Additive Multi-Effort Contests

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Abstract

A rent seeking model is axiomatized and analyzed where players exert multiple additive efforts. An analytical solution is developed when the contest intensity for one effort equals one. Then additional efforts give players higher expected utilities and lower rent dissipation, which contrasts with earlier findings for multiplicative efforts. Players optimize cost effectively across efforts, cutting back on the effort with contest intensity equal to one, and exerting alternative efforts instead. This latter effort eventually decreases towards zero as new efforts are added. It may not be optimal for both players to exert all their available efforts. Accounting for solutions which have to be determined numerically, a Nash equilibrium selection method is provided. For illustration, an example with maximum two efforts for each player is provided. Equilibria are shown where both players choose both efforts, or one player withdraws from its most costly effort. Both players may collectively prefer to exclude one of their efforts, though in equilibrium they may prefer both efforts.

1 Introduction

The rent seeking literature has developed fruitfully for half a century (Congleton et al. 2008). Early developments are by Krueger (1974), Posner (1975), Tullock (1980), etc., reviewed by Nitzan (1994). Skaperdas (1996) axiomatizes the contest success function for symmetric contests, and Clark,Riis (1998) axiomatize asymmetric contests. Examples of rents are R&D budgets, promotions, licenses, privileges, monopoly opportunities, election opportunities, struggles for government support between different industries, competition for budgets by interest groups, and government distribution of public goods. Examples of efforts to obtain rents are multifarious, e.g. lobbying, influence strategies, interference struggles, litigation, strikes and lockouts, political campaigns, commercial efforts to raise rivals' costs (Salop,Scheffman 1983), economic and political maneuvers (Hirshleifer 1995), coaxing, prompting, inducing, urging, extorting, exacting, persuasion techniques, pressure methods, promotions, briberies, skirmishes, battle, combat, and fighting with or without violence.

Earlier research has mostly assumed one effort for each player, which is limiting given the plethora of possible efforts. This paper acknowledges that each player may have available arbitrarily many efforts which may or may not overlap with the contending player's available efforts. Each effort may be of different nature and operate according to its own logic. Formally in this paper, efforts may have three different characteristics, i.e. different unit costs, different impacts, and different contest intensities. Efforts operate additively in the contest success function, which to our knowledge has not been analyzed earlier.

Additive efforts are often descriptive. For example, consider players competing for an elected office position, e.g. U.S president. Each player hires professionals with various kinds of expertise, i.e. political analysts to develop views and positions on issues, media professionals for spin control, social media operatives, business people to recruit donors, telephone operators to convince voters, geographically dispersed ground troops knocking on people's doors, speech writers to tune messages for big rallies and local meetings, gossip developers, and specialists in negative campaigning. These efforts may operate independently of each other and jointly add up to a campaign's effort production function which impacts the contest with the other player(s). It is quite possible for a player's campaign to be successful even if some efforts are missing e.g. due to strategic choice, oversight, lacking competence, or deficient funding. For example, a player may decide to eliminate negative campaigning and ground troops. Alternatively, a player may rely on big colorful rallies applying hitherto unknown influence techniques that the other players are unable or unwilling to apply.

One alternative to additive efforts is multiplicative efforts of the Cobb-Douglas type axiomatized and analyzed excellently by Arbatskaya,Mialon (2010), extended to a two-stage contest by Arbatskaya,Mialon (2012). One of their examples, also provided by Tullock (1980) and Krueger (1974), is that "firms may be able to obtain rents from the government not only by improving their efficiency, but also by lobbying or even bribing government officials" (Arbatskaya,Mialon 2010). Multiplicative efforts can be descriptive of this phenomenon when both improved efficiency and lobbying are mandatory for successful rent seeking. That is, improved efficiency without lobbying guarantees no success, and lobbying without improved efficiency guarantees no success. As the number of efforts increases, Cobb-Douglas type multiplicative efforts become increasingly unrealistic since each effort must be strictly positive to ensure success. The current paper opens for the possibility that improved efficiency without lobbying, or lobbying without improved efficiency, may both constitute successful rent seeking, although both operating additively may be even more successful. The different assumptions of additive and multiplicative efforts cause different results regarding efforts, expected utilities, and rent dissipation. For example, for additional efforts Arbatskaya,Mialon (2010) find increased rent dissipation for symmetric and balanced contests, whereas we find decreased rent dissipation caused by players optimizing more cost effectively across efforts.

Another example of multiplicative efforts, but not of the Cobb-Douglas type, are by Epstein,Hefeker (2003). Assuming two efforts for each player, the first is conventional rent seeking. The second effort may be absent, or it may reinforce the first effort multiplicatively. They find that two efforts strengthen the player with the higher stake and decreases relative rent dissipation.

Supplementing rent seeking with sabotage is another example of multiple efforts. Konrad (2000) assumes that one effort improves the player's contest success whereas a second effort decreases the rival players' success, which may increase lobbying efforts and rent dissipation. Chen (2003) considers competition for promotion involving efforts to enhance one's own performance and efforts to sabotage the opponents' performance. He finds that abler competitors are subject to more attacks. Amegashie,Runkel (2007) study sabotage in a three-stage elimination contest between four players. They find one equilibrium where only the most able contestant engages in sabotage, and one equilibrium without sabotage. Krakel (2005) assumes that each player in the first stage chooses help, sabotage, or no action, and in the second stage chooses effort to win the tournament, which cause a variety of equilibria.

Multiple efforts, i.e. production and appropriation, are also present in the conflict models by Hirshleifer (1995), Skaperdas,Syropoulos (1997), and Hausken (2005), but contest success depends only on appropriation.

In the rent seeking literature the decisiveness or contest intensity parameter is generally considered to be a parameter at the contest level, and thus equivalent for both or all players. The authors are not aware of literature modeling different contest intensity parameters for different players. In this paper each effort operates according to its own logic with an intensity, scaling and impact independent of the other efforts. Hence the contest intensity parameters generally differ across players. Specific efforts by one player are thus not matched against specific efforts by the other player. Instead each player's efforts are added up into an effort production function which competes against the other player's effort production function.

Section 2 axiomatizes the contest assuming multiple additive efforts. Section 3 models additive multi-effort contests. Section 4 analyzes the model. Section 5 concludes.

2 Axiomatization

Player $h \in \{1,2\}$ exerts K efforts y_{hk}, $k \in \{1, \dots, K\}$ at unit cost c_{hk} to increase his probability of winning a rent with value $S \geq 0$. Define $\boldsymbol{y_h} = (y_{h1}, \dots, y_{hK})$ as the vector of player $h's$ K efforts. Rent dissipation is defined as $D = \sum_{h=1}^{2} \sum_{k=1}^{K} c_{hk} y_{hk} / S$. Player $h's$ expected utility is

$$\Pi_h(\boldsymbol{y_1}, \boldsymbol{y_2}) = p_h(\boldsymbol{y_1}, \boldsymbol{y_2})S - \sum_{k=1}^{K} c_{hk} y_{hk} \tag{1}$$

where $p_h(\boldsymbol{y_1}, \boldsymbol{y_2})$ is player $h's$ contest success function, i.e. winning probability, $(\boldsymbol{y_1}, \boldsymbol{y_2}) \in \mathbb{R}_+^{2K}$. In accordance with Arbatskaya,Mialon (2010), we consider subcontests between any two of three players, $h \in H \equiv \{1,2,3\}$. We define $\boldsymbol{y} = (\boldsymbol{y_1}, \boldsymbol{y_2}, \boldsymbol{y_3})$ as the vector of the three players' $3K$ efforts, $\boldsymbol{y_{-h}}$ as the vector of two of the players' $2K$ efforts aside from player $h's$ K efforts, and $p_h(\boldsymbol{y})$ as player $h's$ contest success function.

Axiom 1. (i) For all $h \in H$ and $\boldsymbol{y} \in \mathbb{R}_{+}^{3K}$, $p_h(\boldsymbol{y}) \geq 0$ and $\sum_{h \in H} p_h(\boldsymbol{y}) \leq 1$.
(ii) For all $h \in H$, if $y_{hk} \in \mathbb{R}_{++}^{1}$ for at least one $k \in \{1, \dots, K\}$ and $\boldsymbol{y_{-h}} \in \mathbb{R}_{+}^{2K}$, then $p_h(\boldsymbol{y}) > 0$.
(iii) For all $h \in H$, if $\boldsymbol{y_h} = \boldsymbol{0}$ and $\boldsymbol{y_{-h}} \in \mathbb{R}_{+}^{2K}$, then $p_h(\boldsymbol{y}) = 0$.
(iv) If $y_{hk} \in \mathbb{R}_{++}^{1}$ for at least one $k \in \{1, \dots, K\}$ and for some $h \in H$, then $\sum_{h \in H} p_h(\boldsymbol{y}) = 1$.

Axiom 1 (i) and (iii) are the same as in Arbatskaya,Mialon (2010), whereas (ii) and (iv) are not. Axiom 1 (ii) states that if player h exerts at least one positive effort, then his winning probability is positive regardless of the other players' efforts. This contrasts with Arbatskaya,Mialon (2010) who require that all player $h's$ K efforts have to be positive in order for his winning probability to be positive regardless of the other players' efforts. Analogously, Axiom 1 (iv) states that if at least one player exerts at least one positive effort, then the winning probabilities sum to one. This also contrasts with Arbatskaya,Mialon (2010) who require that all the K efforts by at least one player have to be positive in order for the winning probabilities to sum to one. Axioms 2 and 3 are as in Arbatskaya,Mialon (2010), i.e.

Axiom 2. For all $h \neq r \neq s \in H$, the odds ratio $\frac{p_h(y)}{p_r(y)}$ does not depend on $\boldsymbol{y_k}$ for $\boldsymbol{y_h} \in \mathbb{R}_{+}^{K}$, $\boldsymbol{y_r} \in \mathbb{R}_{++}^{K}$, and $\boldsymbol{y_s} \in \mathbb{R}_{+}^{K}$.

Axiom 3. For all $h \in \{1,2\}$ and $(\boldsymbol{y_1}, \boldsymbol{y_2}) \in \mathbb{R}_{+}^{2K}$, $p_h(\boldsymbol{y_1}, \boldsymbol{y_2}) = p_h(\boldsymbol{y_1}, \boldsymbol{y_2}, \boldsymbol{y_3} = \boldsymbol{0})$.

Lemma 1. If Axioms 1,2,3 hold, then player $h's$ contest success function, $h \in \{1,2\}$, in a two player contest where each player exerts K efforts is

$$p_h(\boldsymbol{y_1}, \boldsymbol{y_2}) \begin{cases} = \dfrac{f_h(\boldsymbol{y_h})}{f_1(\boldsymbol{y_1}) + f_2(\boldsymbol{y_2})} \; if \; y_{1k} \in \mathbb{R}_{++}^{1} \text{ or } y_{2k} \in \mathbb{R}_{++}^{1} \\ \quad \text{for at least one } k \in \{1, \dots, K\} \\ \geq 0 \; and \; p_1(\boldsymbol{y_1}, \boldsymbol{y_2}) + p_2(\boldsymbol{y_1}, \boldsymbol{y_2}) \leq 1 \; if \; y_{1k} \in \mathbb{R}_{+}^{1} \backslash \mathbb{R}_{++}^{1} \text{ and} \\ \quad y_{2r} \in \mathbb{R}_{+}^{1} \backslash \mathbb{R}_{++}^{1} \text{ for at least one } k \in \{1, \dots, K\} \\ \quad \text{and at least one } r \in \{1, \dots, K\} \\ = 0 \; if \; \boldsymbol{y_h} = \boldsymbol{0} \end{cases} \tag{2}$$

where player $h's$ production function $f_h(\boldsymbol{y_h})$ satisfies

$$f_h(\boldsymbol{y_h}) \begin{cases} > 0 \; if \; y_{hk} \in \mathbb{R}_{++}^{1} \text{ for at least one } k \in \{1, \dots, K\} \\ \geq 0 \; if \; y_{hk} \in \mathbb{R}_{+}^{1} \text{ for at least one } k \in \{1, \dots, K\} \\ = 0 \; if \; \boldsymbol{y_h} = \boldsymbol{0} \end{cases} \tag{3}$$

Proof. Appendix A.

Lemma 1 differs from Arbatskaya,Mialon (2010) e.g. in that it is sufficient that one of the K efforts are positive to cause $f_h(\boldsymbol{y_h})$ and $p_h(\boldsymbol{y_1}, \boldsymbol{y_2})$ to be positive.

Axiom 4. For all $h \neq r$, $h \in \{1,2\}$, $r \in \{1,2\}$, $k \in \{1, \ldots, K\}$, and $n \in \{1, \ldots, K\}$, $p_h(\boldsymbol{y_1}, \boldsymbol{y_2})$ is nondecreasing in y_{hk} if $y_{hk} \in \mathbb{R}^1_+$ and $y_{rn} \in \mathbb{R}^1_+$ for at least one $n \in \{1, \ldots, K\}$, and continuous and strictly increasing in y_{hk} if $y_{hk} \in \mathbb{R}^1_{++}$ and $y_{rn} \in \mathbb{R}^1_{++}$ for at least one $n \in \{1, \ldots, K\}$.

Axiom 4 states that each player's success probability is nondecreasing in any given effort by that player if that given effort is positive and at least one effort by the other player is positive. Furthermore, each player's success probability is continuous and strictly increasing in any given effort by that player if that given effort is strictly positive and at least one effort by the other player is strictly positive. This contrasts with Arbatskaya,Mialon (2010) who require that each player's success probability is nondecreasing in any given effort by that player if all $2K$ efforts by both players are positive. Furthermore, they require that each player's success probability is continuous and strictly increasing in any given effort by that player if all $2K$ efforts by both players are strictly positive.

Axiom 5. For all $h \in \{1,2\}$ and $\lambda > 0$, $p_h\big(\lambda f_1(\boldsymbol{y_1}), \lambda f_2(\boldsymbol{y_2})\big) = p_h(\boldsymbol{y_1}, \boldsymbol{y_2})$.

Axiom 5 states that an equiproportionate change in both players' production functions $f_1(\boldsymbol{y_1})$ and $f_2(\boldsymbol{y_2})$ does not impact the players' success probabilities. This is a weaker requirement than Arbatskaya and Mialon's (2010) Axiom 5 which requires that an equiproportionate change in corresponding single efforts, i.e. for any $k \in \{1, \ldots, K\}$, for both players does not impact the players' success probabilities. Their Axiom 5 implies Cobb-Douglas production functions, whereas this paper's Axiom 5 allows any production function.

Axiom 6. For all $h \neq r$, $h \in \{1,2\}$, $r \in \{1,2\}$, if $a \in \mathbb{R}^1_{++}$ or $b > 1$, then $p_h\big(f_h(\boldsymbol{y_1})b + a, f_r(\boldsymbol{y_2})\big) > p_h\big(f_h(\boldsymbol{y_1}), f_r(\boldsymbol{y_2})\big)$.

Axiom 6 states that adding a strictly positive constant to a player's production function, or multiplying a player's production function with a constant strictly larger than one, causes higher success probability.

Axiom 7. For all $h \in \{1,2\}$, $k \neq r$, $k \in \{1, \dots, K\}$, $r \in \{1, \dots, K\}$, assume $f_h(\boldsymbol{y_h}) = f_h(y_{h1}, \dots, y_{hk}, \dots, y_{hK}) = f_h\big(f_{h1}(y_{h1}), \dots, f_{hk}(y_{hk}), \dots, f_{hK}(y_{hK})\big) = \sum_{k=1}^{K} f_{hk}(y_{hk}) = \sum_{k=1}^{K} f_h(y_{hk})$, $\partial f_h(y_{h1}, \dots, y_{hk}, \dots, y_{hK})/\partial y_{hk} = \partial f_h(y_{hk})/\partial y_{hk} \neq 0$, and $\partial f_h(y_{hk})/\partial y_{hr} = 0$.

Axiom 7 assumes that player $h's$ production function $f_h(\boldsymbol{y_h})$ is separable into K additive independent production functions $f_{hk}(y_{hk}) = f_h(y_{hk})$, i.e. one production function for each effort y_{hk}. This differs from the multiplicative, and thus not additive, Cobb-Douglas production function implied by Arbatskaya and Mialon's (2010) Axiom 5.

Axiom 8. For all $h \in \{1,2\}$ and $k \in \{1, \dots, K\}$, if $d_{hk} \in \mathbb{R}_+^1$ and $m_k \in \mathbb{R}_+^1$, then $f_h(y_{hk}) = d_{hk} y_{hk}^{m_k}$ and $f_h(y_{h1}, \dots, y_{hk}) = f_h(y_{h1}, \dots, y_{hk-1}) + f_h(y_{hk})$.

Axiom 8 assumes a functional form for the additive production function.

Lemma 2. When Axioms 1-8 hold, for all $h \in \{1,2\}$ and $k \in \{1, \dots, K\}$, $f_h(\boldsymbol{y_h}) = \sum_{k=1}^{K} d_{hk} y_{hk}^{m_k}$.

Proof. Follows from Axioms 1-8 and Lemma 1.

Axiom 9. For all $h = 1,2$, $p_h(\boldsymbol{y_1}, \boldsymbol{y_2}) = 1/2$ if $\boldsymbol{y_1} = \boldsymbol{y_2} = \boldsymbol{0}$.

Axiom 9 assumes that the players' share the rent equally if both withdraw from exerting effort.

Axiom 10. For any $(\boldsymbol{y_1}, \boldsymbol{y_2}) \in \mathbb{R}_+^{2K}$, $p_1(\boldsymbol{y_1}, \boldsymbol{y_2}) = p_2(\boldsymbol{y_2}, \boldsymbol{y_1})$,

Axiom 10 assumes symmetry or anonymity in the sense that the players' efforts, and not their identities, determine their winning probabilities. Axiom 10 does not apply e.g. if equal efforts by the players cause different winning probabilities.

3 Modeling additive multi-effort contests

Consistently with Axioms 1-9 and Lemmas 1-2 we assume that two players compete for a rent $S \geq 0$. To decrease the number of subscripts we use regular letters for player 1 and capital letters for player 2. Player 1 exerts m rent seeking efforts x_i at unit cost $c_i \geq 0$. Player 2 exerts M rent seeking efforts X_j at unit cost $C_j \geq 0$. We define $K = max\{m, M\}$ which means that if $m \neq M$, then one player exerts fewer efforts than the other player. For the axiomatization in section 2 this means that the efforts not exerted by the player exerting fewest efforts are set to zero. Each player's efforts have additive impact on the contest. Player 1's effort production function is $\sum_{i=1}^{m} d_i x_i^{m_i}$, where d_i is a proportional scaling parameter for impact and $m_i \geq 0$ is player 1's decisiveness or contest intensity which scales as an exponent the impact of each effort x_i. Analogously, player 2's effort production function is $\sum_{j=1}^{M} D_j X_j^{M_j}$, where D_j is a proportional scaling parameter for impact and $M_j \geq 0$ scales the impact of each effort X_j as an exponent.[1] Applying the ratio form contest success function (Tullock 1980; Skaperdas 1996), the probability that player 1 wins the rent is

$$p_1 = p_1(x_1, \dots, x_m, X_1, \dots, X_M) = \frac{\sum_{i=1}^{m} d_i x_i^{m_i}}{\sum_{i=1}^{m} d_i x_i^{m_i} + \sum_{j=1}^{M} D_j X_j^{M_j}} \tag{4}$$

which also can be interpreted as the fraction of the rent earned by player 1 if the rent is sharable, $\partial p_1 / \partial x_i \geq 0$ and $\partial p_1 / \partial X_j \leq 0$. The probability that player 2 wins the rent is $p_2 = p_2(x_1, \dots, x_m, X_1, \dots, X_M) = 1 - p_1$. If the players exert no efforts we set $p_1 = 1/2$. Multiplying each player's probability of winning the rent with the rent S, and subtracting the effort cost expenditures, the expected utilities are

$$\begin{aligned} u &= \frac{\sum_{i=1}^{m} d_i x_i^{m_i}}{\sum_{i=1}^{m} d_i x_i^{m_i} + \sum_{j=1}^{M} D_j X_j^{M_j}} S - \sum_{i=1}^{m} c_i x_i, \\ U &= \frac{\sum_{j=1}^{M} D_j X_j^{M_j}}{\sum_{i=1}^{m} d_i x_i^{m_i} + \sum_{j=1}^{M} D_j X_j^{M_j}} S - \sum_{j=1}^{M} C_j X_j \end{aligned} \tag{5}$$

for player 1 and player 2, respectively. If the players exert no efforts we set $u = U = S/2$. The model has $3m + 3M + 2$ parameters, i.e. $m + M$ unit efforts costs c_i and C_j, $m + M$ scaling parameters d_i and D_j, $m + M$ contest intensities m_i and M_j, and m and M for the numbers of

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[1] When $m_i = M_j = 0$, the efforts have no impact. When $0 < m_i = M_j < 1$, the efforts have less than proportional impact. When $m_i = M_j = 1$, the efforts have proportional impact. When $0 < m_i = M_j < 1$, the efforts have more than proportional impact.

efforts which are the players' strategic choice variables. Both players choose their strategies simultaneously and independently. Appendix B shows the nomenclature.

4 Solving the model

Differentiating (5), the first order conditions are

$$\begin{aligned}\frac{\partial u}{\partial x_i} &= \frac{S m_i d_i x_i^{m_i-1}\sum_{j=1}^{M} D_j X_j^{M_j}}{\left(\sum_{i=1}^{m} d_i x_i^{m_i} + \sum_{j=1}^{M} D_j X_j^{M_j}\right)^2} - c_i = 0 \Leftrightarrow \frac{S\sum_{j=1}^{M} D_j X_j^{M_j}}{\left(\sum_{i=1}^{m} d_i x_i^{m_i} + \sum_{j=1}^{M} D_j X_j^{M_j}\right)^2} = \frac{c_i}{m_i d_i x_i^{m_i-1}},\\ \frac{\partial U}{\partial X_j} &= \frac{S M_j D_j X_j^{M_j-1}\sum_{i=1}^{m} d_i x_i^{m_i}}{\left(\sum_{i=1}^{m} d_i x_i^{m_i} + \sum_{j=1}^{M} D_j X_j^{M_j}\right)^2} - C_j = 0 \Leftrightarrow \frac{S\sum_{i=1}^{m} d_i x_i^{m_i}}{\left(\sum_{i=1}^{m} d_i x_i^{m_i} + \sum_{j=1}^{M} D_j X_j^{M_j}\right)^2} = \frac{C_j}{M_j D_j X_j^{M_j-1}}\end{aligned} \tag{6}$$

Equation (6) allows expressing all efforts for each player as functions of one effort for that player. Without loss of generality we choose that one effort to be x_1 for player 1 and X_1 for player 2. Solving (6) gives

$$\begin{aligned}\frac{c_i}{m_i d_i x_i^{m_i-1}} &= \frac{c_1}{m_1 d_1 x_1^{m_1-1}} \Leftrightarrow x_i = x_1^{\frac{1-m_1}{1-m_i}}\left(\frac{m_i d_i c_1}{m_1 d_1 c_i}\right)^{\frac{1}{1-m_i}}, i = 2,\dots,m\\ \frac{C_j}{M_j D_j X_j^{M_j-1}} &= \frac{C_1}{M_1 D_1 X_1^{M_1-1}} \Leftrightarrow X_j = X_1^{\frac{1-M_1}{1-M_j}}\left(\frac{M_j D_j C_1}{M_1 D_1 C_j}\right)^{\frac{1}{1-M_j}}, j = 2,\dots,M\end{aligned} \tag{7}$$

Inserting (7) into (6) and solving gives

$$\begin{aligned}x_1^{m_1} &= \frac{m_1^2 M_1 C_1 d_1 D_1 X_1^{1-M_1} S}{\left(m_1 C_1 d_1 X_1^{1-M_1} + M_1 c_1 D_1 x_1^{1-m_1}\right)^2} - \frac{1}{d_1}\sum_{i=2}^{m} d_i x_1^{\frac{m_i(1-m_1)}{1-m_i}}\left(\frac{m_i d_i c_1}{m_1 d_1 c_i}\right)^{\frac{m_i}{1-m_i}},\\ X_1^{M_1} &= \frac{m_1 M_1^2 c_1 d_1 D_1 x_1^{1-m_1} S}{\left(m_1 C_1 d_1 X_1^{1-M_1} + M_1 c_1 D_1 x_1^{1-m_1}\right)^2} - \frac{1}{D_1}\sum_{j=2}^{M} D_j X_1^{\frac{M_j(1-M_1)}{1-M_j}}\left(\frac{M_j D_j C_1}{M_1 D_1 C_j}\right)^{\frac{M_j}{1-M_j}}\end{aligned} \tag{8}$$

which are two equations with two unknown x_1 and X_1, where x_i and X_j follow from (7). Differentiating (6), the second order conditions are satisfied e.g. when $m_{\mathrm{i}} \le 1$ and $M_{\mathrm{j}} \le 1$, i.e.

$$\begin{aligned}\frac{\partial^2 u}{\partial x_i^2} &= S m_i d_i x_i^{m_i-2}\sum_{j=1}^{M} D_j X_j^{M_j}\frac{(m_i-1)\left(\sum_{i=1}^{m} d_i x_i^{m_i} + \sum_{j=1}^{M} D_j X_j^{M_j}\right) - 2 m_i d_i x_i^{m_i}}{\left(\sum_{i=1}^{m} d_i x_i^{m_i} + \sum_{j=1}^{M} D_j X_j^{M_j}\right)^3} \le 0,\\ \frac{\partial^2 U}{\partial X_j^2} &= S M_j D_j X_j^{M_j-2}\sum_{i=1}^{m} d_i x_i^{m_i}\frac{(M_j-1)\left(\sum_{i=1}^{m} d_i x_i^{m_i} + \sum_{j=1}^{M} D_j X_j^{M_j}\right) - 2 M_j D_j X_j^{M_j}}{\left(\sum_{i=1}^{m} d_i x_i^{m_i} + \sum_{j=1}^{M} D_j X_j^{M_j}\right)^3} \le 0\end{aligned} \tag{9}$$

Although (7) and (8) are numerically solvable, they are not analytically solvable for all m_i and M_j. The next subsections develop solutions with various assumptions for (7) and (8).

4.1 Removing redundant efforts

If $m_k = m_i$ for at least one $k = 1, \dots, m, k \neq i$ or $M_k = M_j$ for at least one $k = 1, \dots, M, k \neq j$, then at least one other effort, x_k or X_k, has the same decisiveness as either x_i (when $m_k = m_i$) or X_j (when $M_k = M_j$). The least costly of these efforts is retained. That is, for player 1, if $c_i \leq c_k$, effort x_k is removed, efforts $x_{k+1}, \dots, x_m$ are relabeled as efforts $x_k, \dots, x_{m-1}$, and m is decreased by 1. Conversely, if $c_i > c_k$, effort x_i is removed, efforts $x_{i+1}, \dots, x_m$ are relabeled as efforts $x_i, \dots, x_{m-1}$, and m is decreased by 1. Analogously for player 2, if $C_j \leq C_k$, effort X_k is removed, efforts $X_{k+1}, \dots, X_M$ are relabeled as efforts $X_k, \dots, X_{M-1}$, and M is decreased by 1. Conversely, if $C_j > C_k$, effort X_j is removed, efforts $X_{j+1}, \dots, X_M$ are relabeled as efforts $X_j, \dots, X_{M-1}$, and M is decreased by 1. This procedure is repeated until $m_k \neq m_i, k = 1, \dots, m, k \neq i$ and $M_k \neq M_j, k = 1, \dots, M, k \neq j$.

4.2 m efforts against M efforts

This section assumes $m \geq 1, M \geq 1, m_1 = M_1 = 1, 0 < m_i < 1, i = 2, \dots, m; 0 < M_j < 1, j = 2, \dots, M; x_1 \geq 0, X_1 \geq 0$. One common choice for the contest intensity is one, assumed without loss of generality for efforts x_1 and X_1. Inserting $m_1 = M_1 = 1$ into (7) and (8) gives

$$\begin{aligned} x_1 &= \frac{C_1 d_1 D_1 S}{(C_1 d_1 + c_1 D_1)^2} - \frac{1}{d_1} \sum_{i=2}^{m} d_i \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{m_i}{1-m_i}}, x_i = \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{1}{1-m_i}}, i = 2, \dots, m, \\ X_1 &= \frac{c_1 d_1 D_1 S}{(C_1 d_1 + c_1 D_1)^2} - \frac{1}{D_1} \sum_{j=2}^{M} D_j \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{M_j}{1-M_j}}, X_j = \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{1}{1-M_j}}, j = 2, \dots, M \end{aligned} \tag{10}$$

which are inserted into (5) to yield

$$\begin{aligned} u &= \frac{C_1^2 d_1^2 S}{(C_1 d_1 + c_1 D_1)^2} + \sum_{i=2}^{m} c_i \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{1}{1-m_i}} \left(\frac{1}{m_i} - 1\right), \\ U &= \frac{c_1^2 D_1^2 S}{(C_1 d_1 + c_1 D_1)^2} + \sum_{j=2}^{M} C_j \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{1}{1-M_j}} \left(\frac{1}{M_j} - 1\right) \end{aligned} \tag{11}$$

with rent dissipation

$$D=\frac{1}{S}\left(\sum_{i=2}^{m}c_ix_i+\sum_{j=2}^{M}C_jX_j\right)$$
$$=\frac{2c_1C_1d_1D_1}{(C_1d_1+c_1D_1)^2}-\frac{1}{S}\left(\sum_{i=2}^{m}\left(\frac{1}{m_i}-1\right)c_i\left(\frac{m_id_ic_1}{d_1c_i}\right)^{\frac{1}{1-m_i}}+\sum_{j=2}^{M}\left(\frac{1}{M_j}-1\right)C_j\left(\frac{M_jD_jC_1}{D_1C_j}\right)^{\frac{1}{1-M_j}}\right) \tag{12}$$

Differentiating (6), the second order conditions inserting (10) are always satisfied, i.e.

$$\frac{\partial^2u}{\partial x_i^2}=\frac{c_i}{x_i}\left(m_i-1-\frac{2m_id_ix_i^{m_i}(C_1d_1+c_1D_1)}{d_1D_1S}\right)<0,$$
$$\frac{\partial^2U}{\partial X_j^2}=\frac{C_j}{X_j}\left(M_j-1-\frac{2M_jD_jX_j^{M_j}(C_1d_1+c_1D_1)}{d_1D_1S}\right)<0 \tag{13}$$

Proposition 1. Although $u>0$ and $U>0$ in (11), and $x_i>0$ and $X_j>0$ in (10), x_1 and X_1 in (10) are not guaranteed to be positive. When $m_1=M_1=1, 0<m_\mathrm{i}<1, i=2,\dots,m; 0<M_\mathrm{j}<1, j=2,\dots,M; x_1\geq 0, X_1\geq 0$, then $\frac{\partial x_1}{\partial m}<0, \frac{\partial X_1}{\partial M}<0, \frac{\partial x_1}{\partial d_i}<0, \frac{\partial X_1}{\partial D_j}<0, \frac{\partial x_1}{\partial c_1}<0, \frac{\partial X_1}{\partial C_1}<0, \frac{\partial x_1}{\partial c_i}>0, \frac{\partial X_1}{\partial C_j}>0, \frac{\partial x_1}{\partial S}>0, \frac{\partial X_1}{\partial S}>0, \frac{\partial x_1}{\partial m_i}>0\ if\ \ln\left(\frac{m_id_ic_1}{d_1c_i}\right)<m_i-1, \frac{\partial X_1}{\partial M_j}>0\ if\ \ln\left(\frac{M_jD_jC_1}{D_1C_j}\right)<M_j-1$.

Proof. Appendix C.

Proposition 1 states that the players' efforts x_1 and X_1 decrease as the numbers m and M of efforts increase. Additional available efforts x_i and X_i enable a player to optimally and thus cost-effectively choose among these additional efforts, and thus cut back on the extent to which efforts x_1 and X_1 are utilized. But limits exist. Equation (10) shows that x_1 and X_1 can be negative, invalidating the solution, if the negative summation term exceeds the positive term. Player 1's effort x_1 decreases, eventually becoming negative rendering the solution in (10) and (11) invalid, when its unit cost c_1 increases, or the impact d_i of player 1's other efforts x_i increases. Conversely, x_1 increases when the rent S or the unit costs c_i of player 1's other efforts x_i increases. Analogously, player 2's effort X_1 decreases when its unit cost C_1 increases, or the impact D_j of player 2's other efforts X_j increases. Conversely, X_1 increases when the rent S or the unit costs C_j

of player 2's other efforts X_j increases. Appendix C shows some other instances, dependent on various combinations of parameter values, where x_1 or X_1 is negative.

Proposition 2. When $m_1 = M_1 = 1,\ 0 < m_{\mathrm{i}} < 1, i = 2, \dots, m; 0 < M_{\mathrm{j}} < 1, j = 2, \dots, M$;$\mathrm{x}_1 \geq 0$,$X_1 \geq 0$, then $\frac{\partial x_i}{\partial m} = 0, \frac{\partial X_i}{\partial M} = 0, \frac{\partial x_i}{\partial d_1} < 0, \frac{\partial X_j}{\partial D_1} < 0, \frac{\partial x_i}{\partial d_i} > 0, \frac{\partial X_j}{\partial D_j} > 0, \frac{\partial x_i}{\partial c_1} > 0,\ \frac{\partial X_j}{\partial C_1} > 0, \frac{\partial x_i}{\partial c_i} < 0,$ $\frac{\partial X_j}{\partial C_j} < 0, \frac{\partial x_i}{\partial S} = 0, \frac{\partial X_j}{\partial S} = 0, \frac{\partial x_i}{\partial m_i} > 0\ if\ \ln\left(\frac{m_i d_i c_1}{d_1 c_i}\right) > 1 - \frac{1}{m_i}, \frac{\partial X_j}{\partial M_j} > 0\ if\ \ln\left(\frac{M_j D_j C_1}{D_1 C_j}\right) > 1 - \frac{1}{M_j}$.

Proof. Appendix C.

Proposition 2 shows that a player's effort, x_{i} or X_{j}, increases in its impact and the player's first effort's unit cost, c_1 or C_1, and decreases in its unit cost and the player's first effort's impact, d_1 or D_1. Whereas x_1 and X_1 decrease in m and M, and x_{i} and X_{i} are independent of m and M, Arbatskaya,Mialon (2010) find that adding additional efforts decreases the effort amounts for the efforts already in play if the added efforts unbalances the contest, i.e. makes one player sufficiently stronger or more advantaged. However, Epstein,Hefeker (2003) find that if both players use their second efforts, they will invest less in their first efforts, which is more in accordance with our finding.

Proposition 3. When $m_1 = M_1 = 1,\ 0 < m_{\mathrm{i}} < 1, i = 2, \dots, m; 0 < M_{\mathrm{j}} < 1, j = 2, \dots, M$;$\mathrm{x}_1 \geq 0$,$X_1 \geq 0$, then $\frac{\partial U}{\partial m} > 0, \frac{\partial u}{\partial M} > 0, \frac{\partial u}{\partial d_i} > 0, \frac{\partial U}{\partial D_j} > 0, \frac{\partial u}{\partial c_i} < 0, \frac{\partial U}{\partial C_j} < 0, \frac{\partial u}{\partial S} > 0, \frac{\partial U}{\partial S} > 0, \frac{\partial u}{\partial m_i} >$ 0 if $\ln\left(\frac{m_i d_i c_1}{d_1 c_i}\right) > 0, \frac{\partial U}{\partial M_j} > 0$ if $\ln\left(\frac{M_j D_j C_1}{D_1 C_j}\right) > 0$.

Proof. Appendix C.

Proposition 3 shows that a player's expected utility, u or U, increases in the number m or M of available efforts due to increased cost-effectiveness. This useful result combined with Proposition 1 means that if a player' rent seeking is moderately successful by focusing solely on improved efficiency as its single effort, increased success can be obtained by adding e.g. lobbying or bribing as a second effort, and cutting back on the first effort. Further, u or U, increases in the rent S and the impacts d_i or D_j, and decreases in the unit costs c_i or C_j. For example when $\frac{d_i c_1}{d_1 c_i} = \frac{D_j C_1}{D_1 C_j} =$

1 which causes negative logarithms, u or U decreases in the contest intensities m_i or M_j, respectively.

Proposition 4. When $m_1 = M_1 = 1,\ 0 < m_\mathrm{i} < 1, i = 2, \dots, m; 0 < M_\mathrm{j} < 1, j = 2, \dots, M;\mathrm{x}_1 \geq 0, X_1 \geq 0,$ then $\frac{\partial D}{\partial m} < 0, \frac{\partial D}{\partial M} < 0, \frac{\partial D}{\partial d_i} < 0, \frac{\partial D}{\partial D_j} < 0, \frac{\partial D}{\partial c_i} > 0, \frac{\partial D}{\partial C_j} > 0, \frac{\partial D}{\partial S} > 0, \frac{\partial D}{\partial m_i} < 0$ if $\ln\left(\frac{m_i d_i c_1}{d_1 c_i}\right) > 0, \frac{\partial D}{\partial M_j} < 0$ if $\ln\left(\frac{M_j D_j C_1}{D_1 C_j}\right) > 0$. For the special event that $c_1 D_1 = C_1 d_1, \frac{\partial D}{\partial d_1} > 0, \frac{\partial D}{\partial D_1} > 0, \frac{\partial D}{\partial c_1} < 0, \frac{\partial D}{\partial C_1} < 0$.

Proof. Appendix C.

Proposition 4 shows that rent dissipation D decreases in the number m or M of available efforts, and the impacts d_i or D_j, since players with more efforts optimize more cost-effectively across efforts. Rent dissipation D increases in the unit cost c_i or C_j, and the rent S. For example when $\frac{d_i c_1}{d_1 c_i} = \frac{D_j C_1}{D_1 C_j} = 1$ which causes negative logarithms, D increases in the contest intensities m_i and M_j. For unbalanced asymmetric contests, where one player is advantaged, Arbatskaya,Mialon (2010) also find that additional efforts decrease rent dissipation. However, for balanced contests (e.g. when additional efforts are symmetric between players) additional efforts tend to increase rent dissipation since they increase the contest's discriminatory power defined as the sum of the contest intensities (equal for both players) across all efforts (both players have equally many efforts). Their result follows from multiplication of efforts in the contest success function. They thus find that sufficiently symmetric players prefer to eliminate additional efforts, but in equilibrium they utilize the additional efforts.

4.3 *One effort against M efforts*

This section assumes $m = M_1 = 1, M \geq 1, 0 < m_i \leq 1, 0 < M_j < 1, j = 2, \dots, M, X_1 \geq 0$. Another common occurrence is that one player chooses or only has available one effort. Assume that player 1 chooses one effort x_i. Inserting $M_1 = 1$, and $m = 1$ replacing subscript 1 with subscript i for player 1, into (7) and (8), gives

$$x_i^{m_i} = \frac{m_i^2 C_1 d_i D_1 S}{\left(m_i C_1 d_i + c_i D_1 x_i^{1-m_i}\right)^2}, \tag{14}$$

$$X_1 = \frac{m_i c_i d_i D_1 x_i^{1-m_i} S}{\left(m_i C_1 d_i + c_i D_1 x_i^{1-m_i}\right)^2} - \frac{1}{D_1}\sum_{j=2}^{M} D_j \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{M_j}{1-M_j}}, X_j = \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{1}{1-M_j}}, j = 2, \dots, M$$

which simplifies to (10) when $m_i = 1$. When $m_i = 1/2$, the first equation in (14) becomes a third order equation in x_i which impacts X_1.

Proposition 5. When $m = M_1 = 1,\ 0 < M_\mathrm{j} < 1, j = 2, \dots, M; X_1 \geq 0$, then $\frac{\partial x_i}{\partial M} = \frac{\partial x_i}{\partial D_j} = \frac{\partial x_i}{\partial C_j} = 0, \frac{\partial x_i}{\partial S} > 0$.

Proof. Follows from (14).

Proposition 5 states that player 1's single effort x_i does not depend on the number M of efforts exerted by player 2. This follows since player 2 optimizes cost-effectively across efforts so that M does not impact player 1.

4.4 m efforts against one effort

This section assumes $M = m_1 = 1, m \geq 1, 0 < M_j \leq 1, 0 < m_i < 1, i = 2, \dots, m, x_1 \geq 0$. Assume that player 2 chooses one effort X_j. Inserting $m_1 = 1$, and $M = 1$ replacing subscript 1 with subscript j for player 2, into (7) and (8), gives

$$x_1 = \frac{M_j C_j d_1 D_j X_j^{1-M_j} S}{\left(C_j d_1 X_j^{1-M_j} + M_j c_1 D_j\right)^2} - \frac{1}{d_1}\sum_{i=2}^{m} d_i \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{m_i}{1-m_i}}, x_i = \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{1}{1-m_i}}, i = 2, \dots, m,$$

$$X_j^{M_j} = \frac{M_j^2 c_1 d_1 D_j S}{\left(C_j d_1 X_j^{1-M_j} + M_1 c_1 D_1\right)^2} \tag{15}$$

which simplifies to (10) when $M_j = 1$. When $M_j = 1/2$, the second equation in (15) becomes a third order equation in X_j which impacts x_1.

Proposition 6. When $M = m_1 = 1,\ 0 < m_\mathrm{i} < 1, i = 2, \dots, m; x_1 \geq 0$, then $\frac{\partial X_j}{\partial m} = \frac{\partial X_j}{\partial d_i} = \frac{\partial X_j}{\partial c_i} = 0, \frac{\partial X_j}{\partial S} > 0$.

Proof. Follows from (15).

Analogously to Proposition 5, Proposition 6 states that player 2's single effort X_j does not depend on the number m of efforts exerted by player 1.

4.5 One effort against one effort

This section assumes $m = M = 1, u \geq 0, U \geq 0$. Inserting $m = M = 1$ into (6) and solving gives $x_i = \frac{m_i C_j}{M_j c_i} X_j$ causing (6) to be analytically solvable when $m_i = M_j$, which gives

$$x_i = \frac{C_j}{c_i} X_j = \frac{M_j c_i^{M_j - 1} C_j^{M_j} d_i D_j S}{\left(C_j^{M_j} d_i + c_i^{M_j} D_j\right)^2}, X_j = \frac{M_j C_j^{M_j - 1} c_i^{M_j} d_i D_j S}{\left(C_j^{M_j} d_i + c_i^{M_j} D_j\right)^2}, D = \frac{2 M_j c_i^{M_j} C_j^{M_j} d_i D_j}{\left(C_j^{M_j} d_i + c_i^{M_j} D_j\right)^2},$$
$$u = \frac{C_j^{M_j} d_i \left(C_j^{M_j} d_i + (1 - M_j) c_i^{M_j} D_j\right) S}{\left(C_j^{M_j} d_i + c_i^{M_j} D_j\right)^2}, U = \frac{c_i^{M_j} D_j \left(c_i^{M_j} D_j + (1 - M_j) C_j^{M_j} d_i\right) S}{\left(C_j^{M_j} d_i + c_i^{M_j} D_j\right)^2}, \quad (16)$$

which simplifies to (10), (11), and (12) when $M_j = i = j = 1$.

4.6 Optimal numbers of efforts

Although the players have m and M available efforts, it may not be optimal for them to employ all m and M efforts. This section addresses this issue and the issue of $x_1 < 0$ or $X_1 < 0$. When $x_1 < 0$ or $X_1 < 0$ in (10), at least one effort x_{i} or X_{j} has to be removed. Thus all $w = \sum_{z=1}^{m} \binom{m}{z}$ combinations of the m efforts $x_1, \dots, x_m$ should be assessed, matched against all $W = \sum_{Z=1}^{M} \binom{M}{Z}$ combinations of the M efforts $X_1, \dots, X_M$, where $\binom{m}{z}$ and $\binom{M}{Z}$ are the binomial coefficients for the number of ways in which integers z and Z can be selected among m and M, respectively, when the orders of the selections are irrelevant. The integers z and Z are counting parameters where 1,2,…, m, or 1,2,…, M, efforts can be selected among the m and M efforts. As an example, m=4 and M=5 gives w=15 and W=31. In Table 1 player 1 is the row player and player 2 is the column player.

Table 1 $W \times w$ matrix for the players' efforts $x_i(\psi, \Psi)$, $i = 1, \dots, m$, and $X_j(\psi, \Psi)$, $j = 2, \dots, M$, and expected utilities $u(\psi, \Psi)$ and $U(\psi, \Psi)$.

		Player 2		
		$\Psi = 1$	$\cdots$	$\Psi = W$
Player 1	$\psi = 1$			
	$\cdots$		$x_i(\psi, \Psi), X_j(\psi, \Psi), u(\psi, \Psi), U(\psi, \Psi)$	
	$\psi = w$			

The rows are counted from $\psi = 1$ to $\psi = w$. The columns are counted from $\Psi = 1$ to $\Psi = W$. For each cell $x_i(\psi, \Psi), X_j(\psi, \Psi), u(\psi, \Psi), U(\psi, \Psi)$ are calculated. Cells where $x_1 < 0$ or $X_1 < 0$ are excluded from consideration. The remaining cells, where $x_1(\psi, \Psi) \geq 0$ and $X_1(\psi, \Psi) \geq 0$, are used to determine one or several Nash equilibria from which no player prefers to deviate unilaterally, i.e.

$$\begin{aligned} u(\psi^*, \Psi^*) \geq u(\psi, \Psi) \; \forall \; \psi = 1, \dots, w \; and \; \Psi = 1, \dots, W, \\ U(\psi^*, \Psi^*) \geq U(\psi, \Psi) \; \forall \; \psi = 1, \dots, w \; and \; \Psi = 1, \dots, W \end{aligned} \tag{17}$$

4.7 An example: Maximum two efforts for each player

This section assumes $m \leq 2, M \leq 2, m_1 = M_1 = 1, 0 < m_2 < 1, 0 < M_2 < 1$. Two efforts for each player enable illustrating the many instances in Proposition 1 when x_1 and X_1 can be negative. Inserting $m = 2$ and $x_1 = 0$ into (10) and solving with respect to c_2 gives

$$c_2 = \frac{m_2 d_2 c_1}{d_1} \left(\frac{d_2 (C_1 d_1 + c_1 D_1)^2 C_1 d_1^2 D_1 S}{C_1 d_1^2 D_1 S} \right)^{\frac{1-m_2}{m_2}} \tag{18}$$

Figure 1 assumes $c_1 = C_1 = d_1 = D_1 = d_2 = D_2 = m_1 = M_1 = 1$ and $S = 10$ and plots $c_2 = C_2$ as a function of $m_2 = M_2$ when $x_1 = X_1 = 0$.

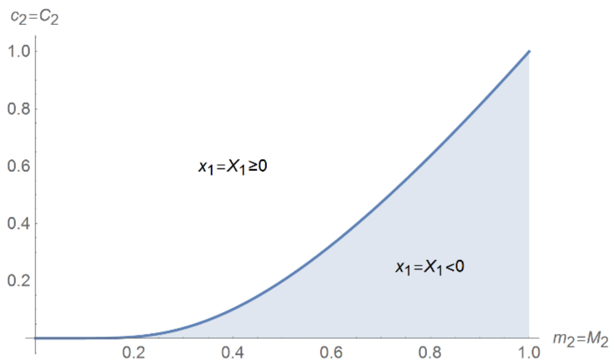

Figure 1 Regions for $x_1 = X_1 \geq 0$ and $x_1 = X_1 < 0$ separated by plotting $c_2 = C_2$ as a function of $m_2 = M_2$ when $x_1 = X_1 = 0$.

Above and to the left of the convex curve the unit costs $c_2 = C_2$ of efforts x_2 and X_2 are sufficiently high, and the contest intensities $m_2 = M_2$ are sufficiently low, making it worth wile for the players to exert the efforts x_1 and X_1 additionally. Conversely, below and to the right of the convex curve the solution in section 4.2 is invalid.

Aside from c_1 which varies, Figure 2 makes the same assumptions as in Figure 1, i.e. $C_1 = d_1 = D_1 = d_2 = D_2 = m_1 = M_1 = 1$ and $S = 10$. Additionally, Figure 2 assumes $c_2 = C_2 = 1$ and $m_2 = M_2 = 0.5$. Figure 2 plots the players' efforts $x_1, x_2, X_1, X_2, etc.$ and expected utilities $u, U, etc.$ as functions of player 1's unit cost c_1 of effort x_1.

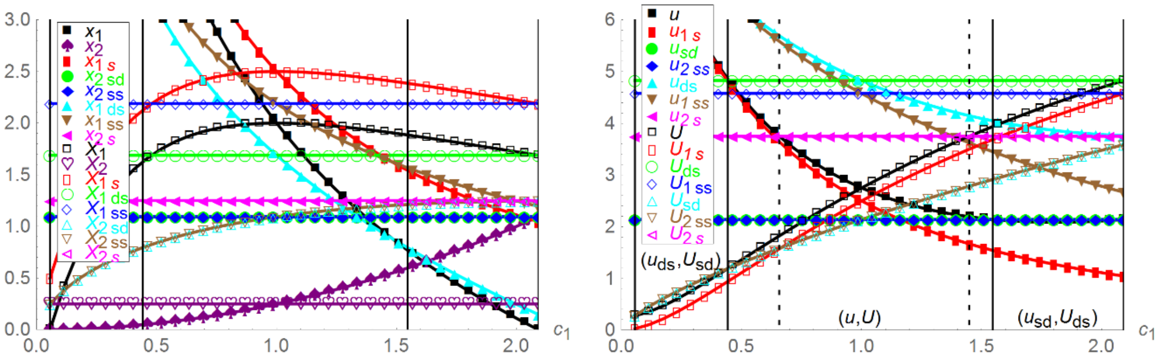

Figure 2 Efforts $x_1, x_2, X_1, X_2, etc.$ and expected utilities $u, U, etc.$ as functions of c_1 when $C_1 = c_2 = C_2 = d_1 = D_1 = d_2 = D_2 = m_1 = M_1 = 1, m_2 = M_2 = 0.5, S = 10$.

Player 1's effort x_1 decreases as its unit effort cost c_1 increases ($\frac{\partial x_1}{\partial c_1} < 0$ in Proposition 1) eventually reaching zero when $c_1 = 2.09$. Player 1 cannot effort the high unit effort cost. This means that $x_1 < 0$ when $c_1 > 2.09$ which invalidates the solution in section 4.2. Conversely, player 1's effort x_2 increases as c_1 increases ($\frac{\partial x_2}{\partial c_1} > 0$ in Proposition 2). Player 1's expected utility u decreases convexly to 2.14 as c_1 increases to 2.09, while player 2's expected utility U increases to 4.83. When c_1 is low, to the advantage of player 1, the first term in the expression for player 2's effort X_1 in (10) is low and cannot compensate for the negative second term. With the given parameter values $X_1 < 0$ when $c_1 < 0.06$, which also invalidates the solution in section 4.2.

Table 2 illustrates the procedure in section 4.6 by presenting a 3×3 matrix accounting for each player's three possibilities. That is, player 1 can choose two efforts x_1 and x_2 where $m_1 = 1$ and $m_2 = 1/2$, one effort x_1 where $m_1 = 1$, or one effort x_2 where $m_2 = 1/2$. Analogously, player 2 can choose two efforts X and X_2 where $M_1 = 1$ and $M_2 = 1/2$, one effort X_1 where $M_1 = 1$, or one effort X_2 where $M_2 = 1/2$.

Table 2 3 × 3 matrix for the players' efforts $x_1, x_2, X_1, X_2, etc.$ and expected utilities $u, U, etc.$

		Player 2		
		$M = 2$	$M = 1, M_1 = 1$	$M = 1, M_2 = 1/2$
Player 1	$m = 2$	$x_1, x_2, X_1, X_2, \boldsymbol{u}, \boldsymbol{U}$	$x_1, x_2, X_{1s}, \boldsymbol{u}, U_{1s}$	$x_{1ds}, x_2, X_{2sd}, \boldsymbol{u_{ds}}, \boldsymbol{U_{sd}}$
	$m = 1, m_1 = 1$	$x_{1s}, X_1, X_2, u_{1s}, \boldsymbol{U}$	$x_{1s}, X_{1s}, u_{1s}, U_{1s}$	$x_{1ss}, X_{2ss}, u_{1ss}, \boldsymbol{U_{2ss}}$
	$m = 1, m_2 = 1/2$	$x_{2sd}, X_{1ds}, X_2, \boldsymbol{u_{sd}}, \boldsymbol{U_{ds}}$	$x_{2ss}, X_{1ss}, \boldsymbol{u_{2ss}}, U_{1ss}$	$x_{2s}, X_{2s}, u_{2s}, U_{2s}$

$\frac{\partial u}{\partial m} > 0$ and $\frac{\partial U}{\partial M} > 0$ in Proposition 3 and (11) in section 4.2 imply that each player prefers the second effort in addition to the first effort when the first effort has contest intensity $m_1 = M_1 = 1$ Thus $u > u_{1s}$ and $U > U_{1s}$ in Figure 2. Each player's second effort as a single effort is not covered by section 4.2 since $m_2 = 1/2$ or $M_2 = 1/2$. Player 1's second effort as a single effort against player 2 exerting both efforts is expressed as x_{2sd} shown in the lower left cell in Table 2. The first subscript "s" means "single effort" by player 1. The second subscript "d" means "double effort" by player 2. Player 1's second effort as a single effort against player 2 exerting only the first effort is expressed as x_{2ss} shown in the lower middle cell in Table 2. Proposition 5 implies $x_{2sd} = x_{2ss}$. Analogously, Proposition 6 implies $X_{2sd} = X_{2ss}$ which is player 2's second effort as a single effort against player 1 exerting both efforts or only the first effort.

An expected utility shown in bold in Table 2 means that this utility is largest for at least one value of c_1. When both expected utilities are in bold within a given cell in Table 2 for the same value of c_1, then the two expected utilities are Nash equilibrium expected utilities as defined in (17).

For the intermediate range $0.44 \leq c_1 \leq 1.55$ in Figure 2, (u, U) is the equilibrium which means that both players choose both efforts. For the upper range $1.55 \leq c_1 \leq 2.09$, the high unit effort cost c_1 makes effort x_1 too costly for player 1 which prefers only the second single effort x_{2sd} with low contest intensity $m_2 = 1/2$. Player 2 still prefers both efforts causing the equilibrium (u_{sd}, U_{ds}). Conversely, for the lower range $0.06 \leq c_1 \leq 0.44$, the low unit effort cost c_1 makes player 1 advantaged. Player 2 can no longer compete cost effectively with both efforts, and settles for the single second effort X_{2sd} with low contest intensity $M_2 = 1/2$. This causes the equilibrium (u_{ds}, U_{sd}).

Between the two dashed vertical lines in Figure 2, $0.66 \leq c_1 \leq 1.45$, the players collectively prefer to exert only their second efforts x_{2s} and X_{2s} as single efforts causing (u_{2s}, U_{2s}), in the lower right cell in Table 2, which is not an equilibrium. For it to arise coordination is needed. Two examples are "burning one's bridges in war" (Schelling 1980) or mutually agreeing on low intensity interaction with $m_2 = M_2 = 1/2$.

5 Conclusion

The paper axiomatizes and analyzes a rent seeking model where players exert multiple additive efforts. Assuming arbitrarily many efforts for each player, an analytical solution exists when the contest intensity for one effort for each player equals one. Then additional efforts enable each player to optimize cost effectively across efforts, cutting back on the effort with contest intensity equal to one. Adding new efforts eventually causes this latter effort eventually to decrease towards zero. Similarly, Epstein,Hefeker (2003) find that if both players use their second of two available efforts, they will invest less in their first efforts, assuming that their second efforts reinforce their first efforts multiplicatively. Interestingly, both additive and multiplicative efforts cause this result.

We find that if one player exerts one effort, this effort does not depend on the number of efforts exerted by the other player which optimizes across efforts. Cost optimization across multiple additive efforts causes lower rent dissipation and higher expected utilities as the number of efforts increases. For symmetric and balanced contests this contrasts with Arbatskaya and Mialon's (2010) finding that additional efforts tend to increase rent dissipation when efforts are multiplicative of the Cobb-Douglas type. It also contrasts with Epstein and Hefeker's (2003) finding of increased rent dissipation when the players' stakes are sufficiently symmetric.

A Nash equilibrium selection method is provided for the event that it may not be optimal for both players to exert all their available efforts, accounting for solutions which have to be determined numerically. An example is provided with maximum two efforts for each player. Nash equilibria are determined where both players choose both efforts, or one player withdraws from its most costly effort to exert only the least costly effort. We also illustrate how both players may collectively prefer to exclude one of their efforts, though in equilibrium they prefer both efforts.

Whereas Arbatskaya,Mialon (2010) as policy implications find that additional socially unproductive efforts may unbalance contests causing increased rent dissipation and decreased expected utilities, we find that both social and non-social additional efforts decrease rent seeking and increase expected utilities due to players' optimization across efforts. For policy careful analysis is required to determine whether multiple efforts are additive or multiplicative. Future research should axiomatize and analyze increasingly general functional forms for the contest success function and analyze empirically which forms are descriptive.

Appendix A Proof of Lemma 1

The proof follows Arbatskaya and Mialon's (2010) template, accounting for Axiom 1 (ii) and (iv) being different. Assume $y_{2k} \in \mathbb{R}^1_{++}$ for at least one $k \in \{1, \dots, K\}$ and $\boldsymbol{y_{-2}} \in \mathbb{R}^{2K}_{+}$, which without loss of generality means that player 2 is the player with at least one positive effort. Axiom 1 (ii) implies $p_2(\boldsymbol{y}) > 0$. Axiom 2 implies that $\frac{p_1(y)}{p_2(y)}$ does not depend on $\boldsymbol{y_3}$. Hence Arbatskaya and Mialon's (2010) subsequent equations apply also for the different Axiom 1 (ii) and (iv), causing Lemma 1.

Appendix B Nomenclature

x_i	player 1's effort, $i = 2, \dots, m$
X_j	player 2's effort, , $j = 2, \dots, M$
m	number of efforts for player 1
M	number of efforts for player 2
S	rent
c_i	player 1's unit cost of effort x_i
C_j	player 2's unit cost of effort X_j
d_i	scaling parameter for player 1's impact of effort x_i
D_j	scaling parameter for player 2's impact of effort X_j
m_i	decisiveness or contest intensity for player 1's effort x_i
M_j	decisiveness or contest intensity for player 2's effort X_j
p_1	probability that player 1 wins the rent
p_2	probability that player 2 wins the rent
u	player 1's expected utility
U	player 2's expected utility

Appendix C First order derivatives for section 3

Differentiating (10) gives

$$
\begin{gathered}
\frac{\partial x_1}{\partial m} < 0, \frac{\partial X_1}{\partial M} < 0, \frac{\partial x_1}{\partial d_1} = \frac{C_1 D_1 (c_1 D_1 - C_1 d_1) S}{(C_1 d_1 + c_1 D_1)^3} + \frac{d_i}{d_1^2 (1-m_i)} \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{m_i}{1-m_i}}, \frac{\partial X_1}{\partial d_1} = \\
\frac{c_1 D_1 (c_1 D_1 - C_1 d_1) S}{(C_1 d_1 + c_1 D_1)^3}, \frac{\partial x_1}{\partial D_1} = \frac{C_1 d_1 (C_1 d_1 - c_1 D_1) S}{(C_1 d_1 + c_1 D_1)^3}, \frac{\partial X_1}{\partial D_1} = \frac{c_1 d_1 (C_1 d_1 - c_1 D_1) S}{(C_1 d_1 + c_1 D_1)^3} + \\
\frac{D_j}{D_1^2 (1-M_j)} \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{M_j}{1-M_j}}, \frac{\partial x_1}{\partial d_i} = \frac{-1}{d_1 (1-m_i)} \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{m_i}{1-m_i}} < 0, \frac{\partial X_1}{\partial d_i} = 0, \frac{\partial x_1}{\partial D_j} = 0, \frac{\partial X_1}{\partial D_j} = \\
\frac{-1}{D_1 (1-M_j)} \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{M_j}{1-M_j}} < 0, \frac{\partial x_1}{\partial c_1} = \frac{-2 C_1 d_1 D_1^2 S}{(C_1 d_1 + c_1 D_1)^3} - \frac{c_i}{c_1^2 (1-m_i)} \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{1}{1-m_i}} < 0, \frac{\partial X_1}{\partial c_1} = \\
\frac{d_1 D_1 (C_1 d_1 - c_1 D_1) S}{(C_1 d_1 + c_1 D_1)^3}, \frac{\partial x_1}{\partial C_1} = \frac{d_1 D_1 (c_1 D_1 - C_1 d_1) S}{(C_1 d_1 + c_1 D_1)^3}, \frac{\partial X_1}{\partial C_1} = \frac{-2 c_1 d_1^2 D_1 S}{(C_1 d_1 + c_1 D_1)^3} - \frac{C_j}{C_1^2 (1-M_j)} \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{1}{1-M_j}} < \\
0, \frac{\partial x_1}{\partial c_i} = \frac{1}{c_1 (1-m_i)} \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{1}{1-m_i}} > 0, \frac{\partial X_1}{\partial c_i} = 0, \frac{\partial x_1}{\partial C_j} = 0, \frac{\partial X_1}{\partial C_j} = \frac{1}{C_1 (1-M_j)} \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{1}{1-M_j}} > \\
0, \frac{\partial x_1}{\partial m_i} = \frac{-d_i \left(1 - m_i + \ln\left(\frac{m_i d_i c_1}{d_1 c_i}\right)\right)}{d_1 (1-m_i)^2} \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{m_i}{1-m_i}}, \frac{\partial X_1}{\partial m_i} = 0, \frac{\partial x_1}{\partial M_j} = 0, \frac{\partial X_1}{\partial M_j} = \\
\frac{-D_j \left(1 - M_j + \ln\left(\frac{M_j D_j C_1}{D_1 C_j}\right)\right)}{D_1 (1-M_j)^2} \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{M_j}{1-M_j}}, \frac{\partial x_1}{\partial S} = \frac{C_1 d_1 D_1}{(C_1 d_1 + c_1 D_1)^2} > 0, \frac{\partial X_1}{\partial S} = \frac{c_1 d_1 D_1}{(C_1 d_1 + c_1 D_1)^2} > 0.
\end{gathered} \tag{19}
$$

and

$$
\begin{gathered}
\frac{\partial x_i}{\partial m} = 0, \frac{\partial X_i}{\partial M} = 0, \frac{\partial x_i}{\partial d_1} = \frac{-1}{d_1 (1-m_i)} \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{1}{1-m_i}} < 0, \frac{\partial X_j}{\partial d_1} = 0, \frac{\partial x_i}{\partial D_1} = 0, \frac{\partial X_j}{\partial D_1} = \\
\frac{-1}{D_1 (1-M_j)} \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{1}{1-M_j}} < 0, \frac{\partial x_i}{\partial d_i} = \frac{1}{d_i (1-m_i)} \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{1}{1-m_i}} > 0, \frac{\partial X_j}{\partial d_i} = 0, \frac{\partial x_i}{\partial D_j} = 0, \frac{\partial X_j}{\partial D_j} = \\
\frac{1}{D_j (1-M_j)} \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{1}{1-M_j}} > 0, \frac{\partial x_i}{\partial c_1} = \frac{1}{c_1 (1-m_i)} \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{1}{1-m_i}} > 0, \frac{\partial X_j}{\partial c_1} = 0, \frac{\partial x_i}{\partial C_1} = 0, \frac{\partial X_j}{\partial C_1} = \\
\frac{1}{C_1 (1-M_j)} \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{1}{1-M_j}} > 0, \frac{\partial x_i}{\partial c_i} = \frac{-1}{c_i (1-m_i)} \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{1}{1-m_i}} < 0, \frac{\partial X_j}{\partial c_i} = 0, \frac{\partial x_i}{\partial C_j} = 0, \frac{\partial X_j}{\partial C_j} = \\
\frac{-1}{C_j (1-M_j)} \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{1}{1-M_j}} < 0, \frac{\partial x_i}{\partial m_i} = \frac{\left(1 - m_i + m_i \ln\left(\frac{m_i d_i c_1}{d_1 c_i}\right)\right)}{m_i (1-m_i)^2} \left(\frac{m_i d_i c_1}{d_1 c_i}\right)^{\frac{1}{1-m_i}}, \frac{\partial X_j}{\partial m_i} = 0, \frac{\partial x_i}{\partial M_j} = \\
0, \frac{\partial X_j}{\partial M_j} = \frac{\left(1 - M_j + M_j \ln\left(\frac{M_j D_j C_1}{D_1 C_j}\right)\right)}{M_j (1-M_j)^2} \left(\frac{M_j D_j C_1}{D_1 C_j}\right)^{\frac{1}{1-M_j}}, \frac{\partial x_i}{\partial S} = 0, \frac{\partial X_j}{\partial S} = 0.
\end{gathered} \tag{20}
$$

Differentiating (11) gives

$$\frac{\partial U}{\partial m} > 0, \frac{\partial u}{\partial M} > 0, \frac{\partial u}{\partial d_1} = \frac{2c_1C_1^2d_1D_1S}{(C_1d_1+c_1D_1)^3} - \frac{c_i}{m_id_1}\left(\frac{m_id_ic_1}{d_1c_i}\right)^{\frac{1}{1-m_i}}, \frac{\partial U}{\partial d_1} = \frac{-2c_1^2C_1D_1^2S}{(C_1d_1+c_1D_1)^3} < 0, \frac{\partial u}{\partial D_1} = \frac{-2c_1C_1^2d_1^2S}{(C_1d_1+c_1D_1)^3} < 0, \frac{\partial U}{\partial D_1} = \frac{2c_1^2C_1d_1D_1S}{(C_1d_1+c_1D_1)^3} - \frac{C_j}{M_jD_1}\left(\frac{M_jD_jC_1}{D_1C_j}\right)^{\frac{1}{1-M_j}}, \frac{\partial u}{\partial d_i} = \frac{c_1}{d_1}\left(\frac{m_id_ic_1}{d_1c_i}\right)^{\frac{m_i}{1-m_i}} > 0, \frac{\partial U}{\partial d_i} = 0, \frac{\partial u}{\partial D_j} = 0, \frac{\partial U}{\partial D_j} = \frac{C_1}{D_1}\left(\frac{M_jD_jC_1}{D_1C_j}\right)^{\frac{M_j}{1-M_j}} > 0, \frac{\partial u}{\partial c_1} = \frac{-2C_1^2d_1^2D_1S}{(C_1d_1+c_1D_1)^3} + \frac{d_i}{d_1}\left(\frac{m_id_ic_1}{d_1c_i}\right)^{\frac{m_i}{1-m_i}}, \frac{\partial U}{\partial c_1} = \frac{2c_1C_1d_1D_1^2S}{(C_1d_1+c_1D_1)^3} > 0, \frac{\partial u}{\partial C_1} = \frac{2c_1C_1d_1^2D_1S}{(C_1d_1+c_1D_1)^3} > 0, \frac{\partial U}{\partial C_1} = \frac{-2c_1^2d_1D_1^2S}{(C_1d_1+c_1D_1)^3} + \frac{D_j}{D_1}\left(\frac{M_jD_jC_1}{D_1C_j}\right)^{\frac{M_j}{1-M_j}}, \frac{\partial u}{\partial c_i} = -\left(\frac{m_id_ic_1}{d_1c_i}\right)^{\frac{1}{1-m_i}} < 0, \frac{\partial U}{\partial c_i} = 0, \frac{\partial u}{\partial C_j} = 0, \frac{\partial U}{\partial C_j} = -\left(\frac{M_jD_jC_1}{D_1C_j}\right)^{\frac{1}{1-M_j}} < 0, \frac{\partial u}{\partial m_i} = \frac{c_i\ln\left(\frac{m_id_ic_1}{d_1c_i}\right)}{m_i(1-m_i)}\left(\frac{m_id_ic_1}{d_1c_i}\right)^{\frac{1}{1-m_i}}, \frac{\partial U}{\partial m_i} = 0, \frac{\partial u}{\partial M_j} = 0, \frac{\partial U}{\partial M_j} = \frac{C_j\ln\left(\frac{M_jD_jC_1}{D_1C_j}\right)}{M_j(1-M_j)}\left(\frac{M_jD_jC_1}{D_1C_j}\right)^{\frac{1}{1-M_j}}, \frac{\partial u}{\partial S} = \frac{C_1^2d_1^2}{(C_1d_1+c_1D_1)^2} > 0, \frac{\partial U}{\partial S} = \frac{c_1^2D_1^2}{(C_1d_1+c_1D_1)^2} > 0. \tag{21}$$

Differentiating (12) gives

$$\frac{\partial D}{\partial m} < 0, \frac{\partial D}{\partial M} < 0, \frac{\partial D}{\partial d_1} = \frac{2c_1C_1D_1(c_1D_1-C_1d_1)}{(C_1d_1+c_1D_1)^3} + \frac{1}{S}\sum_{i=2}^{m}\frac{c_i}{d_1m_i}\left(\frac{m_id_ic_1}{d_1c_i}\right)^{\frac{1}{1-m_i}}, \frac{\partial D}{\partial D_1} = \frac{2c_1C_1d_1(C_1d_1-c_1D_1)}{(C_1d_1+c_1D_1)^3} + \frac{1}{S}\sum_{j=2}^{M}\frac{C_j}{D_1M_j}\left(\frac{M_jD_jC_1}{D_1C_j}\right)^{\frac{1}{1-M_j}}, \frac{\partial D}{\partial d_i} = -\frac{c_1}{d_1S}\left(\frac{m_id_ic_1}{d_1c_i}\right)^{\frac{m_i}{1-m_i}} < 0, \frac{\partial D}{\partial D_j} = -\frac{C_1}{D_1S}\left(\frac{M_jD_jC_1}{D_1C_j}\right)^{\frac{M_j}{1-M_j}} < 0, \frac{\partial D}{\partial c_1} = \frac{2C_1d_1D_1(C_1d_1-c_1D_1)}{(C_1d_1+c_1D_1)^3} - \frac{d_i}{d_1S}\left(\frac{m_id_ic_1}{d_1c_i}\right)^{\frac{m_i}{1-m_i}}, \frac{\partial D}{\partial C_1} = \frac{2c_1d_1D_1(c_1D_1-C_1d_1)}{(C_1d_1+c_1D_1)^3} - \frac{D_j}{D_1S}\left(\frac{M_jD_jC_1}{D_1C_j}\right)^{\frac{M_j}{1-M_j}}, \frac{\partial D}{\partial c_i} = \frac{1}{S}\left(\frac{m_id_ic_1}{d_1c_i}\right)^{\frac{1}{1-m_i}} > 0, \frac{\partial D}{\partial C_j} = \frac{1}{S}\left(\frac{M_jD_jC_1}{D_1C_j}\right)^{\frac{1}{1-M_j}} > 0, \frac{\partial D}{\partial m_i} = -\frac{c_i\ln\left(\frac{m_id_ic_1}{d_1c_i}\right)}{m_i(1-m_i)S}\left(\frac{m_id_ic_1}{d_1c_i}\right)^{\frac{1}{1-m_i}}, \frac{\partial D}{\partial M_j} = -\frac{C_j\ln\left(\frac{M_jD_jC_1}{D_1C_j}\right)}{M_j(1-M_j)}\left(\frac{M_jD_jC_1}{D_1C_j}\right)^{\frac{1}{1-M_j}}, \frac{\partial D}{\partial S} = \frac{1}{S^2}\left(\sum_{i=2}^{m}\left(\frac{1}{m_i}-1\right)c_i\left(\frac{m_id_ic_1}{d_1c_i}\right)^{\frac{1}{1-m_i}} + \sum_{j=2}^{M}\left(\frac{1}{M_j}-1\right)C_j\left(\frac{M_jD_jC_1}{D_1C_j}\right)^{\frac{1}{1-M_j}}\right) > 0. \tag{22}$$

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