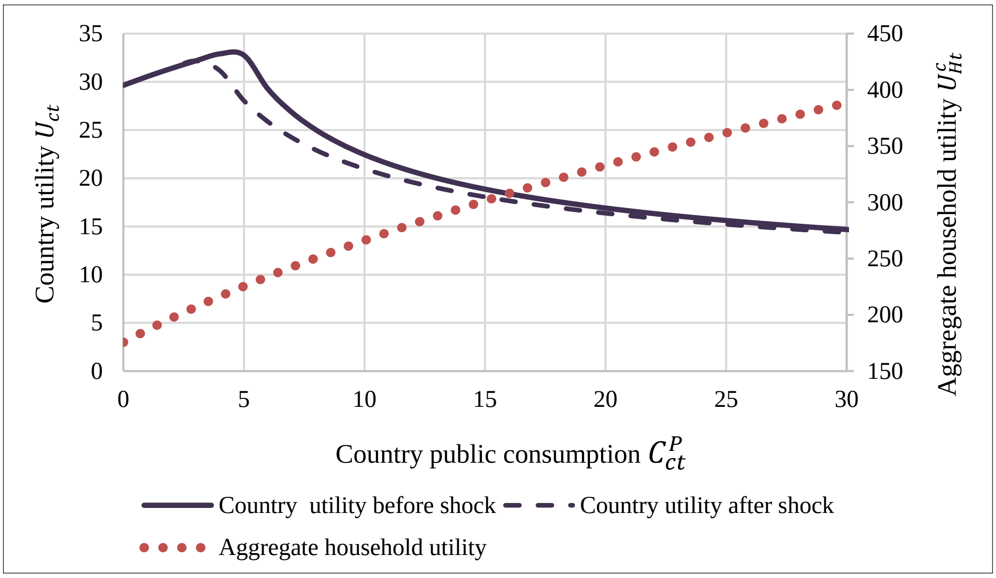

Figure 5: The effect of public consumption C^P_{ct} and a fivefold positive increase in interest rate r_{ct}, from 5% to 25%, on country utility U_{ct}.

3.2 Equilibrium in a Ten Period Model with and without Productivity Shocks

We next consider a ten period model where investment over time is bounded by a 15% absorptive capacity rate (Cohen and Levinthal 1990). Assuming initial investment I^P_{c0}=3, and austerity $\alpha_c = 0$ and $\zeta_c = 1$ so that country c maximizes aggregate household utility in (34), Figure 6 specifies the optimal strategy where the country chooses C^P_{ct}, I^P_{ct}, and B^+_{cjt} in each period to maximize utility. Without a shock (shown in the three top panels in Figure 6) increases in public investment because increased capital, production, household income, consumption, and utility. As household utility increases, the country responds by decreasing spending on public consumption and increasing spending on investment.

However, a -20% shock (shown in the three bottom panels in Figure 6) to productivity A_{ct} (which decreases GDP 20%) in period 3 temporally decreases household consumption 39%. The country's optimal response is a temporary increase in public consumption financed by debt causing household utility in period 3 to decrease only 1.5%. The country repays this debt over the next four periods. The shock decreases total country utility by 0.5% over the 10 periods. As a comparison, a permanent -20% shock to A_{ct} for all periods 3-10 decreases country utility by 11%. The behavior illustrated in this section also applies for common shocks affecting multiple countries.

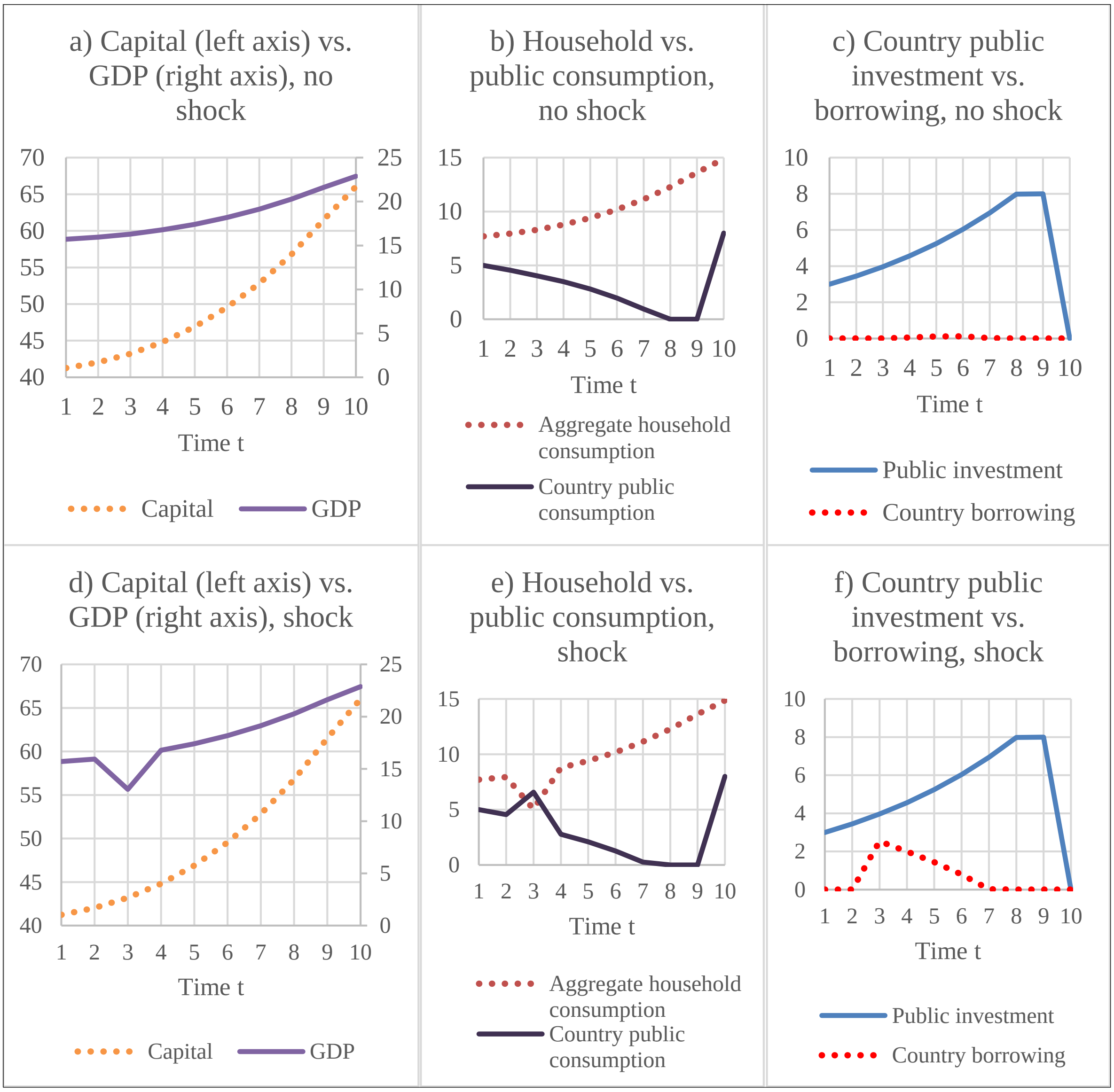

Figure 6: The results of ten period numerical analysis without (top three panels) and with (bottom three panels) a -20% shock to productivity A_{ct} in period 3.

4. Numerical Analysis of Two Countries in One Period Games

Consider a world with country c_1 and country c_2, each with 10 identical firms, 1000 identical households, and parameter values as in Table 6.

4.1 Sensitivity of Utility to Public Consumption with and without Productivity Shocks

The solid brown curve in Figure 7 replicates Figure 2 for both countries. We consider a -40% shock to productivity A_{c_1t} in country c_1, causing decreased imports $M_{c_1t} = 9 = X_{c_2t}$ to country c_1, which is equivalent to decreased exports from country c_2. The shock is to country c_1 for both the dotted purple curve and the blue dashed curve in Figure 7. Both countries suffer lower utilities after the shock showing the risk of contagion through the trade channel. Country c_1 (dotted purple line) suffers especially low utility when consumption is above the optimum whereas country c_2 (dashed blue line) suffers especially low utility when consumption is below the optimum.

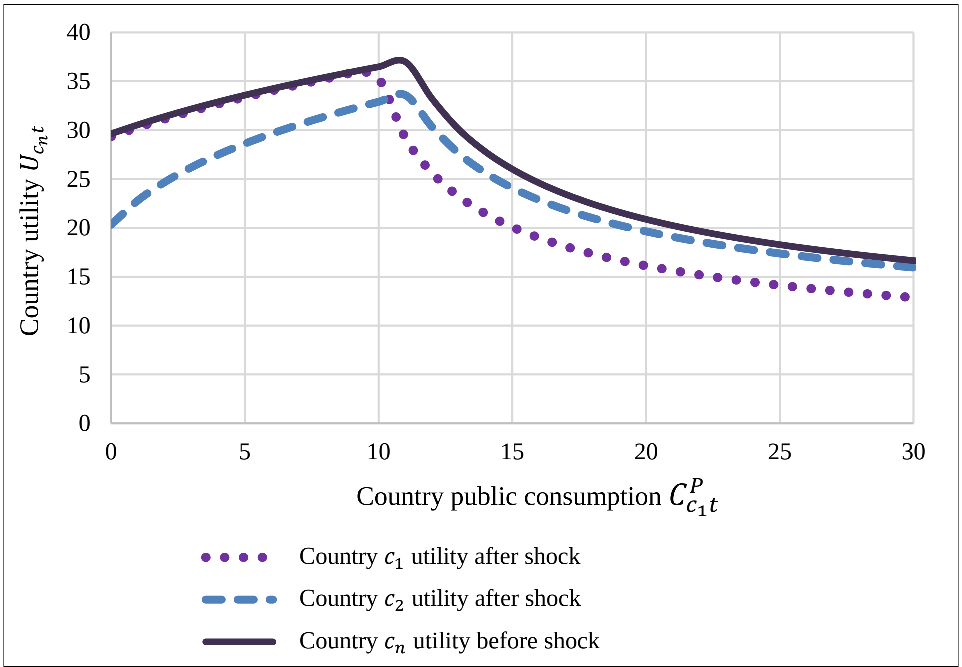

Figure 7: The effect on utility of a -40% shock to country c_1's productivity A_{c_1t} as a function of country c_1's public consumption $C^P_{c_1t}$ assuming two countries.

4.2 Introducing the Threat of Default

We now proceed to consider the threat of default. Consider two countries called Borrower (subscript B) and Lender (subscript L) with the same traits except that Borrower borrows from Lender with initial debt $d^+_{BLt-1} = 3$. Furthermore, assume that Lender optimally consumes $C^P_{Lt} = 11$. Additionally, assume that defaults are not partial (i.e. $\phi_{BLt}D_{BLt} = 1$) and that the default penalty is 20% of Borrower's utility, a sufficiently high penalty to prevent Borrower default. A low default penalty may induce the borrower into default. Figure 8 shows two inverse U-shaped curves for Borrower's utility (solid brown before the shock, dotted purple after the shock). The maximum (approximately the same for both curves) corresponds to the optimum Borrower public consumption C_{Bt}^{P} specified along the horizontal axis. Increasing Borrower public consumption above the optimum gives distinctly lower utilities for both Borrower and Lender. Both utilities are lower after the -40% shock to Borrower's productivity A_{Bt}. As Borrower public consumption increases above the optimum, the impact of the shock on the both utilities decreases. For Borrower public consumption below the optimum, Lender receives the same utility before and after the shock.

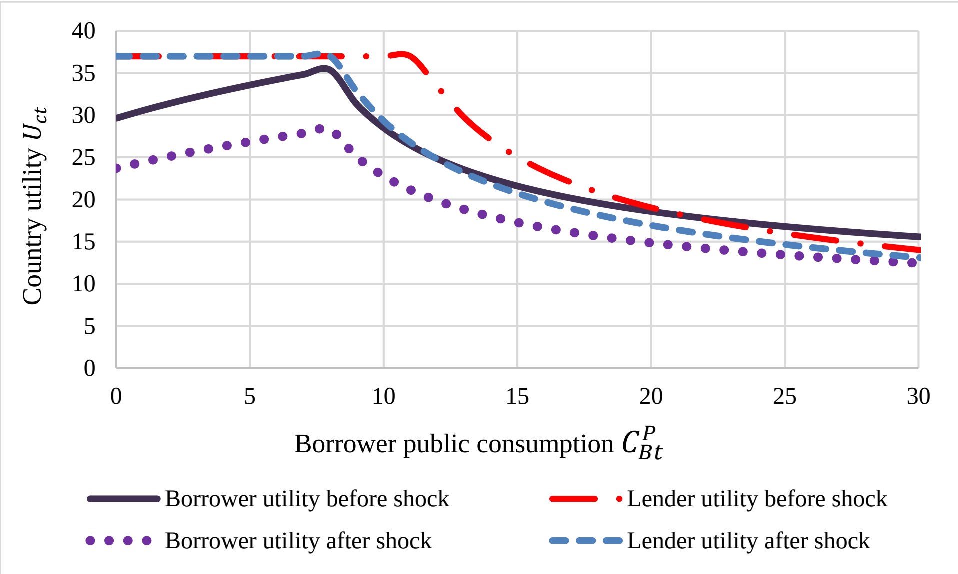

Figure 8: The effect on Borrower's and Lender's utilities of a -40% shock to Borrower's productivity A_{Bt} as a function of Borrower's public consumption C_{Bt}^{P} when default is averted.

5. Numerical Analysis of Greece

We consider a single country environment with Greece (denoted by subscript G), defined with n_h equivalent households and n_f equivalent firms, parametrized with actual 2007 values for Greece in Table 7. In this environment, Greece is the only player that borrows and invests. We model default as a strategic choice and its penalty exogenously as in (3). We perform sensitivity analysis and introduce a negative shock of X% to productivity A_{c_nt} causing an after-shock productivity of $A'_{c_nt} = (1 - X)A_{c_nt}, n = 1$. In order to compute solutions, we compress the large strategy sets presented in Section 2 by using 2007 Greek values in Table 7 to parameterize specific strategies as values. These values are interest rates on borrowing r_{Gt}^{+}, imports M_{Gt}, and the lump sum transfer for households and firms, i.e. $\sum_{h_m \in h^G} N_{h_m Gt}$ & $\sum_{f_l \in h^G} N_{f_l Gt}$. Therefore, Greece's strategy set is reduced to $S_G = \{B, D, C, I\}$, i.e. borrow, default, consume, and invest. Furthermore, we guarantee constant returns to scale by setting $\alpha_G = 1 - \zeta_G$ in (34).

Table 7: Baseline parameter values for Greece.

Parameter	Value
B^{+}_{G2006} (€ billions)	264.06
K_{G2007} (€ billions)	828.16
$\sum_{f_l \in f^G} N_{f_l Gt}$ (€ billions)	5.70
$\sum_{h_m \in h^G} N_{h_m Gt}$(€ billions)	10.82
$\sum N_{i_m Gt}$ (€ billions)	72.57
A_{G2007}	0.0814
L_{Gt} (Thousands)	4916.51
$I^H_{Gt} = I^F_{Gt}$	0
$r^{+}_{Gj_m t} = r^{+}_{Gt}$	4.07%
n_f[5]	4,089,012
β_G[6]	0.90
δ_G	0
α_G	0.5
ζ_G	1-α_G=0.5
ς_G[7]	0.3
ρ_{h_n}	0.5
$\epsilon_{Gq_m t}$ (€ billions)	0
κ	0.2
n_h[8]	858,685

Source: Federal Reserve Economic Data (FRED), Federal Reserve Bank of St. Louis, World Bank Population Data, OECD, based on 2006 and 2007 values for Greece. For the complete explanation for the estimation of productivity, see Appendix A.

5.1 Sensitivity of Utility to Public Consumption in a One Period Game

We start by analyzing the sensitivity of Greek utility to changes in domestic allocations and parameters. Figure 9 specifies Greece's utility $U(x_{G2007}, x_{-G2007})$, defined in (25), and aggregate household utility, $U^H_{G2007} \equiv \sum_{h_n \in h^G} U\left(x_{h_n 2007}, x_{-h_n 2007}\right)$, defined in (13), as a function of Greece's public consumption C^P_{G2007}. Aggregate household utility U^H_{G2007} increases

[5] Total Greek firms with estimated from 2008 data (Commission 2012).
[6] It is often set between 0.88 and 0.95 (Park 2012).
[7] Welburn and Hausken (2015) present a graph (Fig. 13 in Appendix A) to demonstrate that the United States capital-output ratio ς_{c_n} has remained relatively constant around 0.3 from 1960-2012. Hall and Jones (1999) present in Table 7 productivity calculations in ratio to the values of the United States demonstrating a relative similarity of Greece and United States capital-output ratios. We therefore normalize the capital-output ratio to ς_G=0.3 as an approximation.
[8] Number of households estimated using population data and an average household size of 2.73 persons as given by OECD (2015).

as C_{G2007}^{P} increases since households enjoy public consumption in a one-period model when the future is irrelevant. In contrast, Greece's utility has a maximum for intermediate public consumption $C_{G2007}^{P} = \sum N_{i_m Gt} = 72.57$ billion € which is above Greece' 2007 empirical value of public consumption, $C_{G2007}^{P} = 46.4$ billion €. This means that Greece benefits from increased consumption only to a certain extent. Above the optimal consumption, Greece's utility $U(x_{G2007}, x_{-G2007})$ decreases as C_{G2007}^{P} increases, due to incurring debt.

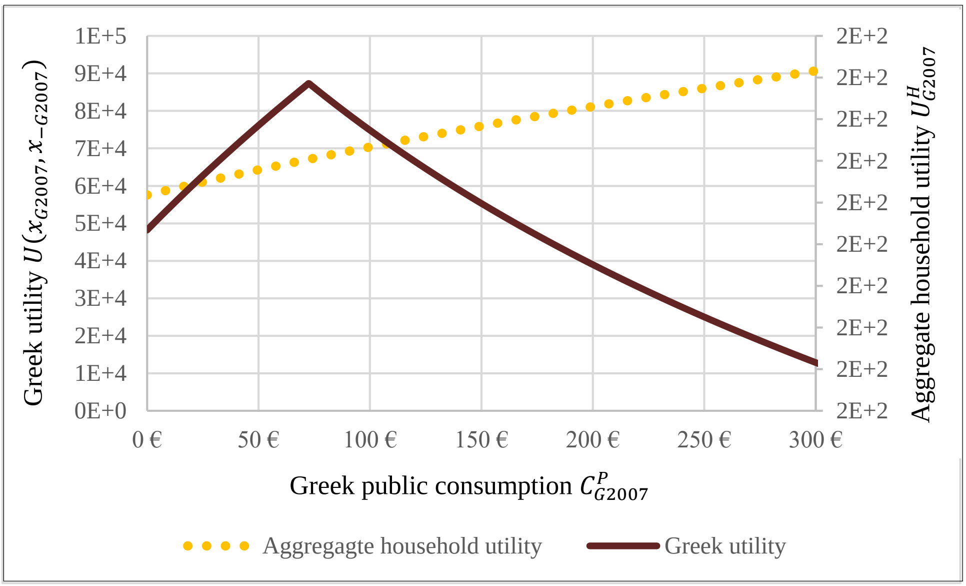

Figure 9: Greece's utility $U(x_{G2007}, x_{-G2007})$ and aggregate household utility U_{G2007}^{H} as functions of Greece's public consumption C_{G2007}^{P} assuming one country and one period.

5.2 Sensitivity of Utility in a Two Period Game

Although the one period model in Figure 9 computes that Greece would have benefitted from higher public consumption, the single period assumption makes the future irrelevant. We thus proceed with the two period model, where 2007 is period 1 and 2008 is period 2.

5.2.1 Sensitivity of Utility to Public Consumption

While two periods still limit the realism, Figure 10 specifies Greece's utility $U(x_{G2007}, x_{-G2007})$ (defined in (25)), and aggregate household utility U_{G2007}^{H} (defined in (13)) as functions of Greece's public consumption C_{G2007}^{P}. These two utilities follow from summing the 2007 utility and the discounted 2008 utility. In 2007, Greece spends its entire revenue from lump sum transfers, $C_{G2007}^{P} = \sum N_{i_m Gt} = 72.57$ billion €, on public consumption C_{G2007}^{P}, and additionally chooses no borrowing, i.e. $B_{G2007}^{+} = 0$. With a lower interest rate r_{Gt}^{+}, or a different discount factor β_G, Greece may borrow in 2007. Then, in 2008, Greece would use its income from lump sum transfers on public consumption and to pay back debt. Borrowing would enable Greece to smooth consumption over the two periods. Greece strikes a balance, and is adversely affected by high interest rates. Due to borrowing costs from accrued interest, borrowing in 2007 stifles future consumption adversely affecting consumption smoothing households. Thus, Greece finds borrowing in 2007 suboptimal in Figure 10. In the absence of shocks, Greece exhausts its revenue from the sum of transfers from households and firms to the government on public consumption $C^P_{G2007} = C^P_{G2008} = \sum N_{i_m Gt} = 72.57$ in both periods, combined with $B^+_{G2007} = 0$, while households consume their income.

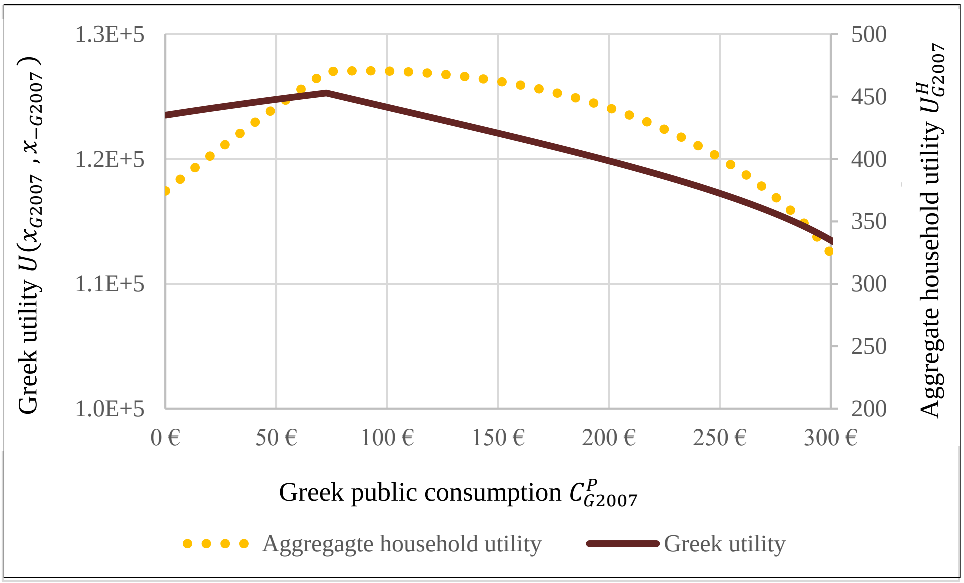

Figure 10: Greece's utility $U(x_{G2007}, x_{-G2007})$ and aggregate household utility U^H_{G2007} as functions of Greece's public consumption C^P_{G2007} assuming one country and two periods.

5.2.2 Sensitivity of Utility to Negative Productivity Shocks

However, the same is not true in the presence of a shock. Consider a temporary shock to productivity, A_{G2007}, only in 2007. Figure 11 shows the effect of no shock, a -30% shock, a -50% shock, and a -70% shock. Shown in Figure 11a, Greece's utility $U(x_{Gt}, x_{-Gt})$, expectedly, decreases with negative shocks. Temporary negative productivity shocks induce Greek borrowing, B^+_{G2007}. More severe shocks cause households to benefit from Greek deficit spending on public consumption, C^P_{G2007}, illustrated with household utility in Figure 11b.

Negative shocks decrease the marginal utility of borrowing. Despite concerns for debt, Greece optimally borrows in 2007 to smooth consumption over the two periods.

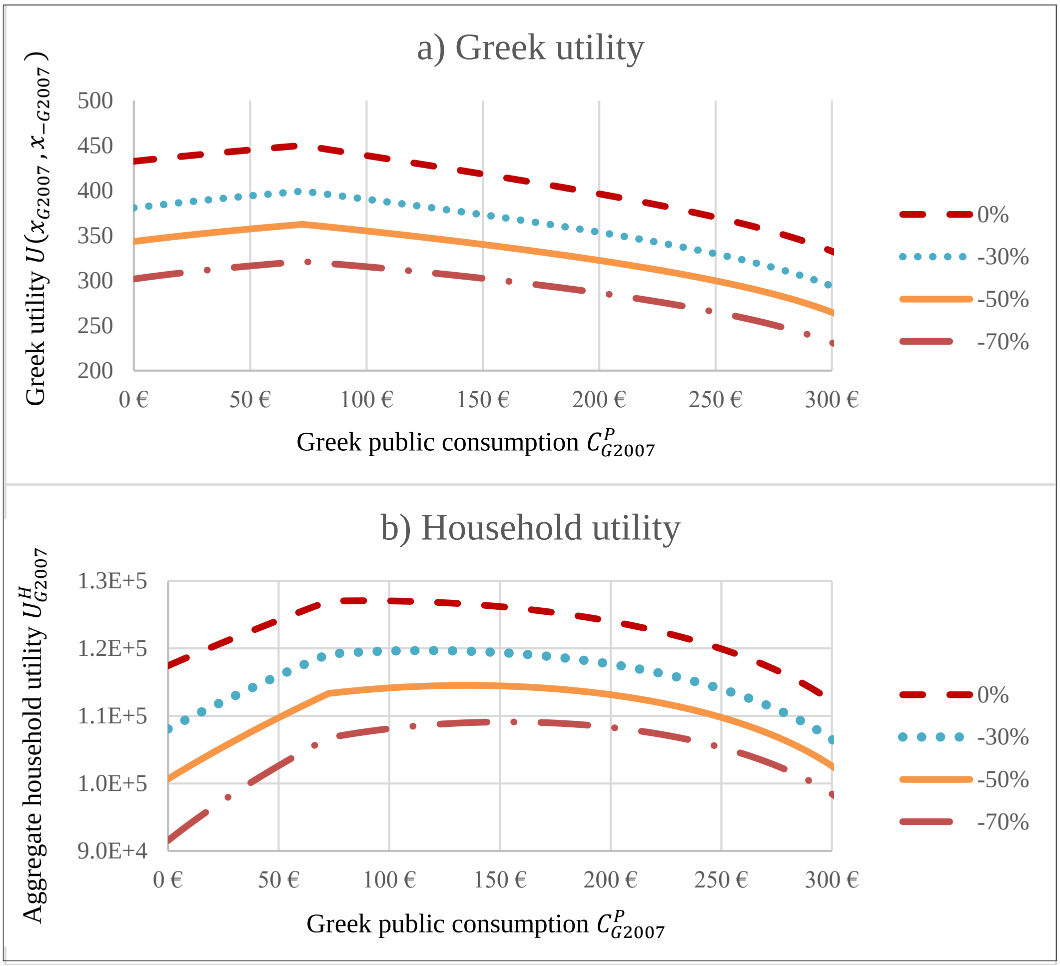

Figure 11: Greece's utility $U(x_{G2007}, x_{-G2007})$ and aggregate household utility U^H_{G2007} as functions of Greece's public consumption C^P_{G2007} assuming one country, two periods and negative productivity shocks, where the percentages express shocks relative to productivity A_{G2007}.

5.2.3 Sensitivity of Utility to Public Consumption and Austerity

Figure 12 plots Greece's utility $U(x_{G2007}, x_{-G2007})$ as a function of public consumption C^P_{G2007} assuming four austerity levels from minimum $\alpha_G = 0$ (entire emphasis on improving social welfare), $\alpha_G = 0.1$, $\alpha_G = 0.9$, to maximum $\alpha_G = 1$ (entire emphasis on constraining debt). All curves are inverse U-shaped. Greece's utility $U(x_{G2007}, x_{-G2007})$ decreases as austerity α_G increases. Furthermore, Greece's optimal public consumption C^P_{G2007}, (causing maximum utility $U(x_{G2007}, x_{-G2007})$) which is $C^P_{G1} = 89$ billion € for minimum austerity ($\alpha_G = 0$), decreases to $C^P_{G2007} = 72.6$ billion € for intermediate austerity ($0.1 \leq \alpha_G \leq 0.9$), and decreases to $C^P_{G2007} = 0$ for maximum austerity ($\alpha_G = 1$). Thus under low austerity Greece prefers consumption at or above $C^P_{G2007} = 72.6$ billion €, while under high austerity Greece prefers consumption at or below $C^P_{G2007} = 72.6$ billion €, assuming two periods.

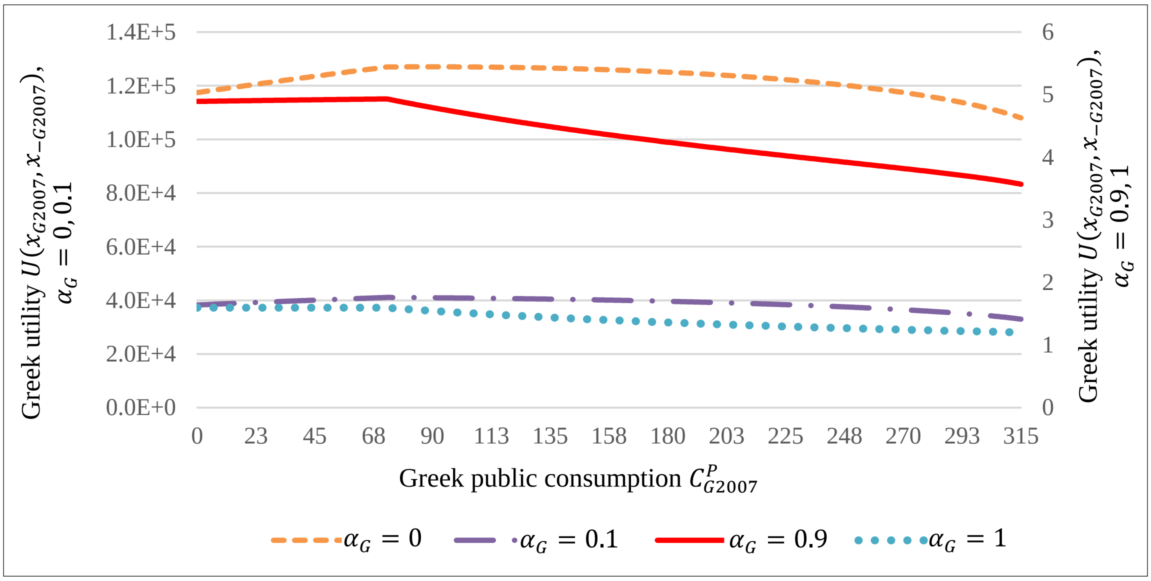

Figure 12: Greece's utility $U(x_{G2007}, x_{-GG2007})$ as a function of Greece's public consumption C^P_{G1} assuming one country, two periods, and four austerity levels from $\alpha_G = 0$ to $\alpha_G = 1$.

Assuming shocks, Figure 13 specifies Greece's utility $U(x_{G2007}, x_{-G2007})$ as a function of Greece's public consumption C^P_{G2007}. Greece's utility $U(x_{G2007}, x_{-G2007})$ decreases as shocks become increasingly negative, with interior optimal public consumption C^P_{G2007} and C^P_{G2008} equivalent to the optima in Figure 12 regardless of austerity α_G for shocks between 0% and -80%. More extreme shocks than -80% are unrealistic.

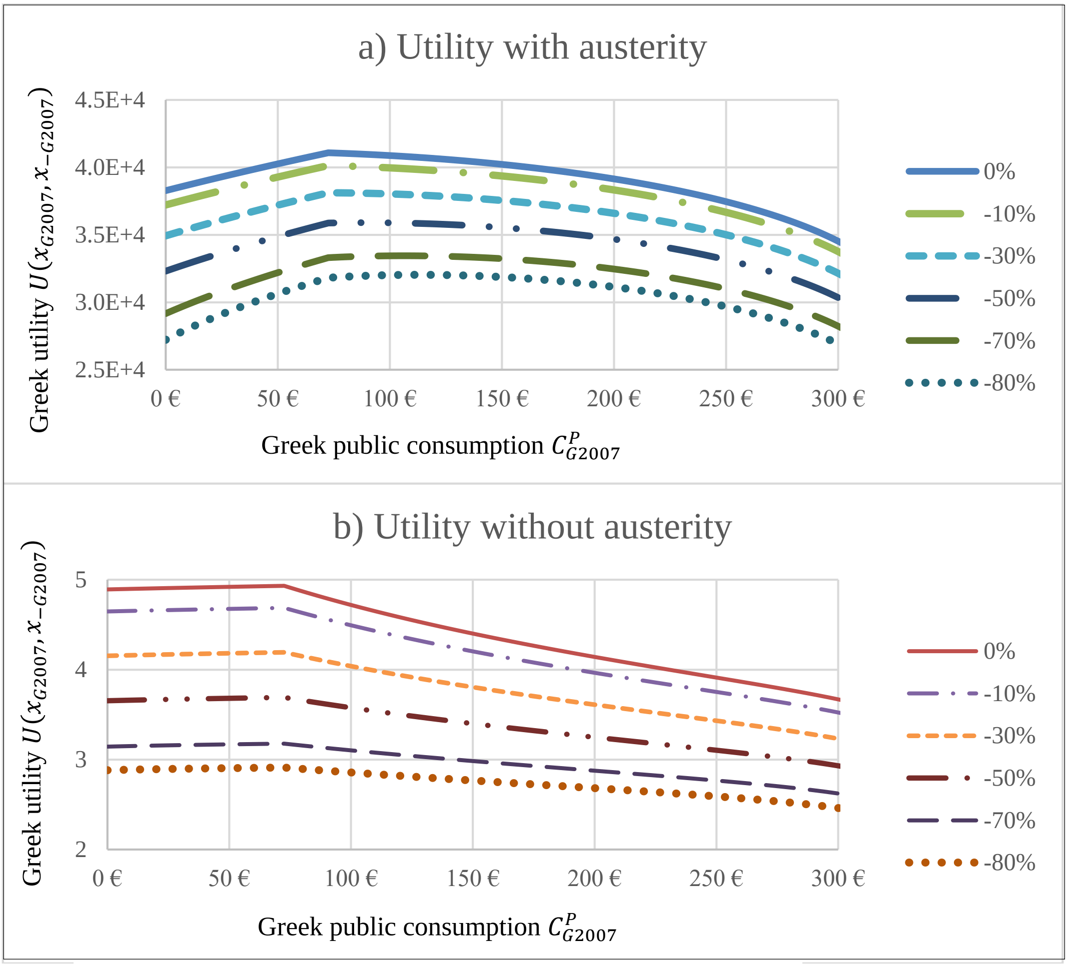

Figure 13: Greece's utility $U(x_{G1}, x_{-G1})$ as a function of Greece's public consumption C^P_{G1} assuming one country, two periods, two austerity levels $\alpha_G = 0.9$ in panel a and $\alpha_G = 0.1$ in panel b, and negative productivity shocks.

5.2.4 Sensitivity of Utility to Public Investment

In Figure 14, Greece selects public investment I^P_{G2007} in 2007 while public consumption is assumed to be held constant at the value of the sum of lump sum transfers, shown in Figure 10 to be $C^P_{G2007} = \sum_{h_m \in h^G} N_{h_m G2007} + \sum_{f_l \in h^G} N_{f_l G2007} = 72.57$ billion €. Public investment in 2008 is $I^P_{G2008} = 0$ since the marginal utility of public investment, $\partial U(x_{G2008}, x_{-G2008}) / \partial I^P_{G2008}$, in 2008 is 0. Thus, consumption is equal to the value of lump sum transfers less debt, i.e. $C^P_{G2008} = \sum_{h_m \in h^G} N_{h_m Gt} + \sum_{f_l \in h^G} N_{f_l Gt} - (1 + r^+_{Gt}) B^+_{G2007}$. Figure 14 shows that as public investment I^P_{G2007} in 2007 increases, Greece's utility $U(x_{G2007}, x_{-G2007})$ decreases while aggregate household utility U^H_{G2007} is U-shaped with an overall weak decrease which drives Greece's decreasing utility. Thus, Greece's utility $U(x_{G2007}, x_{-G2007})$ is maximized at zero investment $I^P_{G2007} = 0$.

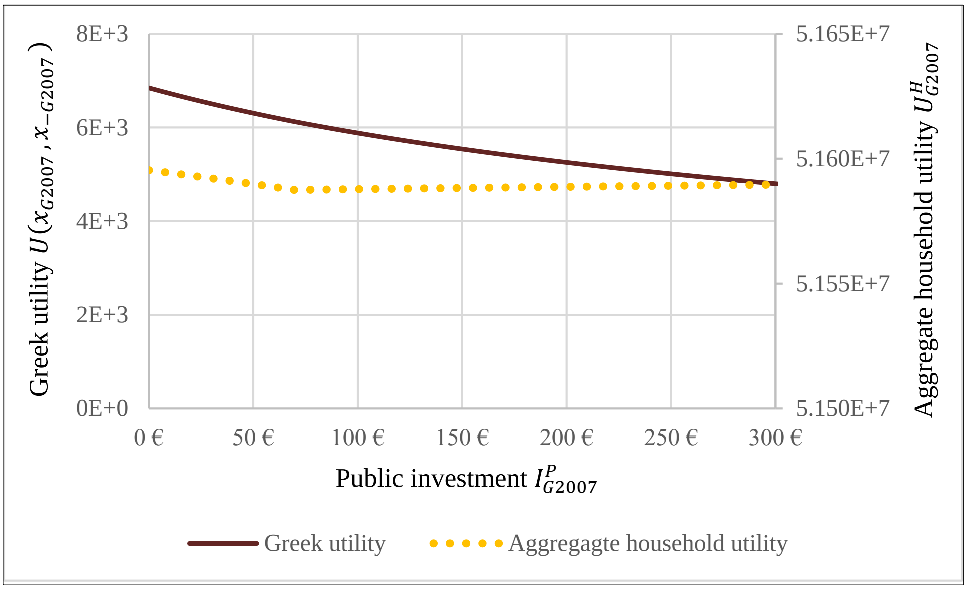

Figure 14: Greece's utility $U(x_{G2007}, x_{-G2007})$ and aggregate household utility U^H_{G2007} as functions of Greece's public investment I^P_{G2007} assuming one country and two periods.

5.3 Equilibrium in a Seven Period Model

Next, we extend the single country environment of Greece, defined with n_h equivalent households and n_f equivalent firms, parametrized with 2007 values in Table 7 to seven periods (2005-2011) parametrized with values in Table 8. Greece invests I_{Gt}, bounded above by a 15% absorptive capacity rate (Cohen and Levinthal 1990). A Mixed-Integer Nonlinear Program (MINLP) is developed in GAMS (gams.com) to solve the game. We assume n_h equivalent households with household utility in (12) & (13), n_f equivalent firms with utility in (21), and no investment $I_{i_n t} = 0$ and no borrowing $B^-_{i_n j_m t} = B^+_{i_n j_m t} = 0$ for households and firms, $i_n \in (h_n, f_n)$. We use production Y_{Gt} in (26), the market clearing condition for goods in (30), the capital law of motion for K_{Gt} in (27), the country budget constraint in (33), and household consumption $C_{h_n t}$ determined by (14). This enables us to find Greece's optimal strategy to maximize utility. Furthermore, in solving the game, we assume that Greece has the sole objective of maximizing social welfare (i.e. Greece has utility in (34) with $\zeta_G = 1$). Table 8 shows the empirical parameter values for Greece 2005-2011.

Table 8 Empirical parameter values for Greece 2005-2011.

	Total tax revenue $\sum N_{i_m Gt}$ *(€ billions)*	*Household transfers* $\sum N_{h_m Gt}$ *(€ billions)*	*Firm transfers* $\sum N_{f_l Gt}$ *(€ billions)*	L_{Gt}	r^+_{Gt} *(%)*	A_{Gt}	A_{Gt} *(% growth)*

2005	62.188	9.090	6.402	4846.1	3.6%	0.082	0.57%
2006	66.068	9.725	5.689	4886.5	4.1%	0.082	-0.08%
2007	72.565	10.821	5.704	4916.5	4.5%	0.081	-0.90%
2008	74.960	11.284	5.875	4937.2	4.8%	0.079	-2.94%
2009	70.562	11.528	5.689	4979.7	5.2%	0.075	-4.56%
2010	70.307	9.905	5.446	5017.3	9.1%	0.074	-2.10%
2011	67.507	9.932	4.377	4967.7	15.8%	0.069	-6.63%

Source: Federal Reserve Economic Data (FRED), Federal Reserve Bank of St. Louis, World Bank Population Data, OECD[9].

5.3.1 Disallowing Default

We first disallow the possibility of default, enabled by assuming a sufficiently high penalty $P^{+}_{Gj_mt}$. Since optimization over seven years leads the country to assume that period 7 is the end of the world, the numerical analysis is computed for 25 years as specified in Appendix B, of which only the first seven periods are plotted. Figure 15 specifies the results with three strategic choice variables for Greece; C^{P*}_{Gt}, I^{P*}_{Gt}, and B^{+*}_{Gt}, and three endogenously determined variables; Y^{*}_{Gt}, K^{*}_{Gt}, and C^{H*}_{Gt}. The model results (displayed as solid blue curves) are presented next to their corresponding empirical values (in grey dashed curves). The model results of GDP, Y_{Gt}, borrowing, B^{+}_{Gt}, and household consumption C^{H}_{Gt} correspond closely to the empirical values. However, public consumption, C^{P}_{Gt}, and investment, I^{P}_{Gt}, deviate from empirical values. Our model shows higher public consumption and lower public investment, than the empirical values, to be optimal in the first time periods following period 2005. The low public investment I^{P}_{Gt} in Figure 15 confirms the findings in Figure 14 where utility $U(x_{G1}, x_{-G1})$ decreases as I^{P}_{Gt} increases. Consequently, our model generates lower capital in equilibrium than the empirical values. Nonetheless, the seven year averages for public consumption (48.0 € billion) and public investment (2.8 € billion) in the model differ from the empirical averages (46.3 € billion and 36.2 € billion, respectively). A possible explanation for the slightly higher public consumption in the early time periods are to account for various un-modeled factors that would otherwise limit high consumption. The model determines higher Greek public consumption than the empirical value before 2008, and lower after 2008, with an overall decreasing trend. Possible explanations for the low public investment in the early time periods

[9] Additionally, we initialize debt through 2004 borrowing and interest rate values such that $B^{+}_{G2004} = 242.17$ € billion, $r^{+}_{G2004} = 4.26\%$.

can be un-modeled factors, lacking Greek rationality, or inability to plan, where Greece did not exploit the available investment opportunities, or that investment opportunities were limited for them.

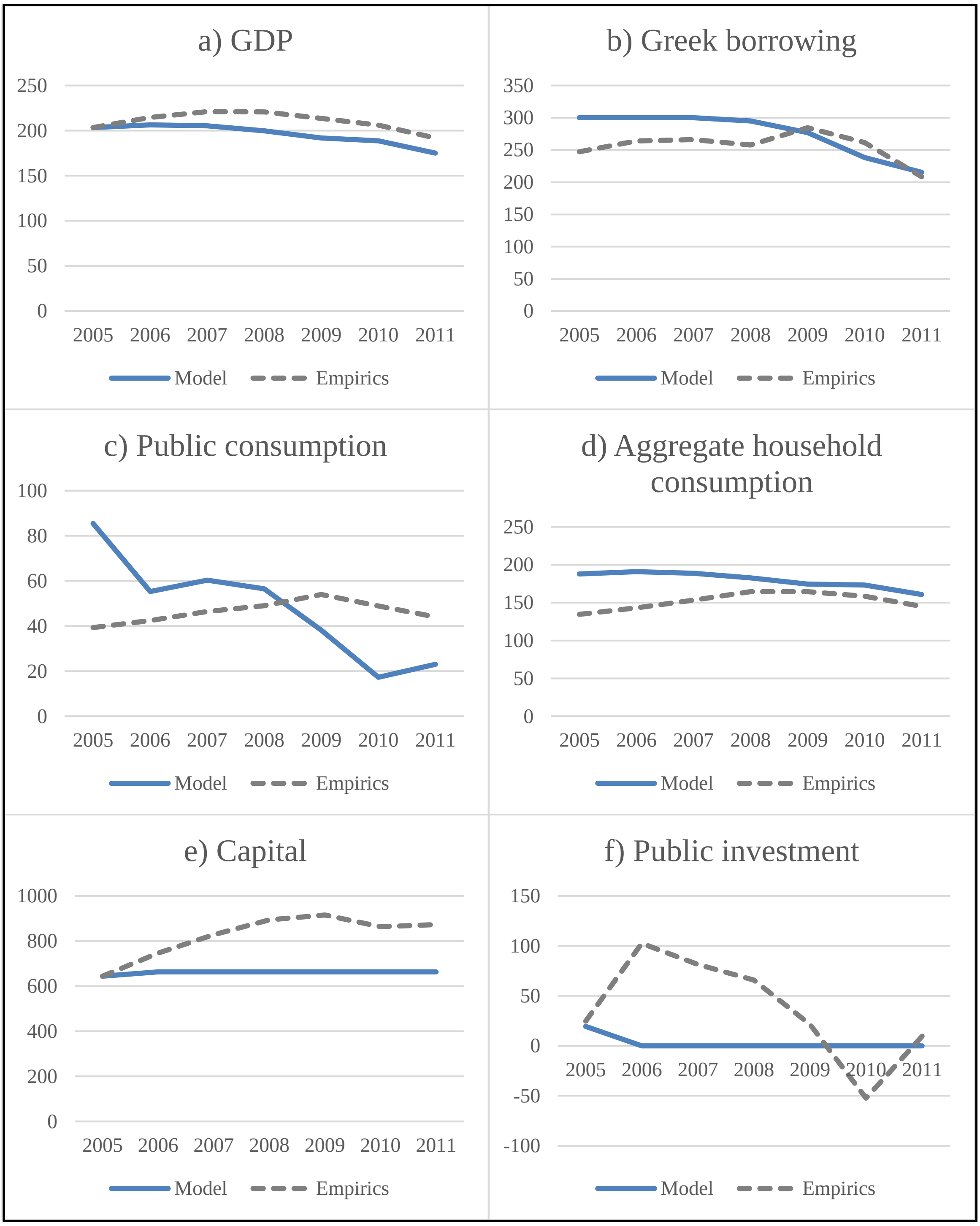

Figure 15: Disallowing default, GDP Y_{Gt}, Greek borrowing B_{Gt}^{+}, public consumption C_{Gt}^{P}, aggregate household consumption C_{Gt}^{H}, capital K_{Gt}, and public investment I_{Gt}^{P} for years 2005-2011. Blue solid curves are for the model. Grey dashed curves are the empirical values.

5.3.2 Allowing Default

We next allow the possibility of default, by lowering the default penalty past the default penalty, $P_{Gj_mt}^{+}$=0.001, finding default in equilibrium. Figure 16 assumes the same empirics and presents the same variables and as in Figure 15 with the main difference that a lower default penalty leads to default in 2005. That is, the model finds that if there would be a default in equilibrium at any point, it would be optimal to default early. Following default, it is optimal to quickly increase borrowing B_{Gt}^{+} to finance public consumption C_{Gt}^{P} and investment I_{Gt}^{P} spending. Although borrowing increases rapidly after default, it stays beneath both the borrowing levels without default and empirical borrowing levels for all periods. Consumption, however, rises well above empirical values in 2006 returning near empirical levels in 2009. Investment rises from zero in 2005 to 28.49 billion € in 2006 falling to zero by 2008 and remaining low relative to empirical values for all periods. Furthermore default has a relatively low impact on other factors; on average capital K_{Gt} is 1.4% lower in Figure 16 than in Figure 15), GDP Y_{Gt} is 0.4% lower, and aggregate household consumption C_{Gt}^{H} is 0.5% lower.

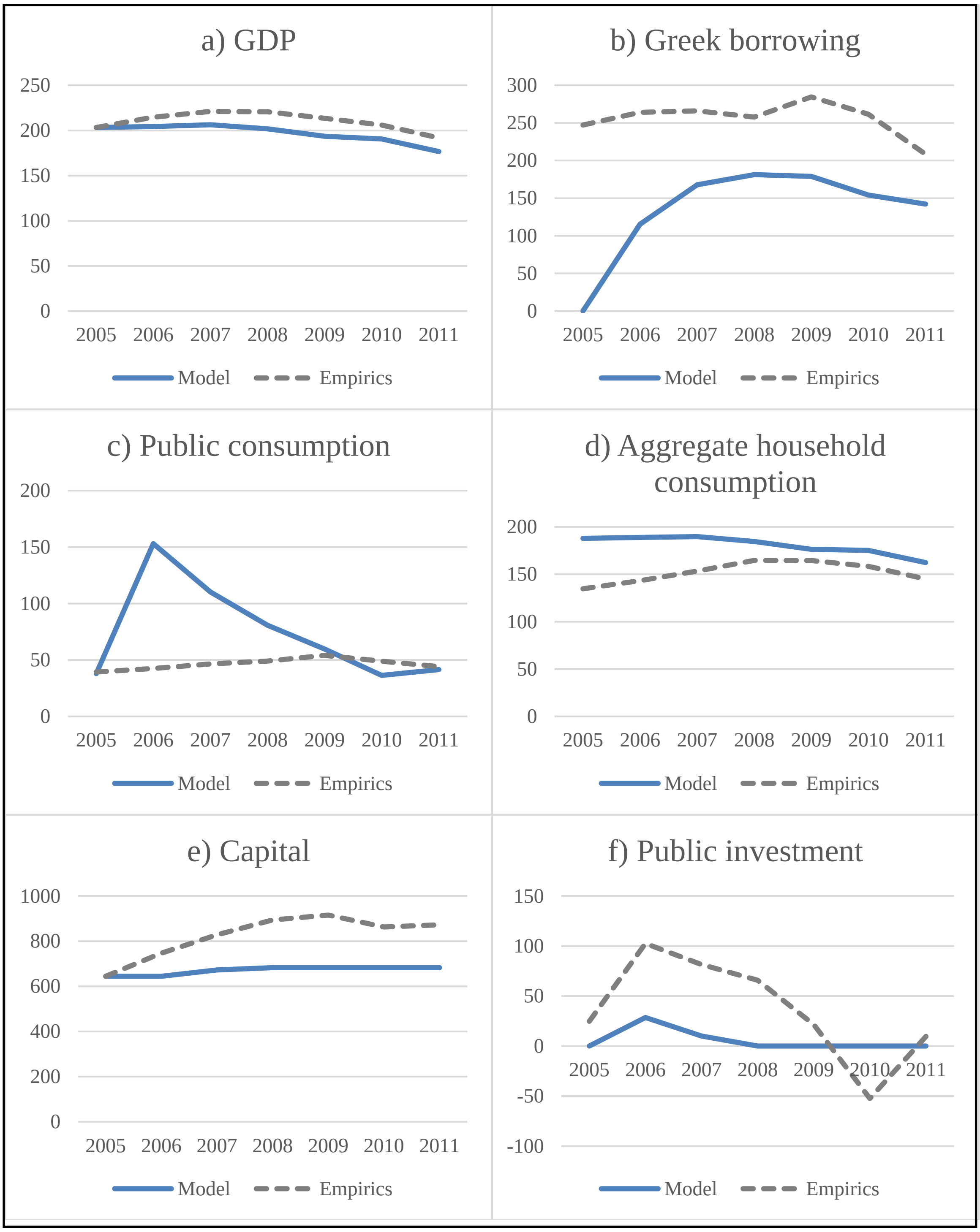

Figure 16: Allowing default, GDP Y_{Gt}, Greek borrowing B_{Gt}^{+}, public consumption C_{Gt}^{P}, aggregate household consumption C_{Gt}^{H}, capital K_{Gt}, and public investment I_{Gt}^{P} for years 2005-2011. Blue solid curves are for the model. Grey dashed curves are the empirical values.

6. Numerical Analysis of Greece and Germany in a Two Period Game

This section considers two countries, c_1 denoted with the letter G for Greece, and c_2 denoted with the letter D for Deutschland, assuming n_f equivalent firms and n_h equivalent households in each country. Greece is a borrower while Germany is a lender. The exports of Germany are the imports of Greece and the imports of Germany are the exports of Greece. The baseline parameter values are shown in Table 9. The equilibrium outcome of a two period model is determined. We impose varying shocks exogenously and determine the changed equilibrium outcome. This section illustrates contagion and how unfortunate events influence the

Table 9: Baseline parameters values for country $c_n \in \{G, D\}$.

Parameter/Country	Greece	Germany
Total tax revenue (€ billions)	72.57	876.74
$\sum_{h_m \in c_n} N_{h_m c_n t}$ (€ billions)	10.82	218.85
$\sum_{f_l \in c_n} N_{f_l c_n t}$ (€ billions)	5.70	54.75
$A_{c_n 2007}$	0.0814	0.1352
$A_{c_n 2008}$	0.0790	0.1327
$L_{c_n 2007}$ (Thousands)	4916.5	41,589.62
$L_{c_n 2008}$ (Thousands)	4937.16	41,589.62
$K_{c_n t}$ (€ billions)	828.16	7842.51
$I^F_{c_n t} + I^H_{c_n t}$ (€ billions)	0	0
$X_{c_n t}$ (€ billions)	60.09	M_{Gt}
$M_{c_n t}$ (€ billions)	13.37	X_{Gt}
B^+_{GD2006} (€ billions)	264.06	0
$r^+_{GD2006} = r^-_{DG2006}$	4.07%	4.07%
$D_{c_n c_m t}$	{0,1}	{0,1}
n_h	4,089,012	39,722,000
n_f	858,685	1,876,543
β_{c_n}	0.90	0.98
δ_{c_n}	0	0
$\zeta_{c_n} = 1 - \alpha_{c_n}$	1	1
ς_{c_n}	0.3	0.3
ρ_{h_n}	0.5	0.5
$\epsilon_{c_n q_m t}$ (€ billions)	0	0
κ	0.2	0.2

Source: Federal Reserve Economic Data (FRED), Federal Reserve Bank of St. Louis, World Bank Population Data, OECD, based on 2007 values for Greece and Germany. Productivity is estimated for 2007 and 2008 as in Appendix A.

Table 9 gives the parameter values for country $c_n \in (G, D) = (Greece, Germany)$, and parameterizes the strategic choice variables; lump sum transfer for households and firms, $\sum_{h_m \in h^{c_n}} N_{h_m c_n t}$ & $\sum_{f_l \in h^{c_n}} N_{f_l c_n t}$, interest imports $M_{c_n t}$, rates on borrowing $r^+_{c_n t}$. For the remainder of this subsection, the strategy sets are a bit more tailored than in Table 1 given the roles of Greece and Germany as borrower and lender, respectively. Greece's strategy set is $S_G = \{B, D, C, I\}$, i.e., borrow, default, consume, and invest, while Germany's strategy set is $S_D = \{B, r, C, I\}$, i.e., lend, set interest rates, consume, and invest.

Germany is the sole creditor to Greece providing Greece's total borrowing B^+_{GDt} in each period. Consequently, the budget constraint of Germany, in (33), is tied to the borrowing behavior of Greece as its lending position is $B^-_{DGt} = B^+_{GDt}$. Now, consider a two period world as in section 4. Both countries trade in both periods while Greece may borrow, B^+_{GDt}, from Germany in 2007 as period 1 to finance investment, I^P_{G2007}, or public consumption, C^P_{G2007}. It must also repay these debts, Δ^+_{GD2008}, in 2008 as period 2 providing income which Germany may use to finance public consumption, C^P_{D2008}. This is illustrated in Figure 17 with convex decrease of both countries' utilities, which largely coincide, when public consumption C^P_{G2007} increases.

6.1 Sensitivity of Greek and German Utilities to Public Consumption and Negative Productivity Shocks

Figure 17 plots Greek and German utilities $U(x_{c_n 2007}, x_{-c_n 2007})$, $c_n = G, D$, as functions of Greece's public consumption C^P_{G2007} assuming two countries and two periods, shown as a green dashed curve and a blue solid curve, respectively. Both utilities decrease as Greece's public consumption C^P_{G2007} increases. Figure 17 also plots the utilities with a -30% shock to Greek productivityA_{G2007}, shown as an orange dash-dotted curve and a red dotted curve, respectively. Compared with no shock, Greece's overall utility decreases, while Germany's utility is virtually unchanged. The latter result follows from the large size difference between the two countries and their economies. Germany's 2007 GDP (2386 Billion, 2005€) is around 10 times larger than Greece's 2007 GDP (221 Billion, 2005€). Furthermore, Germany represents 6% of Greek exports while Greece represents only 1% of total German exports. Germany is thus relatively insensitive to Greece through trade.

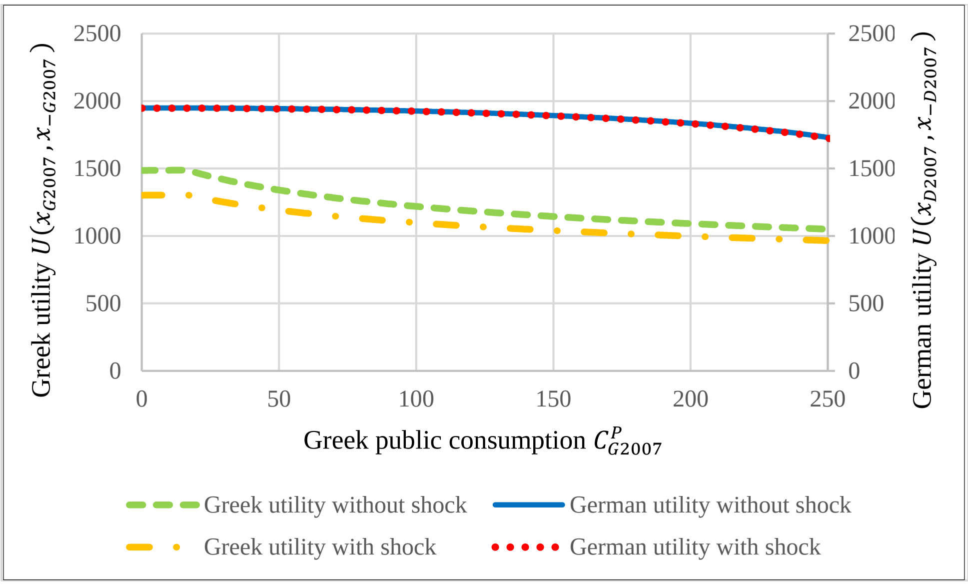

Figure 17: Utility $U\left(x_{c_n 2007}, x_{-c_n 2007}\right)$, $n = 1,2$, of Greece and Germany as functions of Greece's public consumption C^P_{G2007} assuming two countries and two periods, without and with a -30% shock to Greek productivity, A_{G2007}.

6.2 Equilibrium Demonstrating Greek Default when Germany is the Sole Lender

We now assume that Germany is a lender, determining whether or not to lend, how much to lend, and the interest rate on Greece's borrowing. Greece is the borrower, determining whether or not to borrow, how much to borrow, and whether to default. To model this, we assume sequential moves in a two period game between Greece and Germany in 2007 as period 1 and 2008 as period 2. In 2007, Germany (the lender, c_2) moves first, and Greece (the borrower, c_1) moves thereafter, borrows from Germany which causes an expense for Germany, pays debts, $(1 + r^+_{GD2006})d_{G2007}$, carried from 2006 (period 0), incurs new loans, B^+_{GD2007}, consumes, C^P_{G2007}, and invests, I^P_{G2007}. Both Greece and Germany receive income in 2007 and 2008 from production and lump sum transfers. In 2008, Greece pays all debts, providing Germany with income which enables consumption, and spends the remaining income on consumption.

Assuming n_h equivalent households and n_f equivalent firms in Greece and Germany, the two countries choose behavior according to the Nash equilibrium solution in (41), governed by the production function in (26), the market clearing condition for goods in (30), the capital law of motion in (27), and the country budget constraint in (33). The two countries have four strategic choice variables each, i.e. $B^{+*}_{GD2007} = B^{-*}_{DG2007}$, $r^{+*}_{GD2007} = r^{-*}_{DG2007}$, $C^{P*}_{c_n t}, I^{P*}_{c_n t}$, where * means optimal value. The optimal values are determined using GAMS to solve (41). Defining Δ_r=1% and Δ_C=25 as increments on interest rates and consumption, respectively, causing run time proportional to $|\Delta_r \times \Delta_C|$, defining Greece as c_1, Germany as c_2, period 1 as 2007, and period 2 as 2008, the procedure runs through the possible values for each strategy for each country.

Initialize $U_{c_2}^{max}=0$, $C_{c_2 1}^{P,max}=n_h N_{h_i c_2 1} + n_f N_{f_j c_2 1} + \left(1 + r_{c_2 c_1 0}^{-}\right) B_{c_1 c_2 0}^{+}, numNE = 0$
For any $r_{c_2 c_1 1}^{-} = 0, \Delta_r, \ldots, 1 - \Delta_r, 1$
    For any $\Omega = 0,1,2, \ldots, C_{c_2 1}^{P,max} / \Delta_C$
        Set $C_{c_2 1}^{P} = \Omega \Delta_C$
        Solve $\max_{C_{c_1 t}^{P}, I_{c_1 t}^{P}, B_{c_1 c_2 1}^{+}, D_{GDt}} U\left(x_{c_1 1}, x_{-c_1 1}\right)$ *subject to (26), (27)(30), (33), assuming* n_h *equivalent households,* n_f *equivalent firms, and no investment* $I_{i_n t} = 0$ *and no borrowing* $B_{i_n j_m t}^{-} = B_{i_n j_m t}^{+} = 0$ *for households and firms,* $i_n \in (h_n, f_n)$.
        Set $I_{c_2 1}^{P} = n_h N_{h_i c_2 1} + n_f N_{f_j c_2 1} + \left(1 + r_{c_2 c_1 0}^{-}\right) B_{c_1 c_2 0}^{+} - B_{c_1 c_2 1}^{+} - C_{c_2 1}^{P}$
        Set $C_{c_2 2}^{P} = n_h N_{h_i c_2 2} + n_f N_{f_j c_2 2} + \left(1 + r_{c_2 c_1 1}^{-}\right) B_{c_1 c_2 1}^{+}$
        Calculate $U\left(x_{c_2 1}, x_{-c_2 1}\right)$
        if($U_{c_2}^{max} = U\left(x_{c_2 1}, x_{-c_2 1}\right)$, $numNE = numNE + 1$*)*
        If($U_{c_2}^{max} < U\left(x_{c_2 1}, x_{-c_2 1}\right)$, $U_{c_2}^{max} = U\left(x_{c_2 1}, x_{-c_2 1}\right)$, *Set* $x_{c_n t}^{*} = x_{c_n t}, numNE = 0$ *)*
Calculate $U\left(x_{c_n t}^{*}, x_{-c_n t}^{*}\right)$

We present the results of a game between Greece and Germany using parameter values from 2007 and 2008 as shown in Table 9. In the two country game, Germany is the first mover. Germany moves by choosing an interest rate, r_{GD2007}^{+}, at which Greece must repay borrowing, B_{GD2007}^{+}, and chooses public consumption, C_{Dt}^{P}, and investment, I_{Dt}^{P}. Germany's decisions impact its household's utility through the public consumption good thereby driving social welfare. Greece enters the game with high initial debt, d_{G2007}, starting at the brink of crisis. It moves second by choosing whether to repay debt to Germany or to default, $D_{GDt} = 0{,}1$, and choosing public consumption, C_{Gt}^{P}, and investment, I_{Gt}^{P}, which impact the households and social welfare, $\sum_{h_n \in G} U_{h_n t}$. The game, is then, how much Germany should raise interest rates to compensate for lending without driving Greece to default.

Both countries grow and contract over the two periods as described by productivity A_{Ct} in Table 9. The default penalty plays a central role in the game driving repayment by Greece. Consequently, we find Nash equilibria in the two country game with two different penalties, i.e. a low penalty P^{+}_{GDt}=0.0001 and a high penalty P^{+}_{GDt}=0.01, with results depicted in Table 10.

Consider the case where the default penalty is low. In equilibrium, Greece chooses to renege on its large initial debt d_{G2007} to Germany and default. This leaves Greece in a position to greatly expand spending on public consumption C^{P*}_{G2008} in 2008, lifting social welfare. Default, however, shrinks Germany's budget harming its ability to spend on public consumption C^{P*}_{D2007} and C^{P*}_{D2008} and investment I^{P*}_{D2007}. Perhaps most importantly, there is not a single unique Nash equilibrium found in this game. Instead multiple equilibria are found as each possible German strategy for setting interest rates r^{+}_{GD2007} leads to the same outcome; for every feasible choice of interest rate, Germany is unable to stop Greece from defaulting,

Alternatively, consider the case where the default penalty is high. We find a unique Nash equilibrium where Greece does not default. Germany sets the Greek interest rate at r^{+}_{GD2007}=10.2% and Greece borrows, expressed with B^{+*}_{GD2007} and B^{+*}_{GD2008}, moderately during the game. In contrast to the low penalty case, German spending on consumption and investment is lower in 2007 as it lends to Greece but significantly higher in 2008 as it is repaid. Greek spending on public consumption is near flat much lower than in the low penalty case over the two periods it struggles with debt repayment.

Table 10: Results of Greece-Germany two period game.

	Low Penalty $P^{+}_{GDt} = 0.0001$	*High Penalty* $P^{+}_{GDt} = 0.01$
2007 German public consumption, C^{P*}_{D2007} (€ billions)	800	900
2008 German public consumption, C^{P*}_{D2008} (€ billions)	628.27	898.31
2007 German public investment, I^{P*}_{D2007} (€ billions)	76.74	0.18
2007 German household consumption, C^{H*}_{D2007} (€ billions)	3135.82	3135.82
2008 German household consumption, C^{H*}_{D2008} (€ billions)	3145.80	3135.85
2007 Greek public consumption, C^{P*}_{G2007} (€ billions)	95.61	49.13
2008 Greek public consumption, C^{P*}_{G2008} (€ billions)	321.03	50.99
2007 Greek public investment, I^{P*}_{G2007} (€ billions)	0	0
2007 Greek household consumption, C^{H*}_{G2007} (€ billions)	218.04	218.04
2008 Greek household consumption, C^{H*}_{G2008} (€ billions)	211.81	211.81
2007 Greek borrowing, B^{+*}_{GD2007} (€ billions)	300.00	253.5137109
2007 Greek default, D_{GD2007}	0	0
2008 Greek default, D_{GD2008}	1	0
2007 Interest rate, r^{+*}_{GD2007} (%)	0	10.2

2008 Greek borrowing, B^{+*}_{GD2008} (€ billions)[10]	257.80	257.80
Nash equilibrium type	multiple	unique

While we do not explicitly model the full negotiation between the two countries, several conclusions can be drawn that are relevant to the Greek 2010 debt crisis. First, given a sufficiently high shock magnitude and a sufficiently low default penalty, Greece's high debt burdens will make default optimal. Greece entered 2007 with significantly too high debt making it highly vulnerable to negative shocks. The game we present is strictly normative and does not fully explain what occurred during the Greek debt crisis. It suggests that an early Greek default coming in 2007 may have been optimal. Second, our model demonstrates that Germany has no conventional strategies, as specified by Germany's strategy set $\{r_{GDt}, C^P_{Dt}, I^P_{Dt}\}$, to thwart the default, a fact which may lead to a better understanding of what did occur during the debt crisis. That is, default results in multiple equilibria where Greece's best response is to default to every feasible German choice of interest rate r_{GDt}, demonstrating that Germany's interest rate determination is ineffective in preventing default. Consequently, in order to thwart default, Germany must use unconventional tools. The first tool is debt forgiveness. Debt forgiveness, through debt renegotiation, a haircut, or a bailout, could make debt repayment by Greece optimal. The second is changing the default penalty, i.e. increasing the default penalty making default suboptimal. Both strategies have benefits as well as drawbacks. Debt forgiveness is politically challenging but may lead to higher overall utility and stronger Greek growth. Increasing the default penalty is politically advantageous for Germany but will retard Greek growth. In recent developments, increasing the default penalty seems to have been the preferred strategy. The so-called "Grexit" threat that Greece would be forced out of the EU if it chose to default, can be viewed as an increase in the penalty associated with a Greek default. This heightened penalty may have resulted in default as a suboptimal outcome as the results in Table 10 suggest, driving Greece to repayment.

7. Policy Recommendations

One objective of this paper's game-theoretic approach to model financial crises is to further improve macroprudential policy. The game-theoretic model is advantageous in understanding

[10] We assume Greek borrowing B^{+*}_{GD2008} in period 2 bounded above to prevent Ponzi schemes, using the empirical 2008 value B^{+*}_{GD2008} = 257.80 € billions.

how global economic risks can be reduced through policy interventions, including creation of a desirable focal equilibrium among multiple equilibria (Schelling 1980). Our six major policy conclusions are as follows:

1. Following a temporary adverse shock, the best response of a country is to increase borrowing and spending on consumption and investment. This recommendation follows from the results of section 3 (see Figure 6) where we demonstrate the behavior of countries in equilibrium. This behavior enables the country to smooth household consumption and thereby smooth aggregate household utility during the temporary shock. Furthermore, increased investment stimulates growth in future periods ensuring that the drop in production is temporary.
2. Section 3 demonstrates that for a sufficiently large temporary negative productivity shock, the country rationally borrows up to its default threshold in order to smooth aggregate household utility, causing future default. The occurrence of default in equilibrium demonstrates that default follows from rational behavior. The policy recommendation is to be aware of such potential borrowing which may destabilize the financial system through future default.
3. Default crises can follow from a vicious cycle where defaults drive up interest rates which drive up defaults which drive up interest rates, and so on. The reality that a default crisis can result from rational behavior is a risk that necessitates policy intervention.
4. Given that high rates can cause players near the default threshold to default, regulators may want to keep rates low when the threshold is near. That is, the Taylor (1993) rule suggests lowering interests rates following a shock. If many players are near default thresholds, the monetary authority may want to act even faster in lowering rates, to avoid a vicious cycle of increasing rates and defaults that lead to a crisis.
5. The actions of FIGOs can reduce the risk of contagion. Sovereign debt may suffer from the same vicious cycle as for default in point 4. Furthermore, trade may transmit adverse shocks across countries. Lenders may defend against default by restricting borrowing and avoiding the default threshold through borrowing, as shown in section 4.

8. Conclusion

The novel approach in this paper is to model and numerically analyze six kinds of players, i.e. countries, central banks, banks, firms, households, and financial inter-governmental organizations. Countries comprise banks, households and firms. The strategy sets of each player are specified. We account for consumption, investment, borrowing, default, costs of debt, and penalties. Various plausible assumptions are made. Utility functions are developed for each kind of player dependent on the interaction of all the players. Markets for goods, debt, and capital are modeled endogenously.

We first numerically analyze one country illustrating the impacts of public consumption and lump sum transfers on the utilities of the country and its households. We further illustrate the impact of a positive interest rate shock. We next consider a negative shock to productivity over ten time periods. We find that a shock can incentivize short term borrowing to finance increased government spending on public consumption. We further confirm that austerity restrains consumption. Thereafter, we numerically analyze two countries and determine the impact of a productivity shock and default. We find that a shock to the borrower discourages the lender from substantial lending.

Next, we use 2005-2011 empirical data for Greece to determine the impact of public consumption and lump sum transfers on the utilities of Greece and its households. We further illustrate the impact of negative productivity shocks. We compare Greece's utility as a function of public consumption for various austerity levels and negative productivity shocks, finding that… We determine the impact of public investment on utility in a two period model, finding that high debt costs restrain investment. Thereafter, we numerically analyze the game over seven periods, from 2005 to 2011, calculating the development of GDP, borrowing, public consumption, aggregate household consumption, capital, and public investment in equilibrium.

First, by disallowing default through a high default penalty, our model explains the actual trends in Greek borrowing, output, and aggregate household consumption. Our model, however, calculates higher public consumption from 2005 to 2007and lower from 2009 to 2011 than the empirical values. Furthermore, our model calculates lower public investment than the empirical values with the exception of 2010 leading to a lack of capital growth throughout the seven periods.

Second, by allowing default, our model explains that, given the high initial debt, an early default could be optimal by allowing for higher public consumption and higher public investment. Following default, we find that borrowing would quickly resume reaching a peak beneath empirical levels. However, driven by Greece's decision to spend heavily on public consumption and less heavily on investment, our model finds slower production growth following default.

Furthermore, we use 2007-2008 empirical data for Greece and Germany to determine the impact of Greece's 2007 public consumption on both countries' utilities, both with and without negative productivity shocks. Germany's utility is relatively insensitive due to its substantially higher GDP. Assuming Germany is a lender and Greece is a borrower, we find that Greece's high debt burden makes default optimal when the shock magnitude is sufficiently high and the default penalty is sufficiently low. We show that Germany has a limited ability to prevent a Greek default through its available strategies. However, we find that Germany could prevent a Greek default through unconventional tools such debt forgiveness and changing the default penalty.

The model we introduce is intended as a framework and tool to enhance our understanding of the dynamics between players that create the global economic risks resulting in financial crises. By showing how countries can adjust debt and spending in response to adverse events, our framework gives some insight into how crises can be governed and mitigated. The framework allows in future extensions for modeling incomplete information, which has not been illustrated in this paper. For example, future research should address how moral hazard and adverse selection arise in the framework when information asymmetries keep borrowers and lenders from knowing each other's true characteristics (e.g., when the lender cannot easily or accurately assess the credit-worthiness of a borrower).

Appendix A: Estimation of Productivity

We use (26) to estimate productivity $A_{Gt} = \frac{Y_{Gt}}{K_{Gt}^{\varsigma_G} {L_{Gt}}^{1-\varsigma_G}}$ in each period t by using historical data on Greece's real capital stock, real GDP, and civilian labor force for production Y_{Gt}, capital K_{Gt}, labor L_{Gt}, and the value of the capital-output ratio, ς_G=0.3, as specified in Table 7. Inserting the parameter values in Table 7 for 2007 into this expression for A_{Gt} gives A_{G2007}=0.0814. Assuming growth rate g_{c_n}=0.025, determined by inspection of Figure 18 into (28) gives A_{Gt} displayed as a time series in Figure 18.

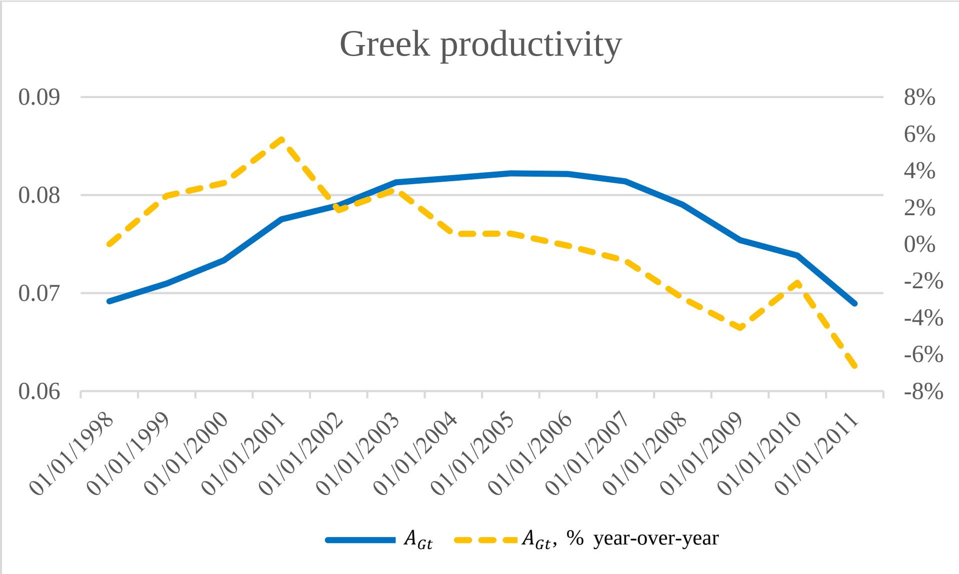

Figure 18: Estimation of productivity and productivity growth for Greece.

Appendix B: Parameter Values for Greece 2012-2029

Table 11 extends Table 8 to 2012-2029. Empirical values are in regular numbers and estimates are in italics. For total tax revenue $\sum N_{i_m Gt}$, household transfers $\sum N_{h_m Gt}$, and firm transfers $\sum N_{f_l Gt}$, empirical values are used for 2012 and a 1% growth rate is assumed for 2013-2029. For labor L_{Gt}, a 1% growth rate is assumed. For interest rates r_{Gt}^{+}, empirical values are used for 2012-2015 and the 2015-2029 values are forecasted assuming $r_{Gt}^{+} = \max\{0.99 r_{Gt-1}^{+}, 4\%\}$ to reflect an anticipation of a gradual reversion to the mean. For productivity A_{Gt}, a growth rate of 2.47% is assumed.

Table 11 Empirical parameter values and forecasted parameter values for Greece 2012-2029.

	Total tax revenue $\sum N_{i_m Gt}$ (€ billions)	*Household transfers $\sum N_{h_m Gt}$ (€ billions)*	*Firm transfers $\sum N_{f_l Gt}$ (€ billions)*	L_{Gt}	r_{Gt}^{+} *(%)*	A_{Gt}	A_{Gt} *(% growth)*
2012	65.473	13.497	2.174	*4967.7*	22.5%	*0.071*	*2.47%*
2013	*66.128*	*13.632*	*2.196*	*5017.3*	10.1%	*0.072*	*2.47%*
2014	*66.789*	*13.768*	*2.218*	*5067.5*	6.9%	*0.074*	*2.47%*
2015	*67.457*	*13.906*	*2.240*	*5118.2*	9.7%	*0.076*	*2.47%*
2016	*68.131*	*14.045*	*2.262*	*5169.4*	*9.6%*	*0.078*	*2.47%*
2017	*68.813*	*14.185*	*2.285*	*5221.1*	*9.5%*	*0.080*	*2.47%*
2018	*69.501*	*14.327*	*2.308*	*5273.3*	*9.4%*	*0.082*	*2.47%*
2019	*70.196*	*14.471*	*2.331*	*5326.0*	*9.3%*	*0.084*	*2.47%*
2020	*70.898*	*14.615*	*2.354*	*5379.3*	*9.2%*	*0.086*	*2.47%*
2021	*71.607*	*14.761*	*2.378*	*5433.1*	*9.2%*	*0.088*	*2.47%*

2022	*72.323*	*14.909*	*2.401*	*5487.4*	*9.1%*	*0.090*	*2.47%*
2023	*73.046*	*15.058*	*2.425*	*5542.3*	*9.0%*	*0.092*	*2.47%*
2024	*73.777*	*15.209*	*2.450*	*5597.7*	*8.9%*	*0.095*	*2.47%*
2025	*74.514*	*15.361*	*2.474*	*5653.7*	*8.8%*	*0.097*	*2.47%*
2026	*75.260*	*15.514*	*2.499*	*5710.2*	*8.7%*	*0.099*	*2.47%*
2027	*76.012*	*15.670*	*2.524*	*5767.3*	*8.6%*	*0.102*	*2.47%*
2028	*76.772*	*15.826*	*2.549*	*5825.0*	*8.5%*	*0.104*	*2.47%*
2029	*77.540*	*15.985*	*2.575*	*5883.2*	*8.4%*	*0.107*	*2.47%*

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References

Acemoglu, D., Ozdaglar, A., & Tahbaz-Salehi, A. (2013). Systemic risk and stability in financial networks. *National Bureau of Economic Research Working Paper Series, No. 18727*, doi:10.3386/w18727.

Acharya, V. V., & Yorulmazer, T. (2008). Information contagion and bank herding. *Journal of Money, Credit and Banking, 40*(1), 215-231, doi:10.1111/j.1538-4616.2008.00110.x.

Aguiar, M., & Gopinath, G. (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics, 69*(1), 64-83.

Allen, F., & Gale, D. (2000). Financial contagion. *Journal of Political Economy, 108*(1), 1-33, doi:10.1086/262109.

Andritzky, J. (2006). *Sovereign default risk valuation: implications of debt crises and bond restructurings* (Vol. 582). New York: Springer Science & Business Media.

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *The American Economic Review, 98*(3), 690-712, doi:10.2307/29730092.

Arellano, C., & Bai, Y. (2013). Linkages across sovereign debt markets. *National Bureau of Economic Research Working Paper Series, No. 19548*, doi:10.3386/w19548.

Baral, P. (2013). *Essays on formation of networks and its applications*. Bloomington, Indiana: Indiana University.

Broner, F. A. (2008). Discrete devaluations and multiple equilibria in a first generation model of currency crises. *Journal of Monetary Economics, 55*(3), 592-605.

Bulow, J. I., & Rogoff, K. (1988). Sovereign debt: is to forgive to forget? *National Bureau of Economic Research Working Paper Series, No. 2623*, doi:10.3386/w2623.

Chari, V. V., & Kehoe, P. J. (1999). Optimal fiscal and monetary policy. In B. T. John, & W. Michael (Eds.), *Handbook of macroeconomics* (Volume 1, Part C, pp. 1671-1745): San Diego, CA: Elsevier.

Chari, V. V., & Kehoe, P. J. (2004). Financial crises as herds: overturning the critiques. *Journal of Economic Theory, 119*(1), 128-150.

Cohen, W. M., & Levinthal, D. A. (1990). Absorptive capacity: a new perspective on learning and innovation. *Administrative Science Quarterly, 35*(1), 128-152.

Commission, E. (2012). SME Performance Review. http://ec.europa.eu/growth/smes/business-friendly-environment/performance-review/index_en.htm. Accessed November 9, 2015.

Das, S. R., Duffie, D., Kapadia, N., & Saita, L. (2007). Common failings: how corporate defaults are correlated. *The Journal of Finance*, *62*(1), 93-117, doi:10.1111/j.1540-6261.2007.01202.x.

Duffie, D., & Singleton, K. (1999). Modeling term structures of defaultable bonds. *Review of Financial studies*, *12*(4), 687-720, doi:10.1093/rfs/12.4.687.

Eaton, J., & Gersovitz, M. (1981). Debt with potential repudiation: theoretical and empirical analysis. *The Review of Economic Studies*, *48*(2), 289-309, doi:10.2307/2296886.

Eaton, J., Gersovitz, M., & Stiglitz, J. E. (1986). The pure theory of country risk. *European Economic Review, 30*(3), 481-513.

Edwards, S. (1999). How effective are capital controls? *National Bureau of Economic Research Working Paper Series, No. 7413*, doi:10.3386/w7413.

Eggertsson, G. B. (2003). Zero bound on enterest rates and optimal monetary policy. *Brookings Papers on Economic Activity, 2003*(1), 139-233.

Eggertsson, G. B., & Krugman, P. (2012). Debt, deleveraging, and the liquidity trap: a fisher-minsky-koo approach. *The Quarterly Journal of Economics, 127(3)*, 1469–1513, doi:10.1093/qje/qjs023.

Forbes, K., & Rigobon, R. (2001). Measuring contagion: conceptual and empirical issues. In S. Claessens, & K. Forbes (Eds.), *International Financial Contagion* (pp. 43-66): New York: Springer US.

Forbes, K., & Rigobon, R. (2002). No contagion, only interdependence: measuring stock market comovements. *The Journal of Finance, 57*(5), 2223-2261, doi:10.1111/0022-1082.00494.

Gaillard, N. (2011). *A century of sovereign ratings*. New York: Springer Science & Business Media.

Gaillard, N. (2014). What is the value of sovereign ratings? *German Economic Review, 15*(1), 208-224, doi:10.1111/geer.12018.

Galati, G., & Moessner, R. (2013). Macroprudential policy – a literature review. *Journal of Economic Surveys, 27*(5), 846-878, doi:10.1111/j.1467-6419.2012.00729.x.

Giesecke, K., & Weber, S. (2006). Credit contagion and aggregate losses. *Journal of Economic Dynamics and Control, 30*(5), 741-767.

Greenwood, J., Hercowitz, Z., & Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. *The American Economic Review, 78*(3), 402-417, doi:10.2307/1809141.

Hall, R. E., & Jones, C. I. (1999). Why do some countries produce so much more output per worker than others? *National Bureau of Economic Research Working Paper Series, No. 6564,* doi:10.3386/w6564.

Hausken, K., & Plumper, T. (2002). Containing contagious financial crises: the political economy of joint intervention into the asian crisis. *Public Choice, 111*(3-4), 209-235.

Hernández, L. F., & Valdés, R. O. (2001). What drives contagion: trade, neighborhood, or financial links? *International Review of Financial Analysis, 10*(3), 203-218.

Kaminsky, G. L., Reinhart, C., & Vegh, C. A. (2003). The unholy trinity of financial contagion. *National Bureau of Economic Research Working Paper Series, No. 10061,* doi:10.3386/w10061.

Kiyotaki, N., & Moore, J. (1995). Credit cycles. *National Bureau of Economic Research Working Paper Series, No. 5083,* doi:10.3386/w5083.

Krugman, P. (2013). How the case for austerity has crumbled. *The New York Review of Books, 60(10). http://www.nybooks.com/articles/archives/2013/jun/06/how-case-austerity-has-crumbled/*. Accessed November 9, 2015.

Lorenzoni, G., & Werning, I. (2013). Slow moving debt crises. *National Bureau of Economic Research Working Paper Series, No. 19228*, doi:10.3386/w19228.

Mendoza, E. G., & Yue, V. Z. (2011). A general equilibrium model of sovereign default and business cycles. *National Bureau of Economic Research Working Paper Series, No. 17151*, doi:10.3386/w17151.

Morris, S., & Shin, H. (2000). *Global games: theory and applications*. New Haven, Connecticut: Cowles Foundation for Research in Economics.

Morris, S., & Shin, H. (2012). Contagious adverse selection. *American Economic Journal: Macroeconomics, 4*(1), 1-21.

Obstfeld, M. (1984). Rational and self-fulfilling balance-of-payments crises. *National Bureau of Economic Research Working Paper Series, No. 1486*, doi:10.3386/w1486.

OECD (2015). Families and children. http://www.oecd.org/els/family/. Accessed November 9, 2015.

Park, J. (2012). Contagion of sovereign default risk: the role of two financial frictions. In: working paper, National University of Singapore.

Reinhart, C. M., Kirkegaard, J. F., & Sbrancia, M. B. (2011). Financial repression redux. *Finance and Development*, 22-26. http://ssrn.com/abstract=1864806. Accessed November 9, 2015.

Reinhart, C. M., & Rogoff, K. (2009). *This time is different: eight centuries of financial folly*: Princeton, New Jersey: Princeton University Press.

Roubini, N., & Sala-i-Martin, X. (1992). Financial repression and economic growth. *Journal of Development Economics, 39*(1), 5-30.

Schelling, T. C. (1980). *The strategy of conflict*. Cambridge, Massachusetts: Harvard University Press.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy, 39*, 195-214.

Uribe, M., & Schmitt-Grohé, S. (2013). Open economy macroeconomics. Columbia University. http://www.columbia.edu/~mu2166/book/oem.pdf. Accessed November 9, 2015.

Welburn, J. W., & Hausken, K. (2015). A game theoretic model of economic crises. *Applied Mathematics and Computation*, *266*, 738-762.

Yue, V. Z. (2010). Sovereign default and debt renegotiation. *Journal of International Economics, 80*(2), 176-187.