

The Liquidity of the Oslo Stock Exchange – A Source Book 1980-2016

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Abstract

This paper describes measures of liquidity at the Oslo Stock Exchange. It collects in one place descriptives for the trading process at the OSE.

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Introduction

This paper describes measures of liquidity at the Oslo Stock Exchange. It collects in one place descriptives for the trading process at the OSE.

The focus of the current version of the paper is to communicate numbers, discussion is almost non-existent. This will hopefully improve, but even in its current state, the manuscript contains a lot of useful information for students, academics and practitioners dealing with data from the Oslo Stock Exchange. I have chosen to make the manuscript public in the hope that it will be useful for those users.

Many of the analyses are updates of those done in Næs, Skjeltorp, and Ødegaard (2008). The reader is referred to that paper for more comprehensive discussions. In addition to the analyses in that paper, this paper has analyses that utilizes trade by trade data from the trading process at the OSE.

A couple of words of warning, though. If you have little prior knowledge of market micro-structure, this is not the paper for you. Also, the analysis is in places preliminary, and will be improved. The paper is continuously being updated, if you are missing something, check back to my homepage for a new version.

1 Data

Data for this paper is from three sources. First, I use data from the OSE data service (OBI) which provides daily observations of stock prices, quantities traded, corporate events, etc. This data is available from 1980 onwards. This data is updated through 2016:06.

Second, I use data from the trade process at the OSE. This is a microstructure dataset provided by the “Market Watch” department at the Oslo Stock Exchange. This dataset provides information about all trades and order at the exchange in the period 1999–2012, allowing one to construct the order book at any point of time. The data comes with separate feeds that contains orders, trades, and the state of the order book (the best five levels) at any time with a change. The data contains a wealth of flags of different types. For example, for each trade there are flags for whether the participating traders are automated (algorithms) or not. The data feed also allow us to distinguish different traders.

Third, I use trade by trade data on Norwegian shares on European Exchanges sourced from Reuters. For all exchanges I have similar, albeit less detailed data, as I have for the process at the OSE, i.e. prices, times and quantities of trades, and state of the order book (the best nine levels) at each time with a change in the state of the order book (order, order withdrawal, etc). This data is from 2006–2012.

Some Norwegian stock also trade outside of Europe, but this trading is in other timesones, which I view as a justification for not including it here, but it does mean that some of the totals are incomplete.

The analysis uses data for all but the least liquid stocks. Stocks traded less than 100 days of the year are removed.

2 Volume measures

2.1 Annual estimates of Turnover

Table 1 Descriptive, Turnover

Period	min	Q1	med	mean	(std)	Q3	max	n	Size quartiles(means)			
									1(small)	2	3	4
1980-2016	0.00	0.20	0.46	0.80	(1.09)	0.98	21.19	850	0.96	0.79	0.77	0.75
1980	0.02	0.03	0.04	0.04	(0.02)	0.05	0.10	14				0.04
1981	0.02	0.03	0.04	0.06	(0.04)	0.07	0.18	22				0.06
1982	0.02	0.03	0.05	0.05	(0.02)	0.06	0.08	24			0.06	0.04
1983	0.01	0.11	0.17	0.20	(0.13)	0.25	0.64	54		0.40	0.23	0.15
1984	0.02	0.14	0.27	0.33	(0.26)	0.42	1.44	86	0.53	0.41	0.37	0.23
1985	0.02	0.12	0.23	0.39	(0.61)	0.45	5.50	114	0.55	0.79	0.28	0.29
1986	0.01	0.09	0.15	0.24	(0.27)	0.29	1.84	113	0.47	0.46	0.18	0.18
1987	0.03	0.11	0.22	0.32	(0.32)	0.39	2.51	106	0.58	0.32	0.26	0.30
1988	0.07	0.22	0.34	0.43	(0.33)	0.49	1.82	74	0.45	0.53	0.31	0.42
1989	0.09	0.33	0.55	0.60	(0.36)	0.81	1.88	100	0.69	0.52	0.64	0.59
1990	0.04	0.30	0.52	0.59	(0.42)	0.77	2.36	102	1.03	0.41	0.57	0.57
1991	0.06	0.29	0.50	0.58	(0.41)	0.76	1.81	95		0.39	0.54	0.63
1992	0.04	0.24	0.51	0.62	(0.48)	0.77	2.31	83		0.43	0.47	0.79
1993	0.11	0.57	0.91	1.15	(0.78)	1.53	3.94	100	1.66	1.39	1.05	0.84
1994	0.00	0.38	0.64	0.78	(0.61)	1.03	4.27	111	1.15	0.88	0.63	0.66
1995	0.04	0.47	0.74	0.94	(0.79)	1.06	4.24	122	1.19	1.38	1.19	0.65
1996	0.11	0.49	0.72	1.03	(0.80)	1.36	4.60	143	1.82	1.10	0.84	0.76
1997	0.05	0.42	0.77	1.02	(0.79)	1.46	5.27	168	1.31	1.25	0.91	0.84
1998	0.04	0.34	0.57	0.72	(0.51)	1.00	2.37	170	0.68	0.82	0.79	0.63
1999	0.04	0.33	0.56	0.79	(0.71)	1.02	4.81	172	0.75	0.65	0.94	0.84
2000	0.08	0.29	0.63	0.93	(0.90)	1.28	5.59	172	1.07	1.05	0.87	0.74
2001	0.03	0.23	0.52	0.76	(0.85)	0.94	5.76	163	0.45	0.79	0.69	0.87
2002	0.04	0.22	0.48	0.74	(0.78)	0.92	3.59	139	0.60	0.65	0.64	0.89
2003	0.01	0.28	0.67	1.08	(1.14)	1.38	5.50	134	1.34	0.97	0.90	1.13
2004	0.01	0.32	0.91	1.31	(1.64)	1.66	11.72	151	2.24	0.89	1.08	0.88
2005	0.01	0.38	0.89	1.52	(1.95)	1.93	13.45	184	3.55	1.69	1.45	1.23
2006	0.01	0.30	0.70	1.35	(2.06)	1.56	21.19	205	2.45	1.39	1.35	1.13
2007	0.01	0.22	0.61	1.09	(1.49)	1.29	15.06	223	1.33	0.87	0.95	1.28
2008	0.00	0.15	0.39	0.90	(1.24)	1.02	8.91	223	0.77	0.41	0.89	1.25
2009	0.00	0.19	0.43	0.85	(1.12)	1.05	7.41	198	0.55	0.71	0.67	1.36
2010	0.01	0.15	0.36	0.77	(1.04)	0.85	5.72	207	0.74	0.54	0.79	1.08
2011	0.00	0.12	0.28	0.58	(0.78)	0.70	4.13	210	0.48	0.39	0.48	0.77
2012	0.00	0.11	0.26	0.56	(0.74)	0.74	5.37	198	0.57	0.58	0.47	0.62
2013	0.01	0.15	0.35	0.76	(1.53)	0.81	16.39	197	0.80	0.54	1.15	0.49
2014	0.00	0.14	0.31	0.64	(1.02)	0.71	9.65	202	0.68	0.53	0.72	0.63
2015	0.01	0.12	0.33	0.61	(0.82)	0.77	5.72	205	0.47	0.63	0.59	0.69
2016	0.00	0.16	0.37	0.70	(0.96)	0.87	5.56	207	0.67	0.74	0.63	0.75

Figure 1 Histogram Turnover - Subperiod

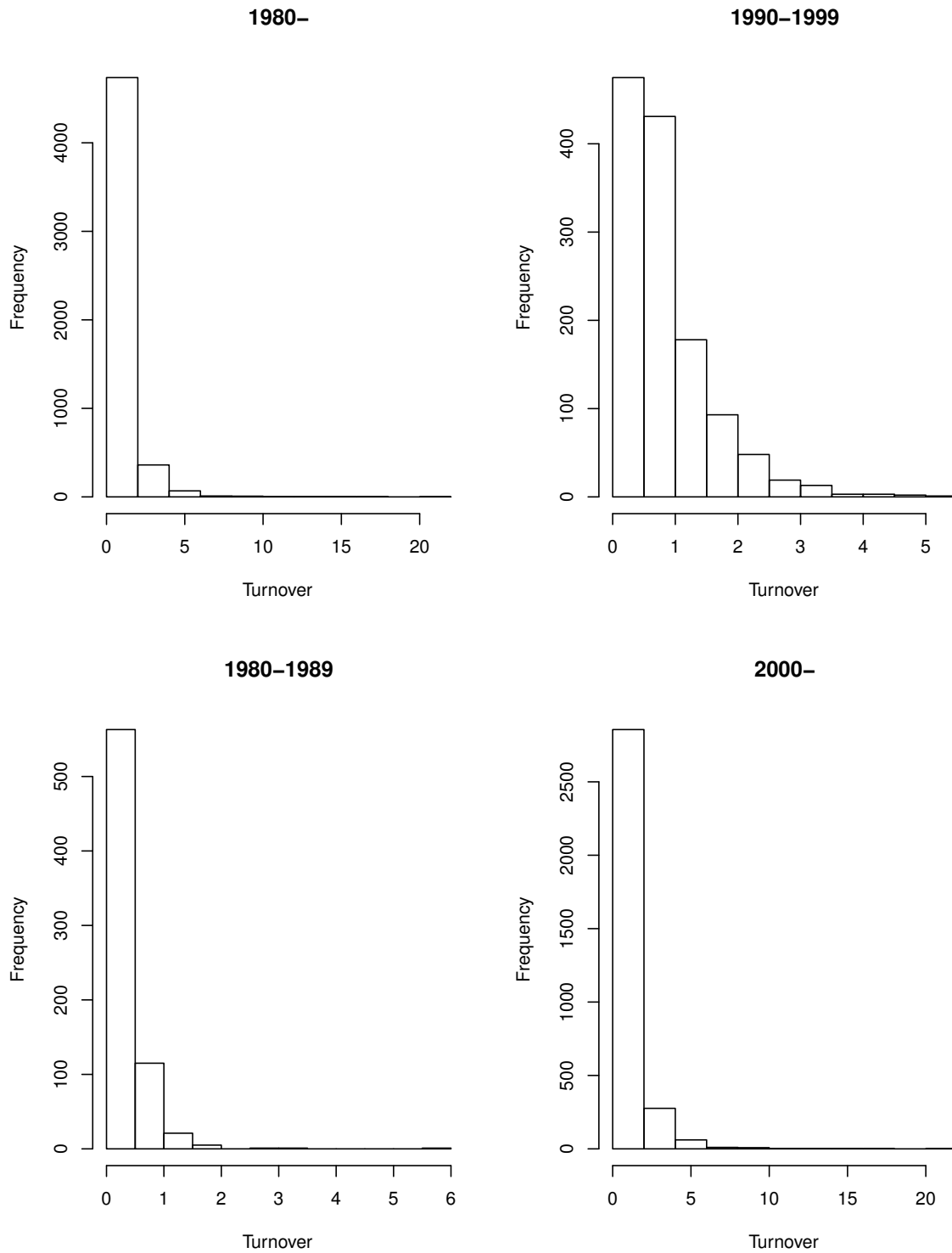


Figure 2 Histogram Turnover - By Size

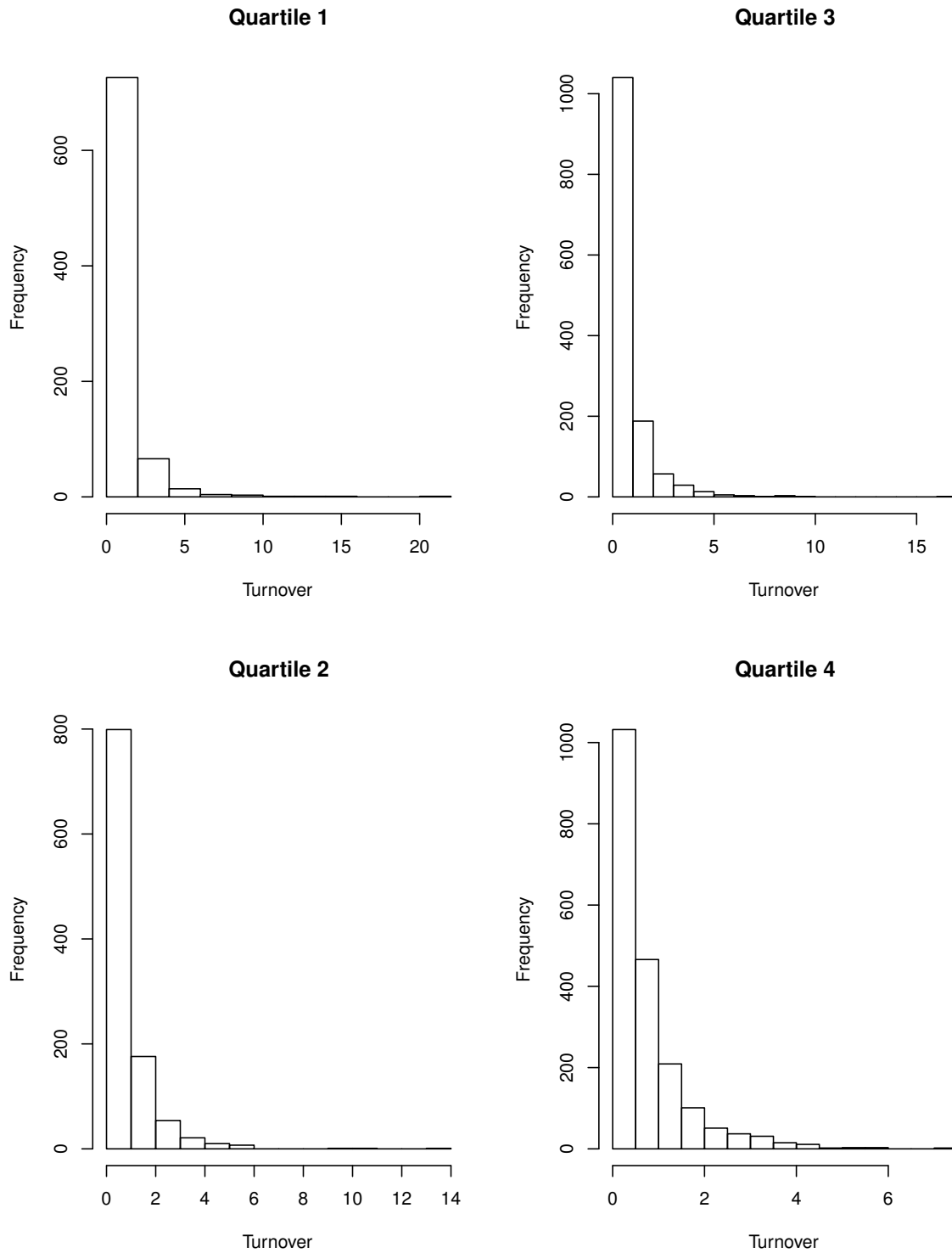
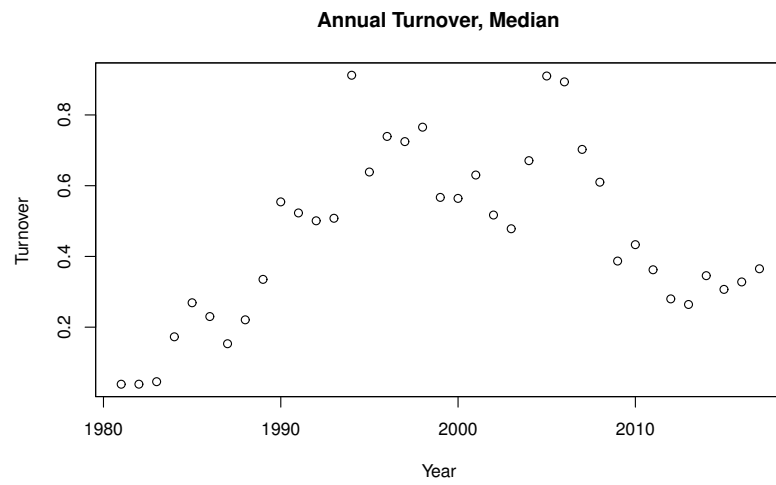
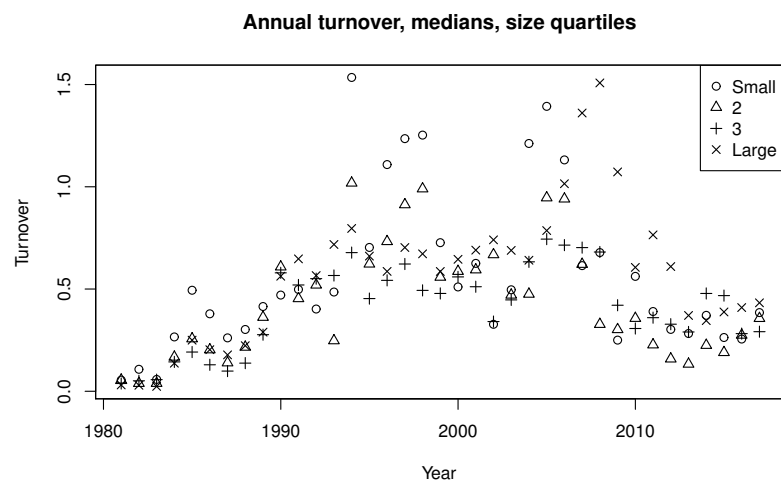


Figure 3 Time Series, Turnover, Monthly Median

Panel A: Crossectional Median



Panel B: Crossectional Median, Size sorted portfolios.

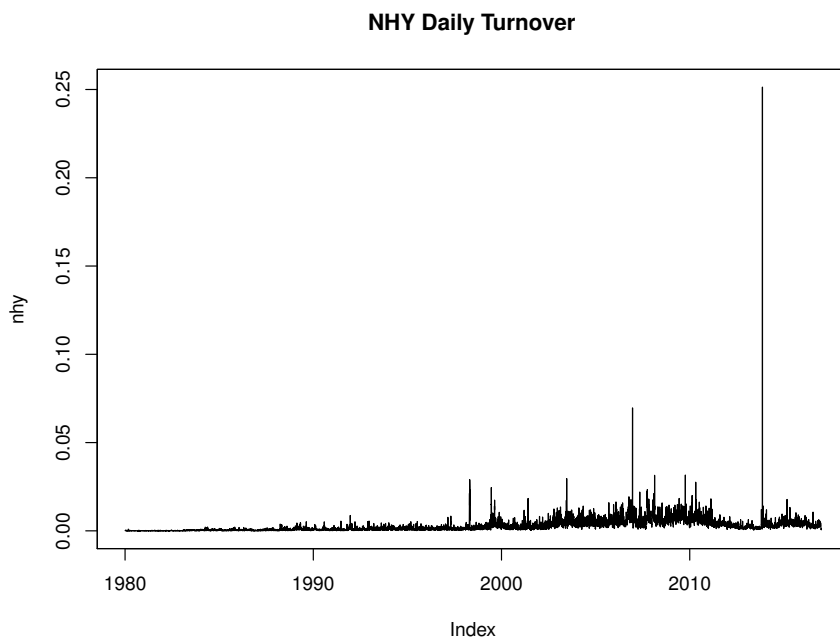


Time series plot of median volume across stocks on the Oslo Stock Exchange.

Figure 4 Example, Single companies daily turnover

This figure illustrates the evolution of daily turnover for two example stocks, Norsk Hydro and Statoil

Panel A: Norsk Hydro



Panel B: Statoil

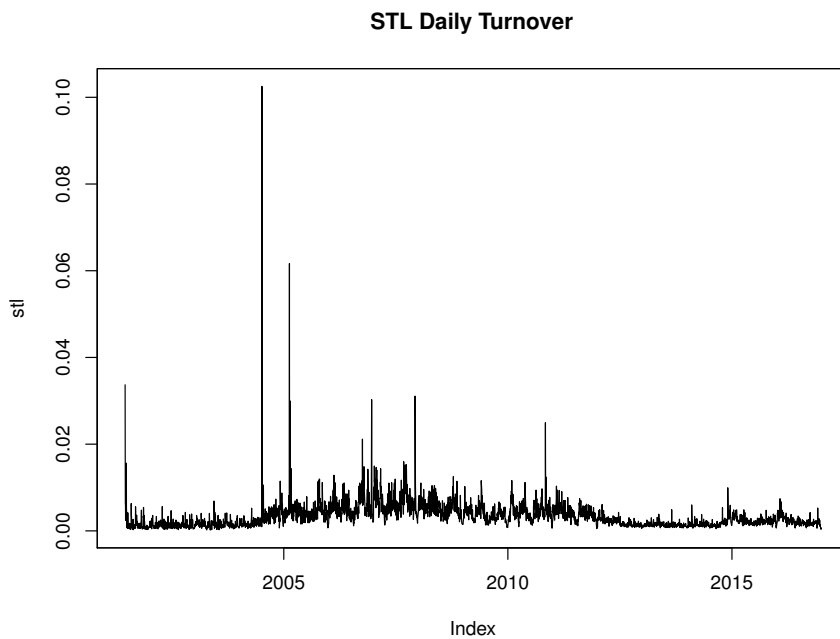
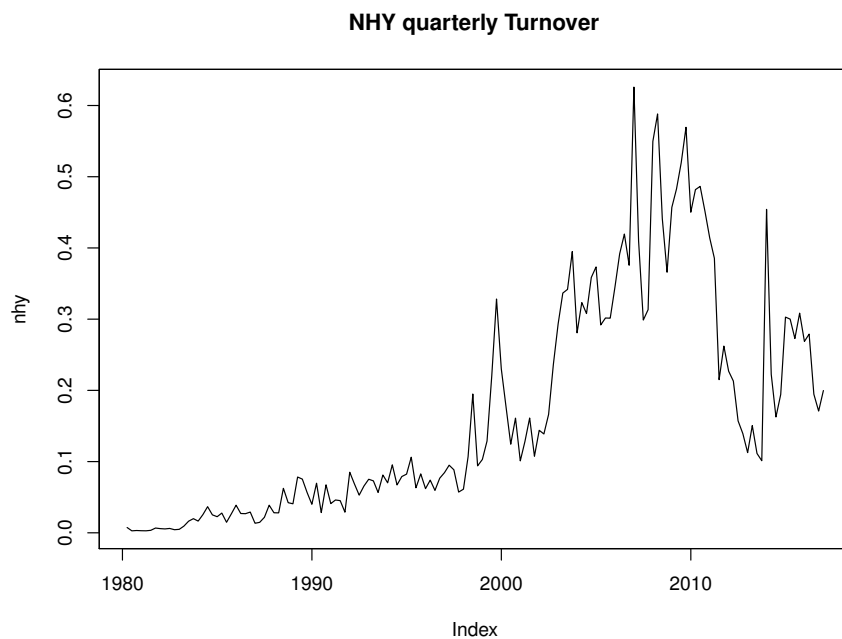


Figure 5 Example, Single companies quarterly turnover

This figure illustrates the evolution of quarterly turnover for two example stocks, Norsk Hydro and Statoil

Panel A: Norsk Hydro



Panel B: Statoil

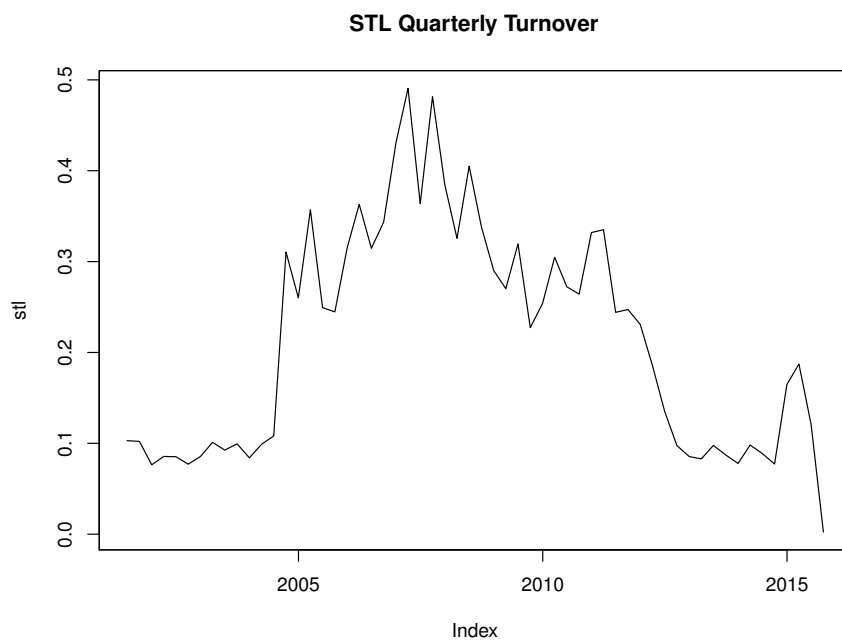
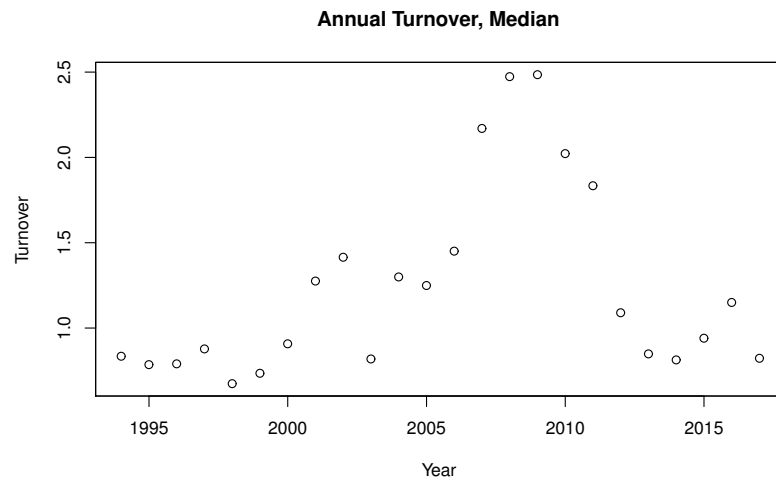
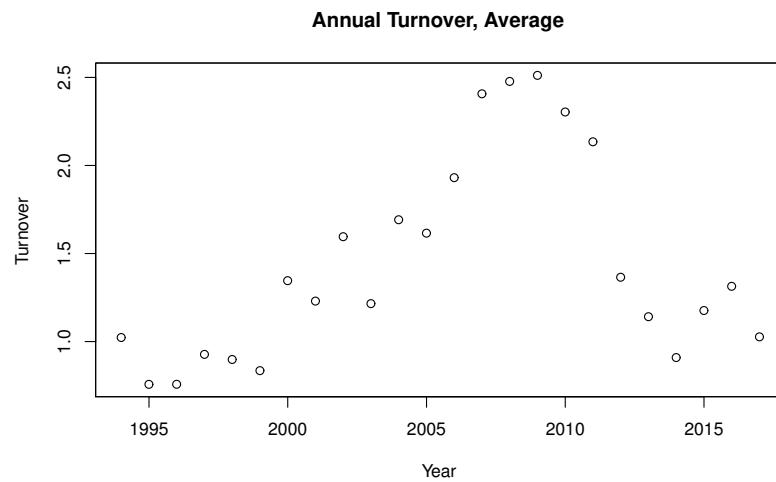


Figure 6 Time Series, Turnover, Monthly, OBX Constituents

Panel A: Crossectional Median



Panel B: Crossectional Mean (trimmed)



Time series plot of median volume across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

2.2 No Trading Days in Year

We count the number of days with trades for each stock, and describe these data. Recall that we have filtered out stocks with less than 100 trades in a year, which explains why the minimum is exactly 100.

Table 2 Descriptive, No Trading Days

Period									Size quartiles(means)			
	min	Q1	med	mean	(std)	Q3	max	n	1(small)	2	3	4
1980-2016	99	180	237	213	(46)	250	253	884	194	200	210	229
1980	107	128	176	171	(41)	192	243	14				171
1981	103	126	172	172	(47)	207	247	22				172
1982	99	118	175	171	(47)	211	251	24			117	182
1983	106	190	228	216	(37)	249	253	54		213	197	225
1984	105	144	192	190	(51)	242	250	86	172	175	182	205
1985	105	144	206	192	(50)	243	249	114	154	181	174	210
1986	101	144	196	189	(48)	233	249	113	165	185	168	215
1987	100	150	211	194	(48)	241	251	106	169	164	185	215
1988	114	182	226	211	(44)	251	253	74	180	191	191	231
1989	108	158	197	200	(46)	248	251	100	179	182	194	216
1990	109	155	224	202	(47)	244	249	102	187	154	191	221
1991	109	172	209	202	(44)	244	249	95		140	192	217
1992	108	150	205	198	(49)	246	252	83		157	182	225
1993	118	198	237	218	(39)	250	252	100	189	210	223	234
1994	111	176	235	215	(41)	250	253	111	192	213	200	228
1995	121	182	224	212	(39)	248	251	122	147	185	214	223
1996	110	199	238	219	(39)	249	250	143	217	203	224	227
1997	114	198	235	218	(39)	250	250	168	202	219	204	233
1998	105	182	231	213	(40)	248	250	170	184	196	218	226
1999	105	171	230	208	(47)	251	252	172	172	201	212	231
2000	101	172	240	213	(47)	250	251	172	200	209	215	227
2001	102	166	223	204	(49)	249	249	163	162	188	200	221
2002	100	172	230	208	(46)	248	249	139	173	188	208	231
2003	100	188	231	210	(47)	249	250	134	178	191	216	232
2004	102	215	247	224	(43)	253	253	151	229	200	232	230
2005	100	206	249	222	(47)	253	253	184	227	201	219	236
2006	100	197	242	216	(46)	251	251	205	197	208	207	234
2007	100	192	244	218	(45)	250	250	223	195	214	207	240
2008	102	182	239	214	(46)	252	252	223	197	186	213	241
2009	100	188	238	216	(44)	251	251	198	188	198	222	242
2010	100	192	245	220	(45)	252	252	207	211	216	228	237
2011	100	180	247	218	(48)	253	253	210	192	200	205	242
2012	101	182	247	216	(47)	251	251	198	189	197	215	241
2013	104	197	245	219	(44)	249	249	197	187	215	235	244
2014	101	194	246	220	(44)	250	250	202	185	208	233	245
2015	103	197	249	221	(44)	251	251	205	186	200	228	245
2016	107	218	252	228	(41)	253	253	207	209	226	225	249

Figure 7 Histogram No Trading Days - Subperiod

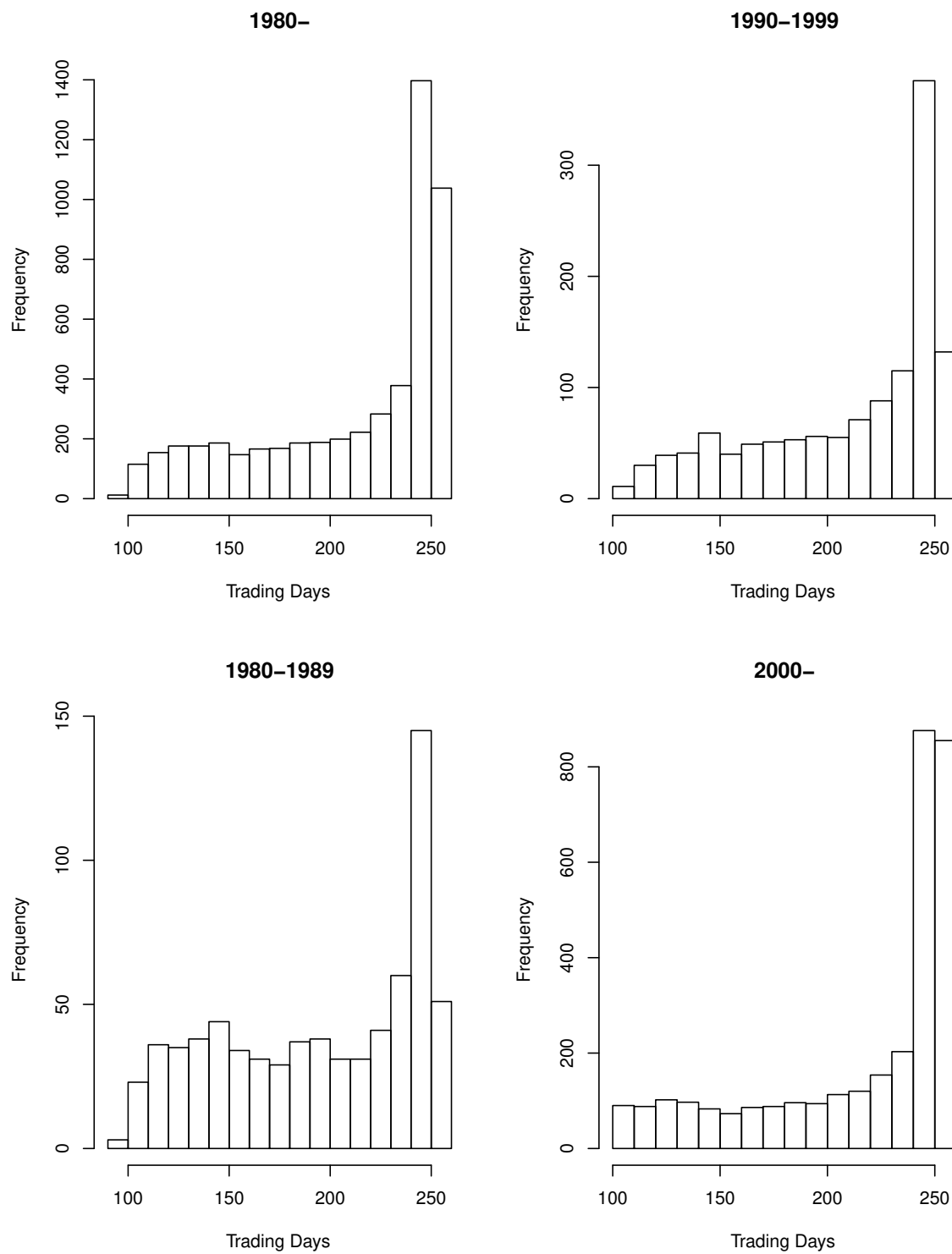


Figure 8 Histogram No Trading Days - By Size

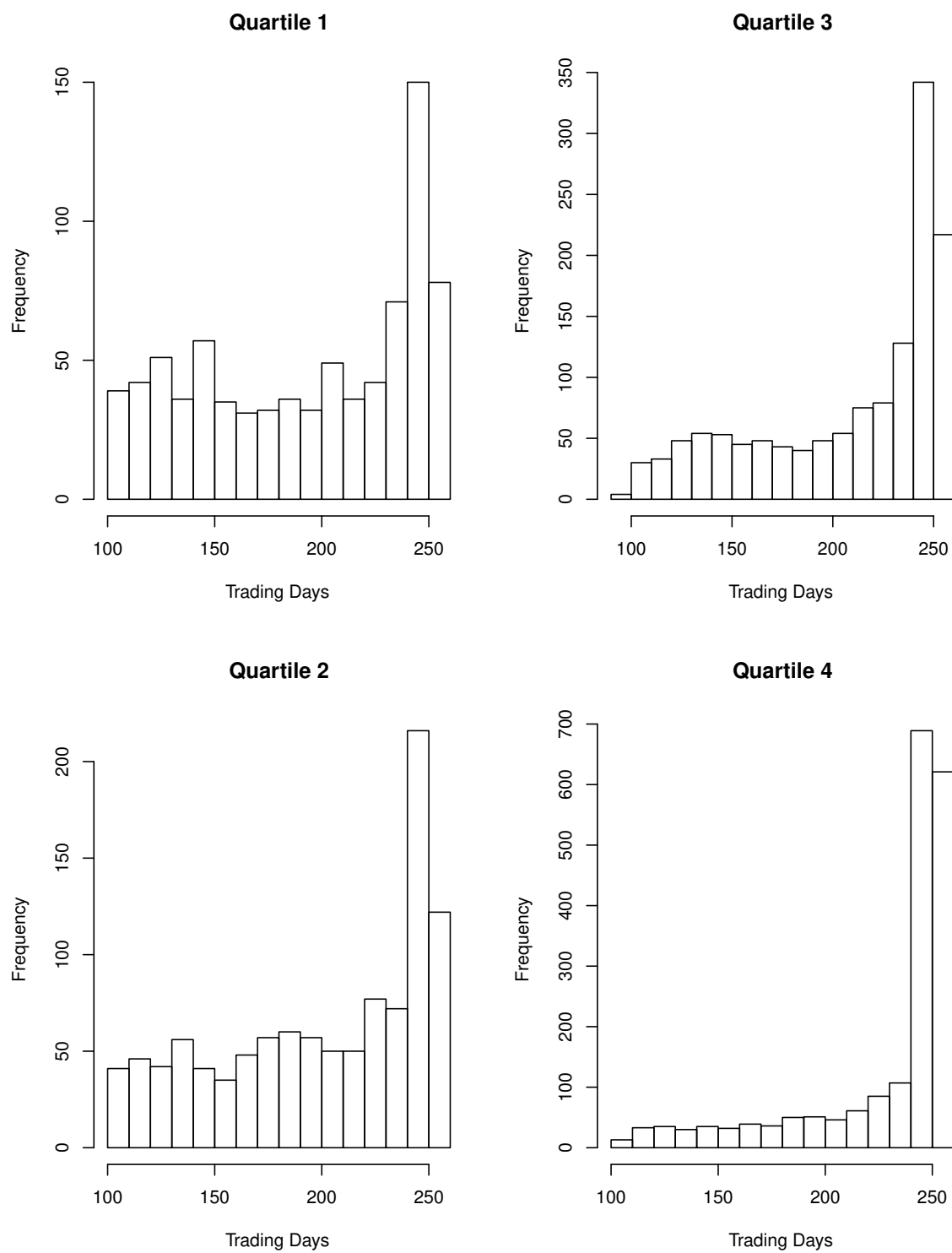
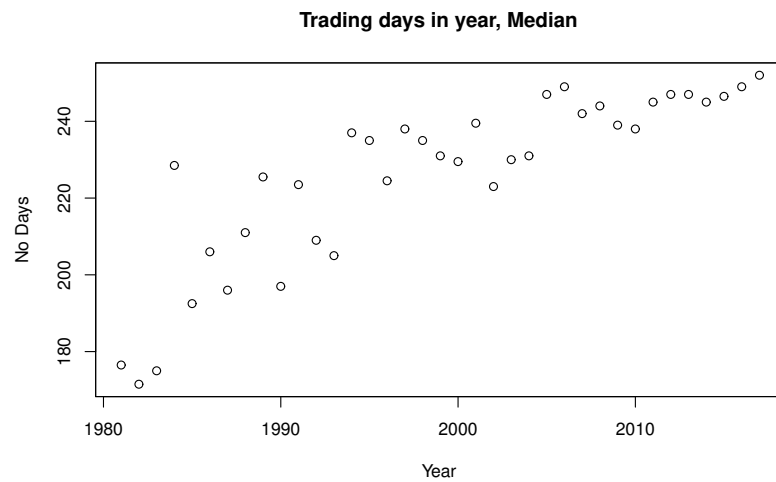
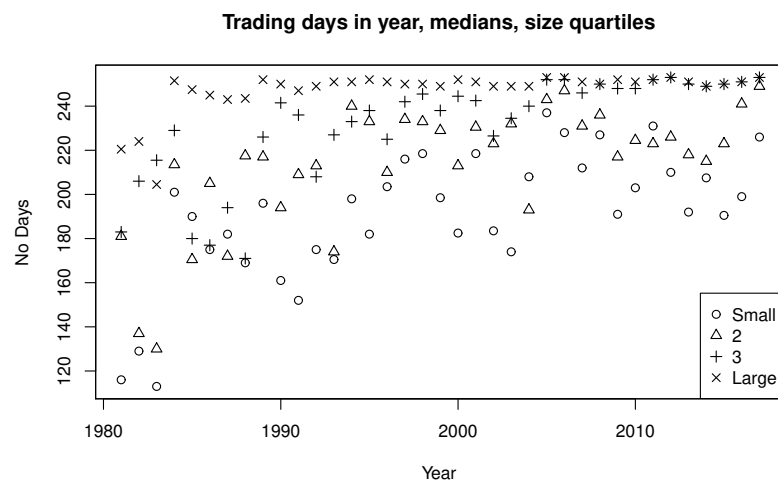


Figure 9 Time Series, No Trading Days, Monthly Median

Panel A: Crossectional Median



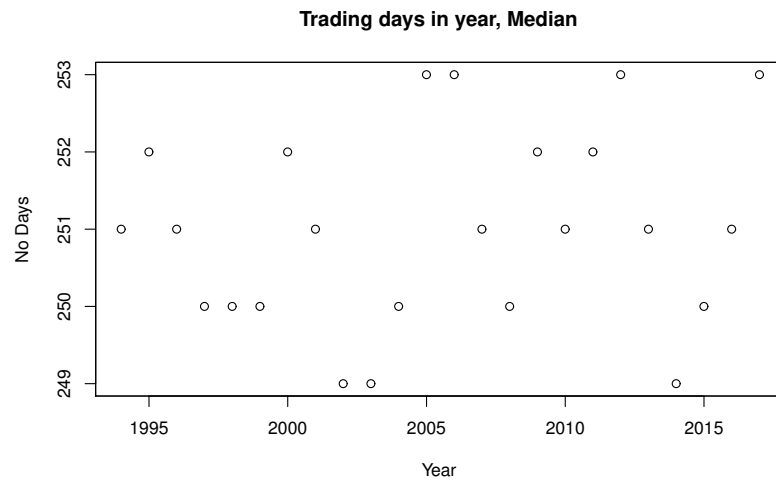
Panel B: Crossectional Median, Size sorted portfolios.



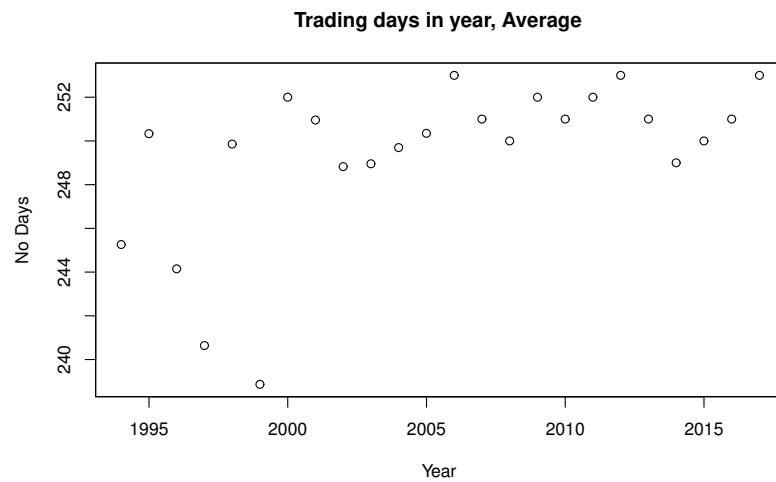
Time series plot of median volume across stocks on the Oslo Stock Exchange.

Figure 10 Time Series, No Trading Days, Monthly, OBX Constituents

Panel A: Crossectional Median



Panel B: Crossectional Mean (trimmed)



Time series plot of median volume across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

3 Variations on Spreads

A *spread* is a measure of the cost of trading an asset. It is the difference between the price desired by a seller and the price a buyer is willing to pay. For trade to happen one of the parties have to “cross the spread.” The spread thus measures how much one party is willing to lose in order to finalize the trade.

There are many different variations on spread.

3.1 Closing Spread

This uses the last registered buy and sell quotes in the day, as reported daily. The Spread is the difference between the buy and sell quote.

$$Spread = p_{sell} - p_{buy}$$

Table 3 Descriptive, Bid Ask Spread

Period									Size quartiles(means)			
	min	Q1	med	mean	(std)	Q3	max	n	1(small)	2	3	4
1980-2016	0.0	0.2	0.8	1.6	(3.0)	1.9	64.9	882	1.1	1.6	1.6	1.9
1980	0.8	1.1	2.3	2.4	(1.3)	3.2	5.0	14				2.4
1981	1.0	1.8	4.0	7.1	(10.4)	7.6	52.2	22				7.1
1982	0.9	1.9	3.6	5.2	(4.3)	6.5	16.7	23			9.3	4.3
1983	0.8	2.0	3.6	6.4	(9.4)	7.4	63.1	54		5.8	6.4	6.6
1984	0.4	1.6	3.3	5.5	(6.7)	6.0	47.8	85	9.0	9.1	4.3	3.6
1985	0.3	1.2	2.0	3.5	(4.1)	4.1	26.8	114	1.7	3.4	4.4	3.0
1986	0.2	1.0	1.7	2.9	(2.9)	3.9	14.7	113	2.0	2.6	4.2	1.9
1987	0.1	0.9	1.7	2.9	(3.0)	4.0	17.0	106	2.9	3.5	3.4	2.5
1988	0.1	1.3	2.1	2.8	(3.0)	3.5	18.6	74	3.7	2.4	4.0	2.4
1989	0.1	1.4	2.8	4.1	(5.5)	4.8	38.8	100	2.2	6.1	4.3	3.6
1990	0.1	1.7	3.1	4.4	(6.7)	5.3	64.9	102	4.0	8.0	3.3	3.8
1991	0.1	1.1	2.5	2.9	(2.3)	4.1	16.0	95		3.4	2.6	2.9
1992	0.1	0.8	1.7	2.0	(1.7)	2.8	11.2	83		2.0	2.3	1.9
1993	0.1	0.8	1.6	1.9	(1.6)	2.4	10.5	100	2.1	1.9	2.1	1.5
1994	0.1	0.8	1.5	1.8	(1.4)	2.3	7.0	111	1.7	1.6	2.4	1.7
1995	0.0	0.6	1.2	1.4	(1.1)	1.8	5.7	122	1.0	1.2	1.3	1.5
1996	0.0	0.6	1.2	1.4	(1.5)	1.8	14.2	143	1.2	1.5	1.5	1.5
1997	0.0	0.7	1.5	1.8	(1.6)	2.3	11.2	168	1.4	1.5	2.1	1.9
1998	0.0	0.7	1.3	1.8	(1.8)	2.4	12.9	170	1.0	1.8	1.6	2.2
1999	0.0	0.5	1.2	1.6	(1.6)	2.1	9.5	172	1.1	1.6	1.5	1.9
2000	0.0	0.6	1.3	1.9	(2.0)	2.4	11.1	172	1.1	1.9	2.7	2.0
2001	0.0	0.4	0.9	1.6	(2.2)	2.0	16.6	163	0.4	1.3	1.4	2.1
2002	0.0	0.2	0.6	1.2	(1.4)	1.4	7.2	139	0.6	1.1	1.2	1.4
2003	0.0	0.1	0.5	1.0	(1.4)	1.1	6.2	134	0.7	1.2	1.1	1.0
2004	0.0	0.1	0.5	1.0	(1.5)	1.3	7.5	151	0.7	1.3	1.0	1.3
2005	0.0	0.2	0.4	1.3	(3.3)	1.3	38.2	184	0.5	0.9	0.9	1.9
2006	0.0	0.2	0.6	1.4	(3.5)	1.3	31.9	205	0.3	1.3	1.3	1.9
2007	0.0	0.3	0.6	1.2	(2.0)	1.2	20.7	222	0.7	1.1	1.2	1.3
2008	0.0	0.2	0.5	1.2	(2.3)	1.2	25.7	222	0.8	1.5	1.1	1.2
2009	0.0	0.1	0.2	0.8	(1.8)	0.7	21.7	197	0.6	0.7	0.7	1.1
2010	0.0	0.1	0.3	0.8	(1.7)	0.8	15.8	206	0.8	0.8	0.6	1.0
2011	0.0	0.1	0.2	0.8	(1.7)	0.8	18.6	209	1.1	0.7	0.7	0.8
2012	0.0	0.1	0.2	0.6	(1.2)	0.6	11.9	197	0.6	0.6	0.5	0.7
2013	0.0	0.1	0.2	0.5	(1.0)	0.6	11.0	196	0.5	0.4	0.4	0.7
2014	0.0	0.1	0.2	0.6	(1.0)	0.7	8.9	201	0.8	0.8	0.4	0.5
2015	0.0	0.1	0.3	0.7	(1.0)	0.7	6.5	204	0.8	0.8	0.7	0.5
2016	0.0	0.1	0.2	0.5	(0.8)	0.7	4.9	206	0.6	0.6	0.6	0.4

Figure 11 Histogram Bid Ask Spread - Subperiod

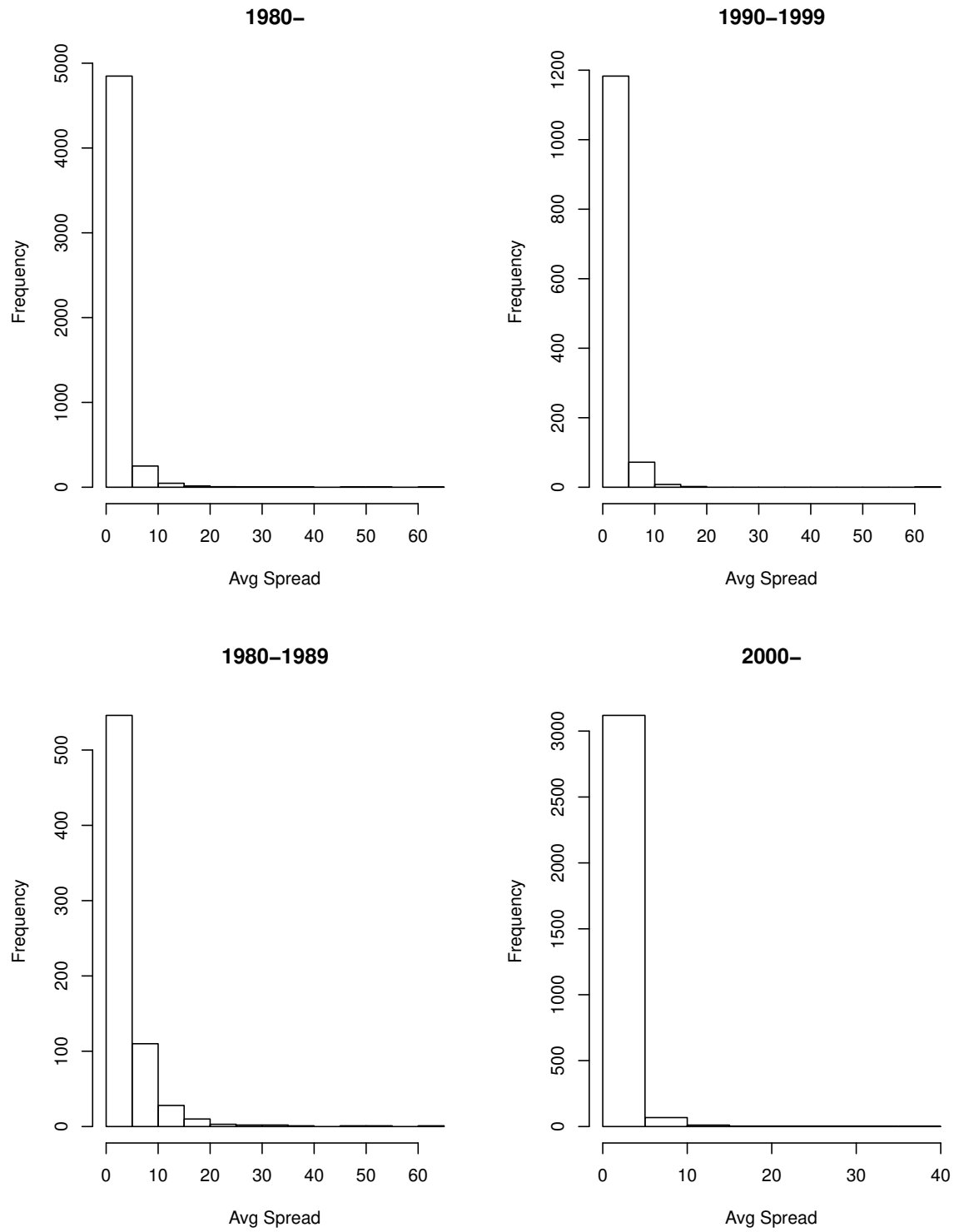


Figure 12 Histogram Bid Ask Spread - By Size

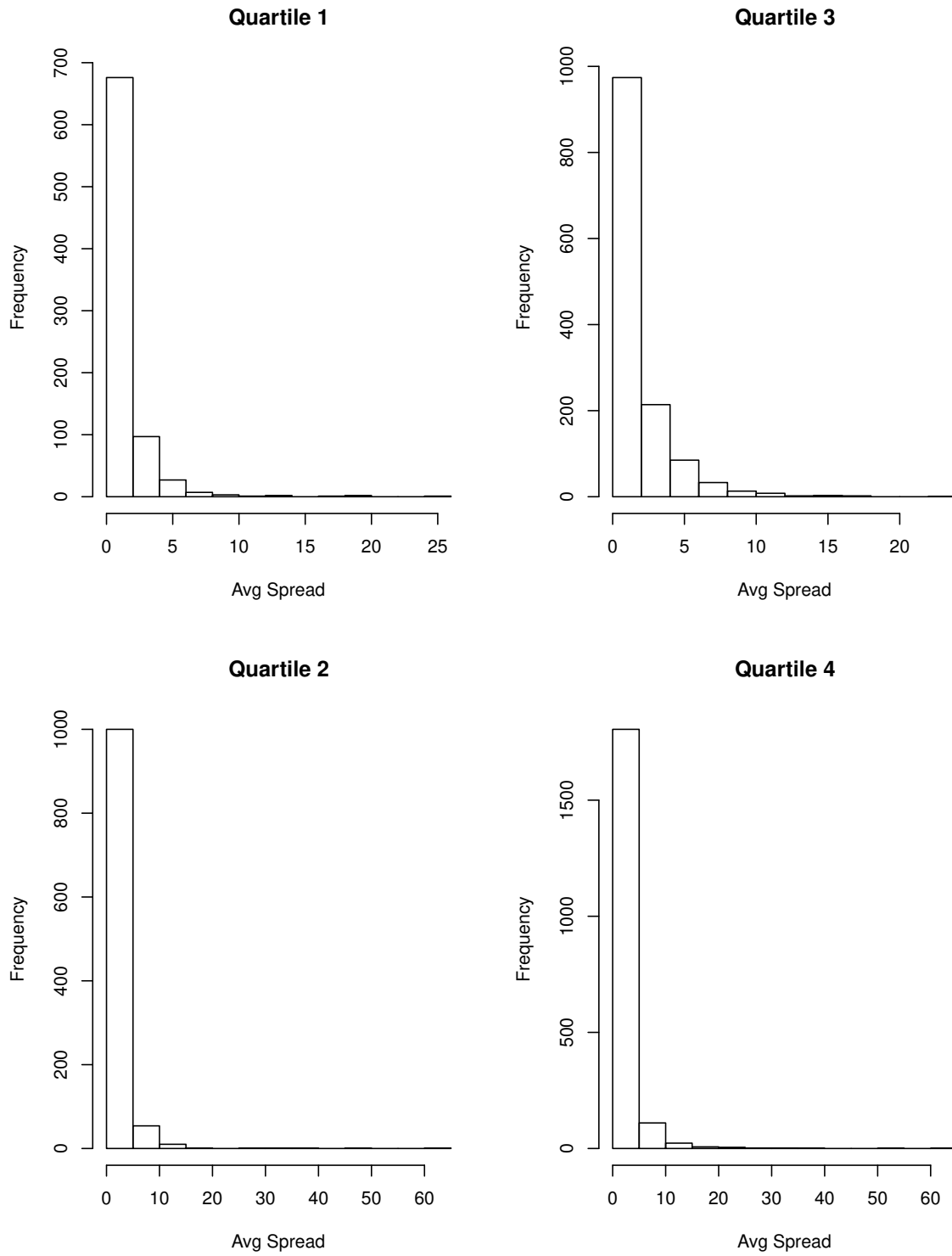
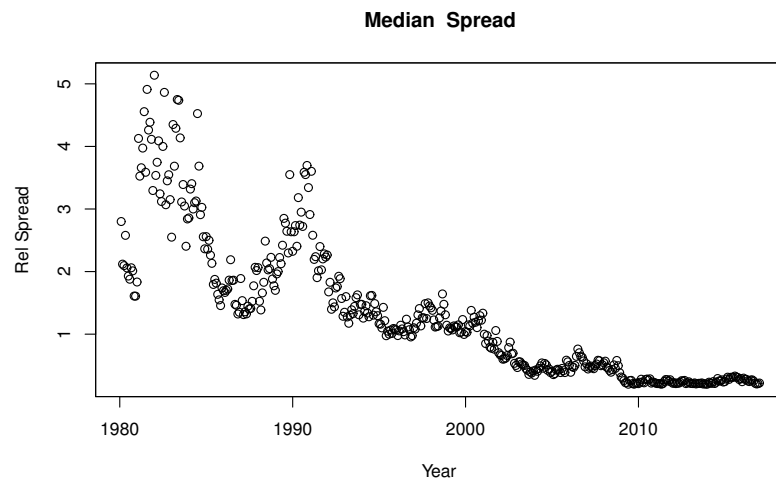
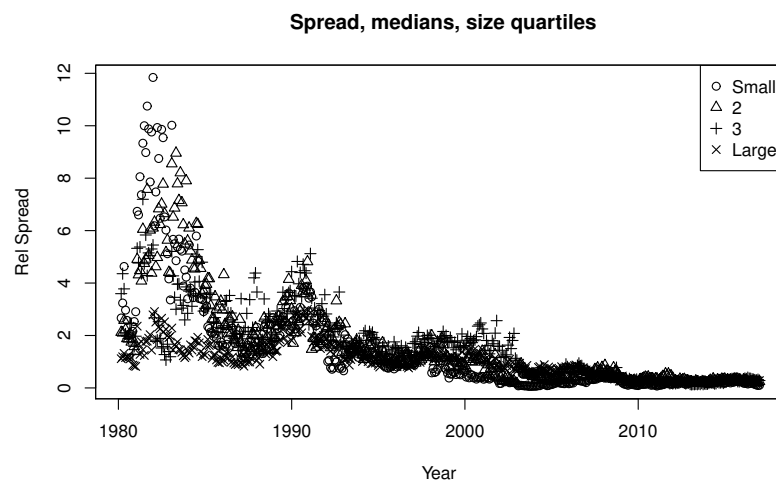


Figure 13 Time Series, Bid Ask Spread, Monthly Median

Panel A: Crossectional Median



Panel B: Crossectional Median, Size sorted portfolios.



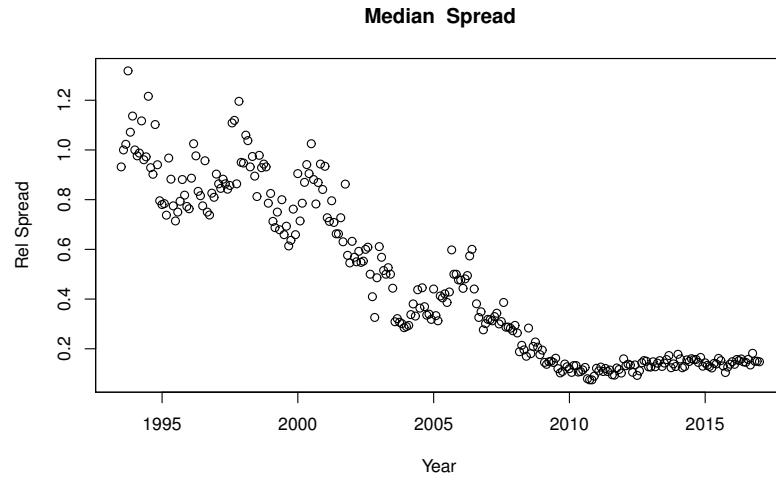
Time series plot of median spread across stocks on the Oslo Stock Exchange.

3.1.1 OBX constituents and others

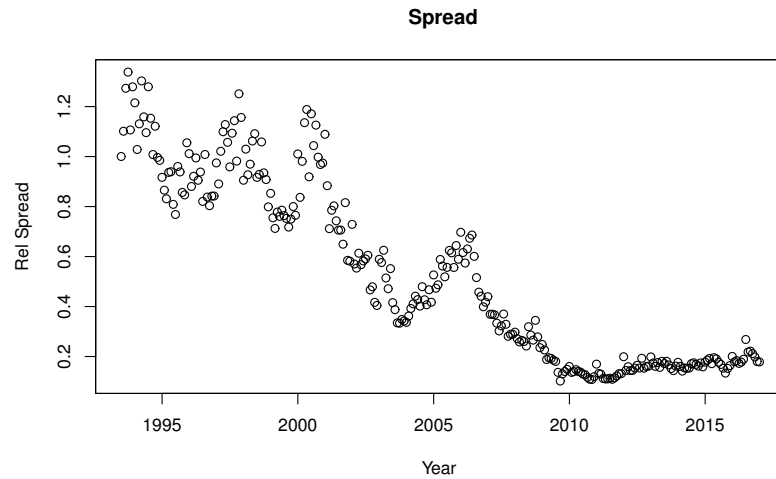
We now differentiate between firms in the OBX index, and others (still maintaining restriction that there is a minimum of 100 trading days)

Figure 14 Time Series, Bid Ask Spread, Monthly Median. OBX Constituents

Panel A: Crossectional Median

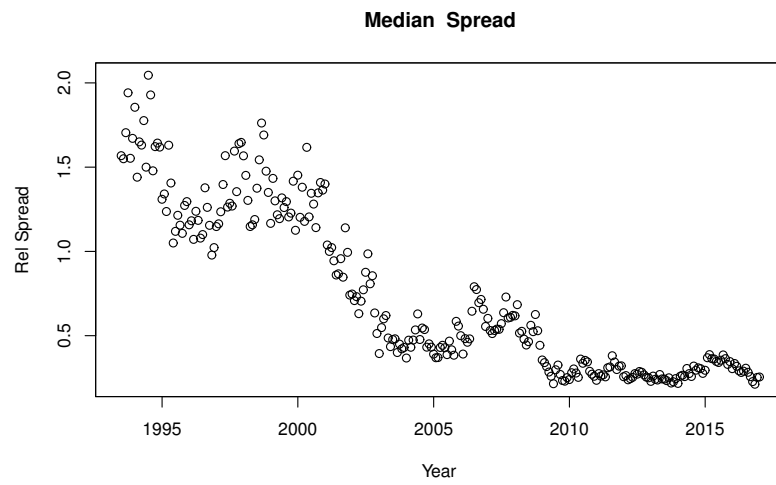


Panel A: Crossectional Average

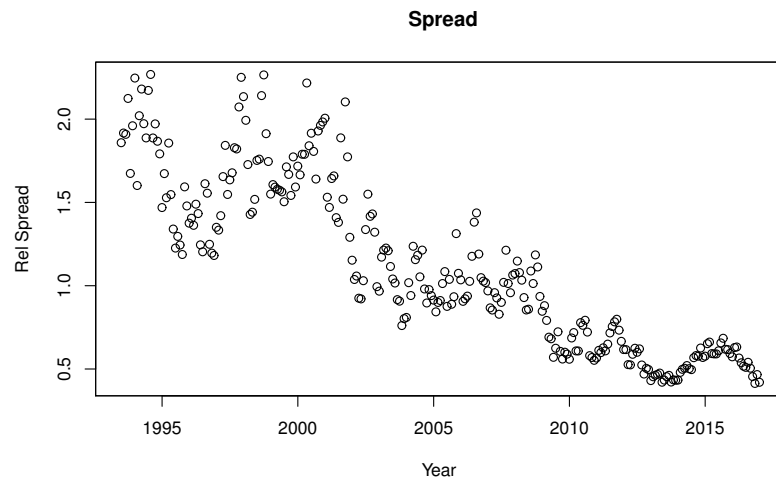


Time series plot of median spread across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

Figure 15 Time Series, Bid Ask Spread, Monthly Median. Non-OBX Constituents
Panel A: Crossectional Median



Panel A: Crossectional Average



Time series plot of median spread across stocks on the Oslo Stock Exchange.

3.2 Closing Relative Spread

The relative spread calculates the spread relative to the current price.

$$RelSpread = \frac{p_{sell} - p_{buy}}{\hat{P}}$$

One typically uses the average of the buy and sell quotes as an estimate of the price.

$$\hat{P} = \frac{p_{sell} + p_{buy}}{2}$$

Table 4 Descriptive, Relative Bid Ask Spread

Period	min	Q1	med	mean	(std)	Q3	max	n	Size quartiles(means)			
									1(small)	2	3	4
1980-2016	0.001	0.011	0.021	0.028	(0.024)	0.037	0.159	882	0.048	0.037	0.027	0.015
1980	0.006	0.007	0.021	0.021	(0.012)	0.025	0.052	14				0.021
1981	0.007	0.016	0.028	0.027	(0.012)	0.034	0.057	22				0.027
1982	0.007	0.016	0.024	0.028	(0.015)	0.035	0.071	23			0.046	0.024
1983	0.004	0.014	0.025	0.030	(0.020)	0.037	0.094	54		0.046	0.036	0.022
1984	0.004	0.012	0.024	0.025	(0.015)	0.032	0.084	85	0.044	0.035	0.023	0.017
1985	0.005	0.011	0.018	0.020	(0.014)	0.026	0.093	114	0.040	0.029	0.026	0.012
1986	0.004	0.012	0.021	0.022	(0.014)	0.030	0.066	113	0.043	0.030	0.025	0.014
1987	0.003	0.012	0.023	0.025	(0.018)	0.032	0.109	106	0.038	0.047	0.024	0.015
1988	0.004	0.015	0.029	0.036	(0.025)	0.054	0.100	74	0.062	0.051	0.040	0.024
1989	0.004	0.014	0.027	0.033	(0.022)	0.050	0.102	100	0.057	0.048	0.031	0.021
1990	0.004	0.018	0.028	0.033	(0.018)	0.043	0.090	102	0.046	0.052	0.032	0.025
1991	0.004	0.018	0.031	0.037	(0.023)	0.050	0.115	95		0.059	0.044	0.029
1992	0.004	0.019	0.038	0.046	(0.033)	0.061	0.126	83		0.076	0.060	0.026
1993	0.004	0.014	0.025	0.029	(0.019)	0.036	0.084	100	0.047	0.040	0.029	0.014
1994	0.003	0.013	0.021	0.025	(0.017)	0.033	0.086	111	0.039	0.034	0.027	0.017
1995	0.002	0.011	0.018	0.023	(0.016)	0.030	0.087	122	0.053	0.035	0.022	0.017
1996	0.002	0.010	0.015	0.019	(0.014)	0.024	0.080	143	0.027	0.025	0.021	0.010
1997	0.002	0.011	0.018	0.021	(0.013)	0.026	0.076	168	0.033	0.024	0.023	0.012
1998	0.003	0.015	0.027	0.032	(0.021)	0.044	0.102	170	0.059	0.041	0.034	0.019
1999	0.003	0.014	0.028	0.037	(0.028)	0.054	0.117	172	0.070	0.042	0.030	0.019
2000	0.003	0.014	0.024	0.033	(0.025)	0.047	0.104	172	0.050	0.033	0.029	0.018
2001	0.003	0.014	0.028	0.040	(0.031)	0.059	0.130	163	0.085	0.056	0.040	0.023
2002	0.003	0.015	0.029	0.042	(0.034)	0.062	0.141	139	0.073	0.062	0.042	0.020
2003	0.002	0.014	0.024	0.034	(0.029)	0.045	0.145	134	0.059	0.050	0.031	0.014
2004	0.002	0.008	0.016	0.020	(0.015)	0.025	0.069	151	0.025	0.028	0.016	0.010
2005	0.002	0.008	0.014	0.017	(0.015)	0.022	0.090	184	0.028	0.024	0.019	0.011
2006	0.002	0.008	0.016	0.019	(0.015)	0.025	0.090	205	0.034	0.023	0.021	0.012
2007	0.002	0.008	0.015	0.019	(0.016)	0.025	0.085	222	0.033	0.022	0.021	0.009
2008	0.001	0.012	0.029	0.034	(0.028)	0.050	0.147	222	0.054	0.049	0.035	0.015
2009	0.001	0.015	0.027	0.036	(0.030)	0.054	0.157	197	0.065	0.049	0.032	0.013
2010	0.001	0.008	0.022	0.028	(0.027)	0.038	0.149	206	0.040	0.032	0.016	0.007
2011	0.001	0.009	0.023	0.030	(0.026)	0.043	0.119	209	0.055	0.043	0.036	0.011
2012	0.001	0.009	0.019	0.029	(0.028)	0.040	0.141	197	0.067	0.042	0.025	0.009
2013	0.001	0.007	0.015	0.024	(0.026)	0.032	0.159	196	0.048	0.026	0.013	0.005
2014	0.001	0.007	0.014	0.023	(0.024)	0.030	0.121	201	0.044	0.031	0.017	0.006
2015	0.001	0.007	0.017	0.025	(0.025)	0.032	0.127	204	0.051	0.036	0.021	0.008
2016	0.001	0.006	0.015	0.022	(0.023)	0.028	0.118	206	0.040	0.027	0.021	0.005

Figure 16 Histogram Relative Bid Ask Spread - Subperiods

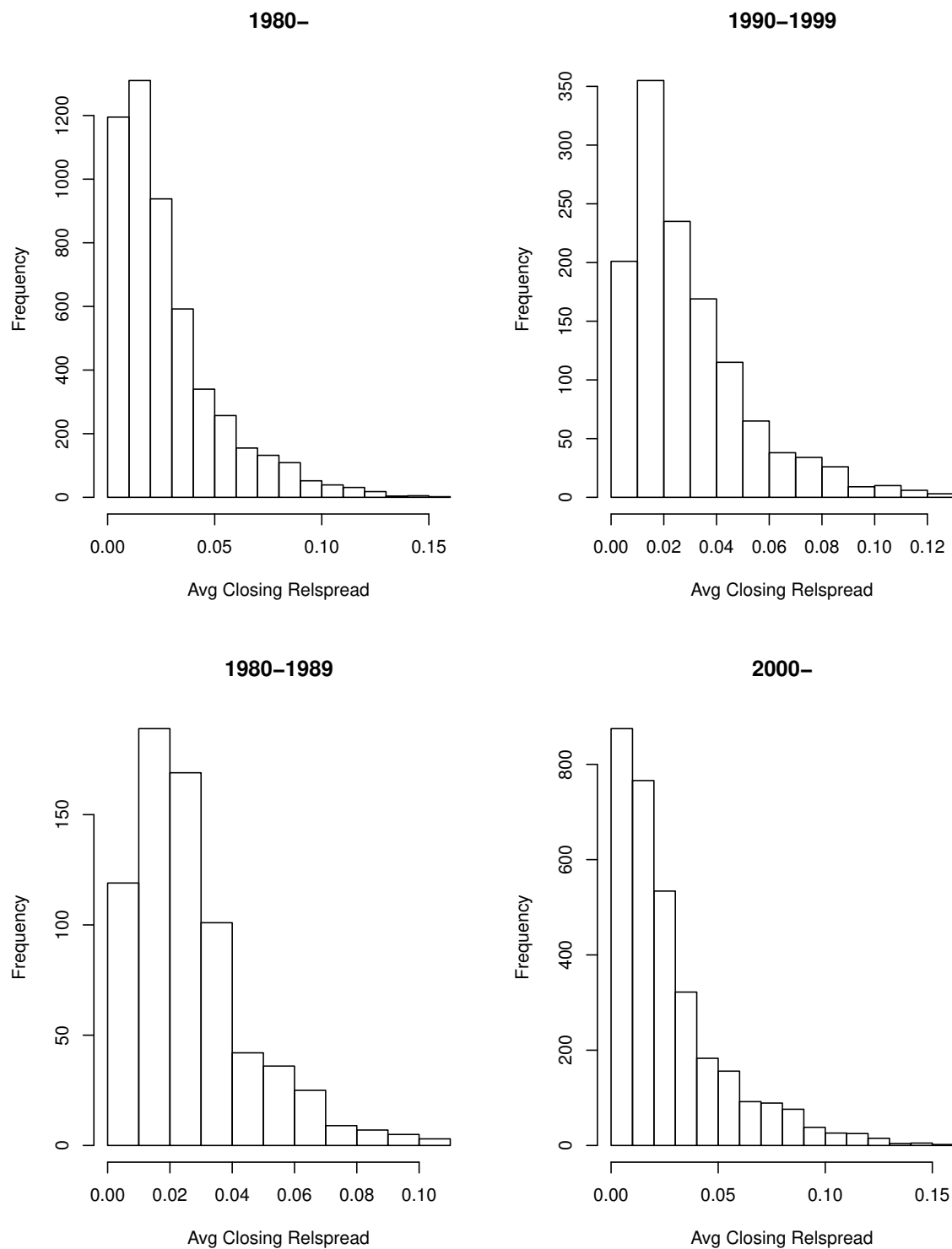


Figure 17 Histograms Relative Bid Ask Spread - By Size

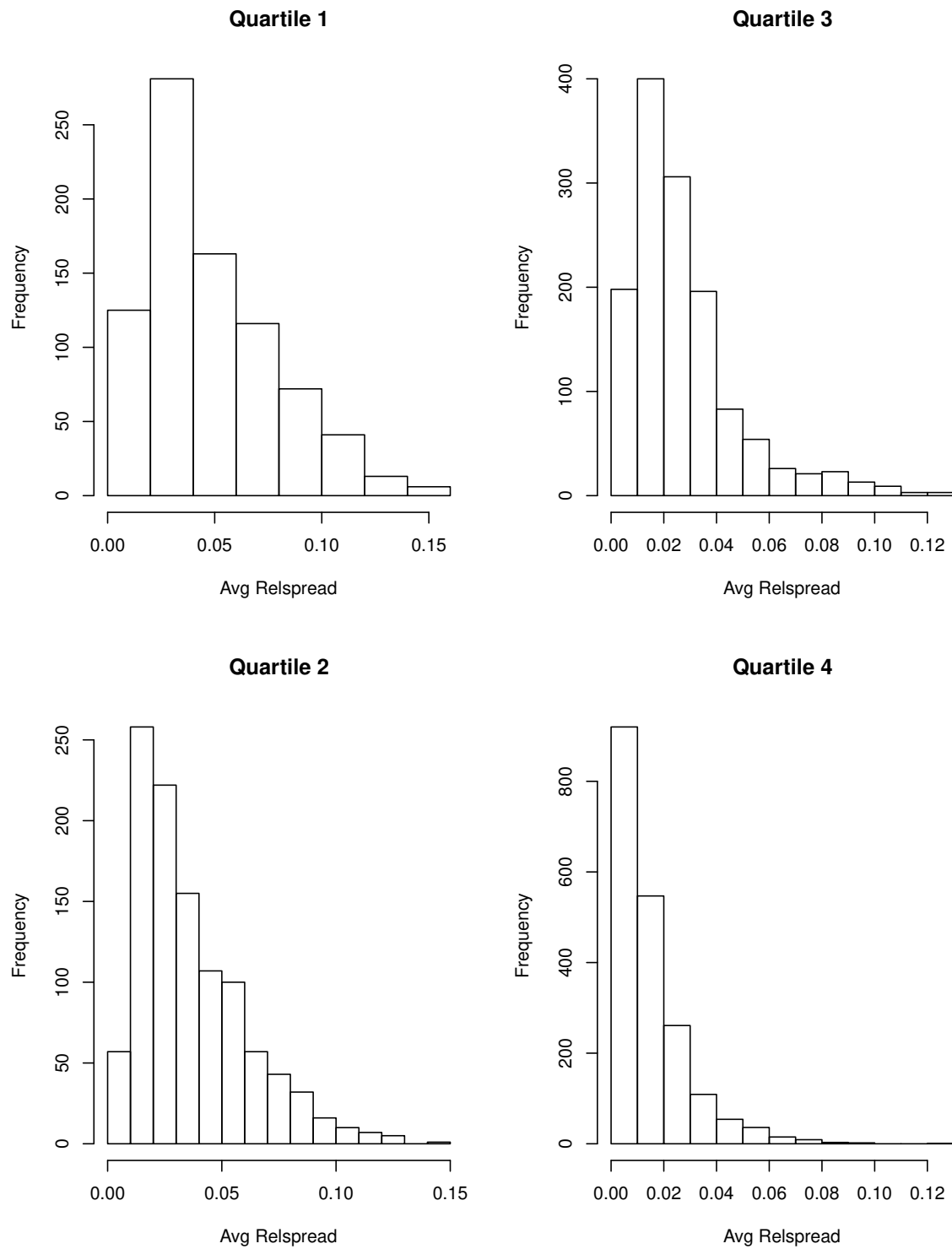
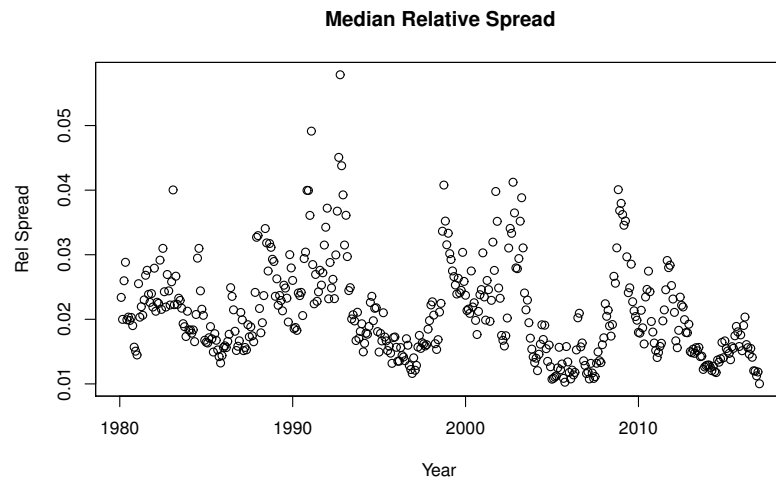
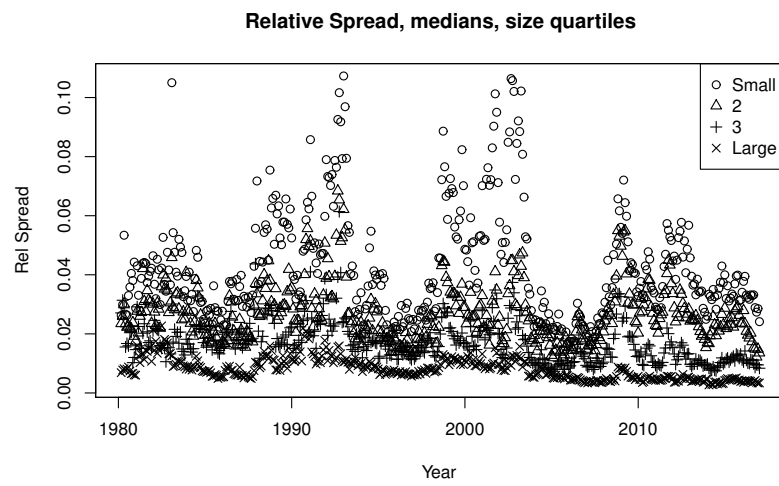


Figure 18 Time Series, Closing Relative Bid/Ask Spread, Monthly Median

Panel A: Crossectional Median



Panel B: Crossectional Median, Size sorted portfolios.



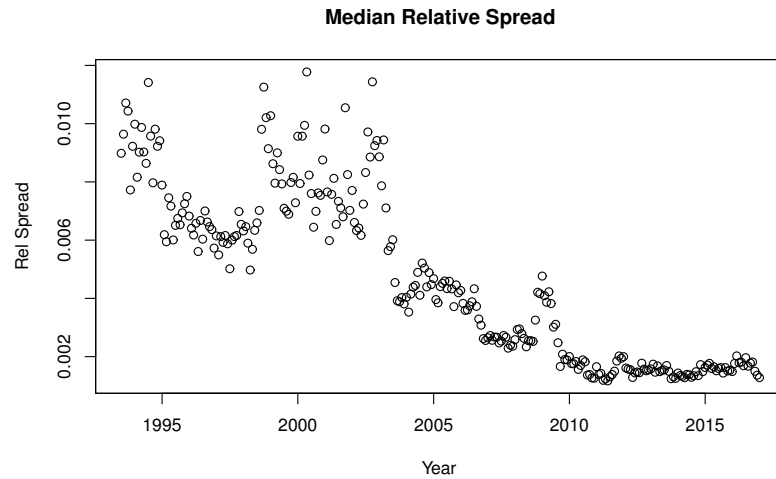
Time series plot of median relative spread across stocks on the Oslo Stock Exchange.

3.2.1 OBX constituents and others

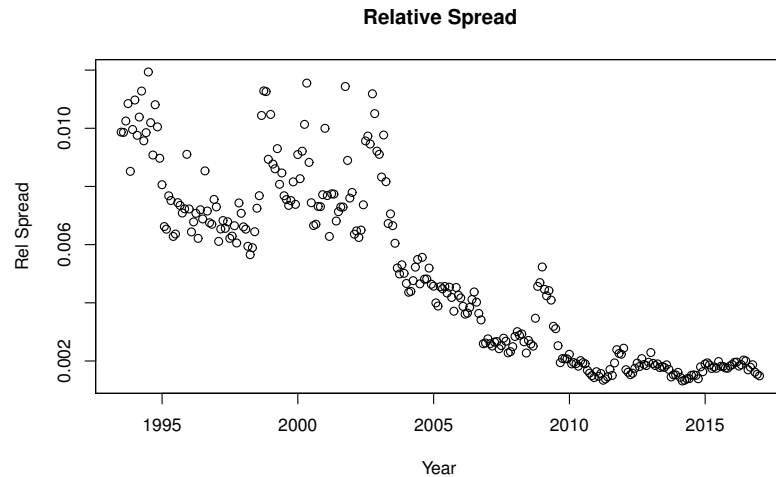
We now differentiate between firms in the OBX index, and others (still maintaining restriction that there is a minimum of 100 trading days)

Figure 19 Time Series, Closing Relative Bid/Ask Spread, Monthly Median, OBX constituents

Panel A: Crossectional Median



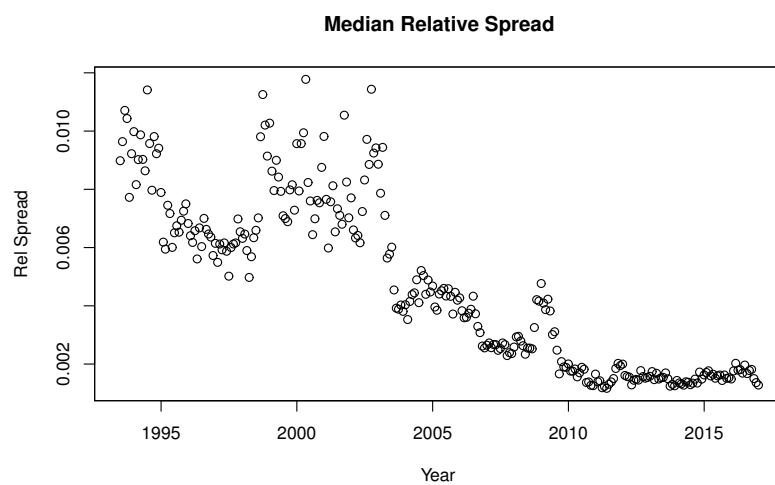
Panel B: Crossectional Mean



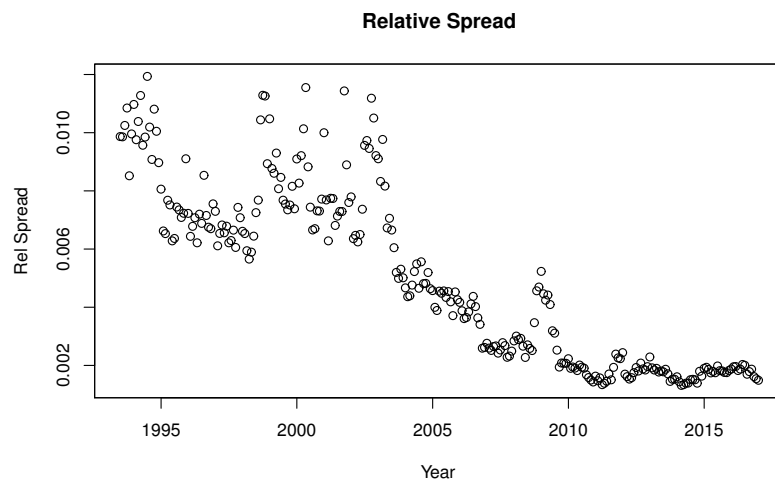
Time series plot of median relative spread across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

Figure 20 Time Series, Closing Relative Bid/Ask Spread. Non-OBX constituents

Panel A: Crossectional Median



Panel B: Crossectional Mean

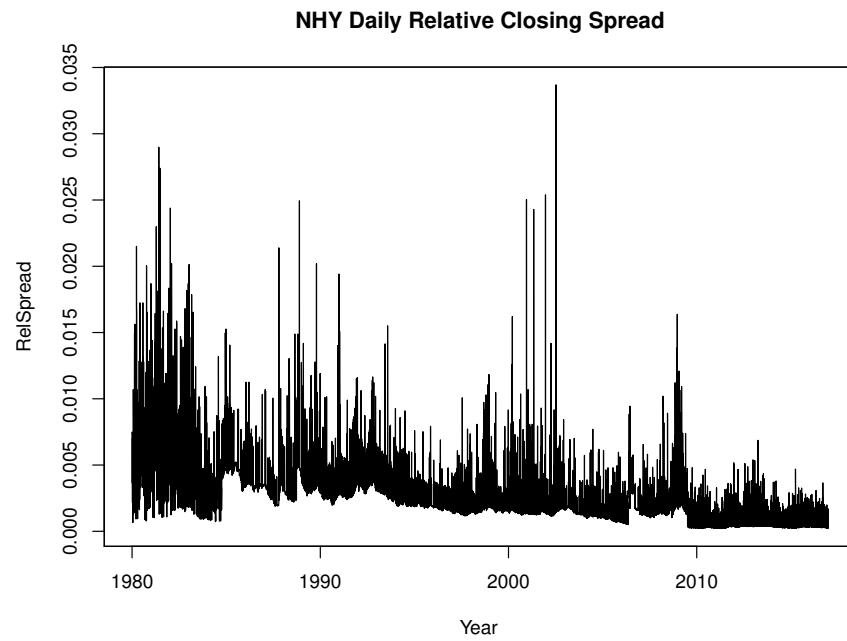


Time series plot of relative spread across stocks on the Oslo Stock Exchange.

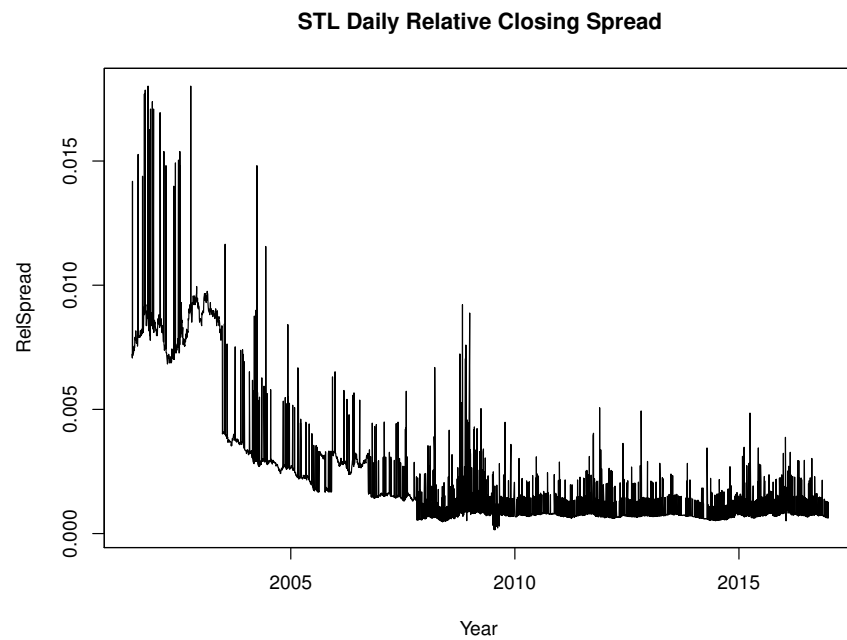
Figure 21 Example, Norsk Hydro closing relative spread

This figure illustrates the evolution of relative spread (calculated using closing prices) for the stock Norsk Hydro.

Panel A: Norsk Hydro



Panel B: Statoil



3.3 Relative Spread - intraday

When one has access to the full orderbook, one can also observe the buy and sell quotes in the order book at any point of time during the trading day, and calculate the spread as difference between the two.

Table 5 Descriptive, Relative Bid Ask Spread

Period	min	Q1	med	mean	(std)	Q3	max	n	Size quartiles(means)			
									1(small)	2	3	4
1999-2012	0.001	0.011	0.024	0.029	(0.022)	0.043	0.104	3066	0.048	0.038	0.030	0.015
1999	0.003	0.016	0.031	0.036	(0.023)	0.054	0.095	171	0.063	0.042	0.032	0.021
2000	0.003	0.015	0.027	0.032	(0.021)	0.046	0.083	169	0.048	0.032	0.027	0.019
2001	0.002	0.016	0.031	0.037	(0.024)	0.055	0.096	156	0.072	0.050	0.039	0.024
2002	0.002	0.017	0.032	0.039	(0.026)	0.060	0.104	139	0.064	0.056	0.040	0.021
2003	0.002	0.016	0.028	0.033	(0.022)	0.048	0.097	134	0.051	0.045	0.032	0.016
2004	0.001	0.009	0.018	0.022	(0.015)	0.030	0.072	151	0.028	0.031	0.019	0.011
2005	0.001	0.008	0.015	0.019	(0.014)	0.024	0.071	182	0.027	0.026	0.021	0.011
2006	0.002	0.008	0.018	0.021	(0.016)	0.029	0.088	202	0.037	0.025	0.024	0.012
2007	0.001	0.007	0.016	0.020	(0.016)	0.028	0.073	212	0.034	0.023	0.024	0.008
2008	0.001	0.013	0.028	0.031	(0.022)	0.046	0.092	200	0.047	0.044	0.035	0.014
2009	0.001	0.013	0.028	0.031	(0.022)	0.048	0.094	179	0.050	0.043	0.032	0.013
2010	0.001	0.011	0.028	0.031	(0.024)	0.046	0.093	204	0.043	0.035	0.019	0.009
2011	0.001	0.011	0.029	0.033	(0.024)	0.050	0.093	208	0.053	0.048	0.043	0.013
2012	0.001	0.009	0.028	0.033	(0.026)	0.051	0.092	196	0.065	0.048	0.032	0.011

Figure 22 Histogram Bid Ask Spread - Subperiod

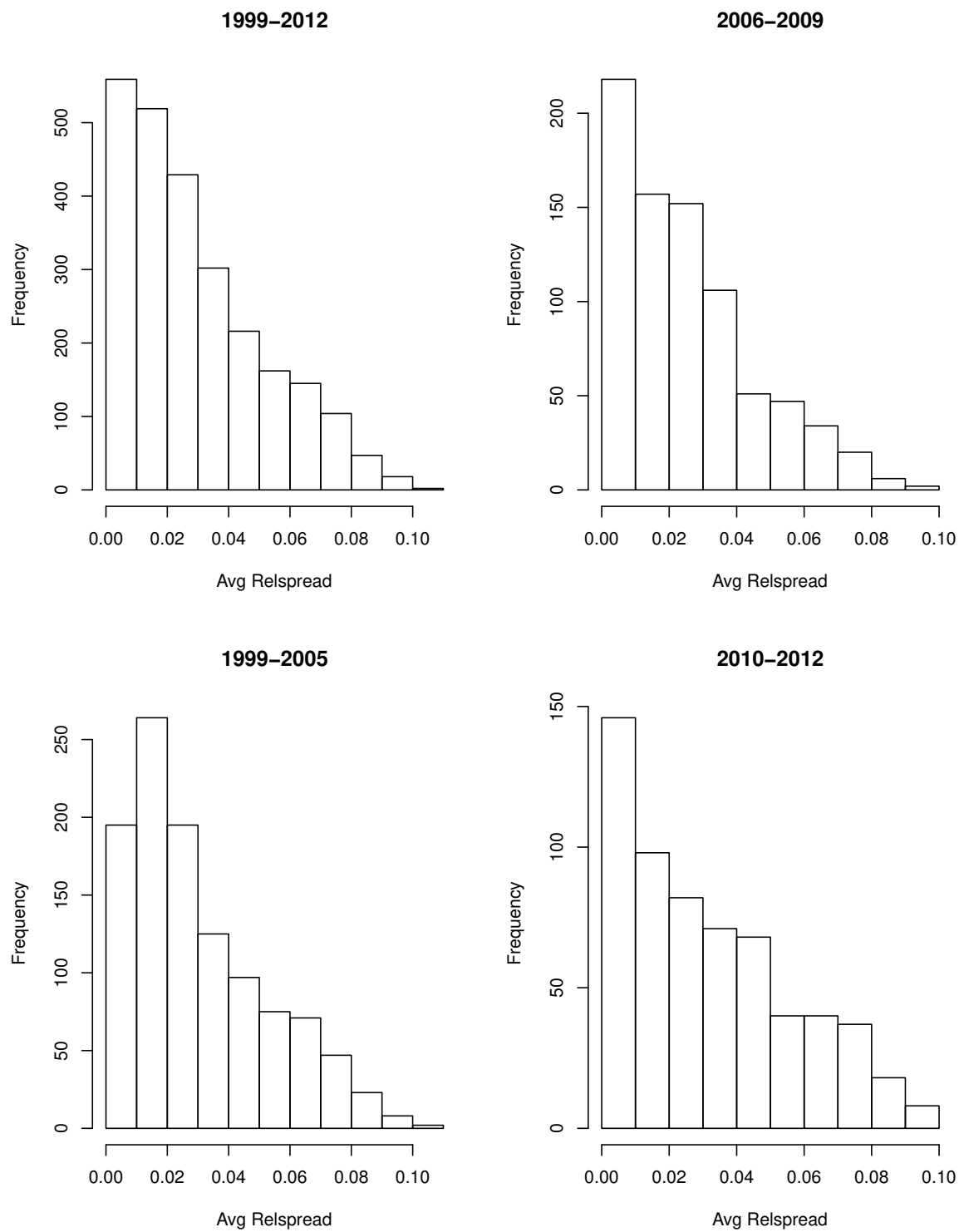


Figure 23 Histogram Bid Ask Spread - By Size

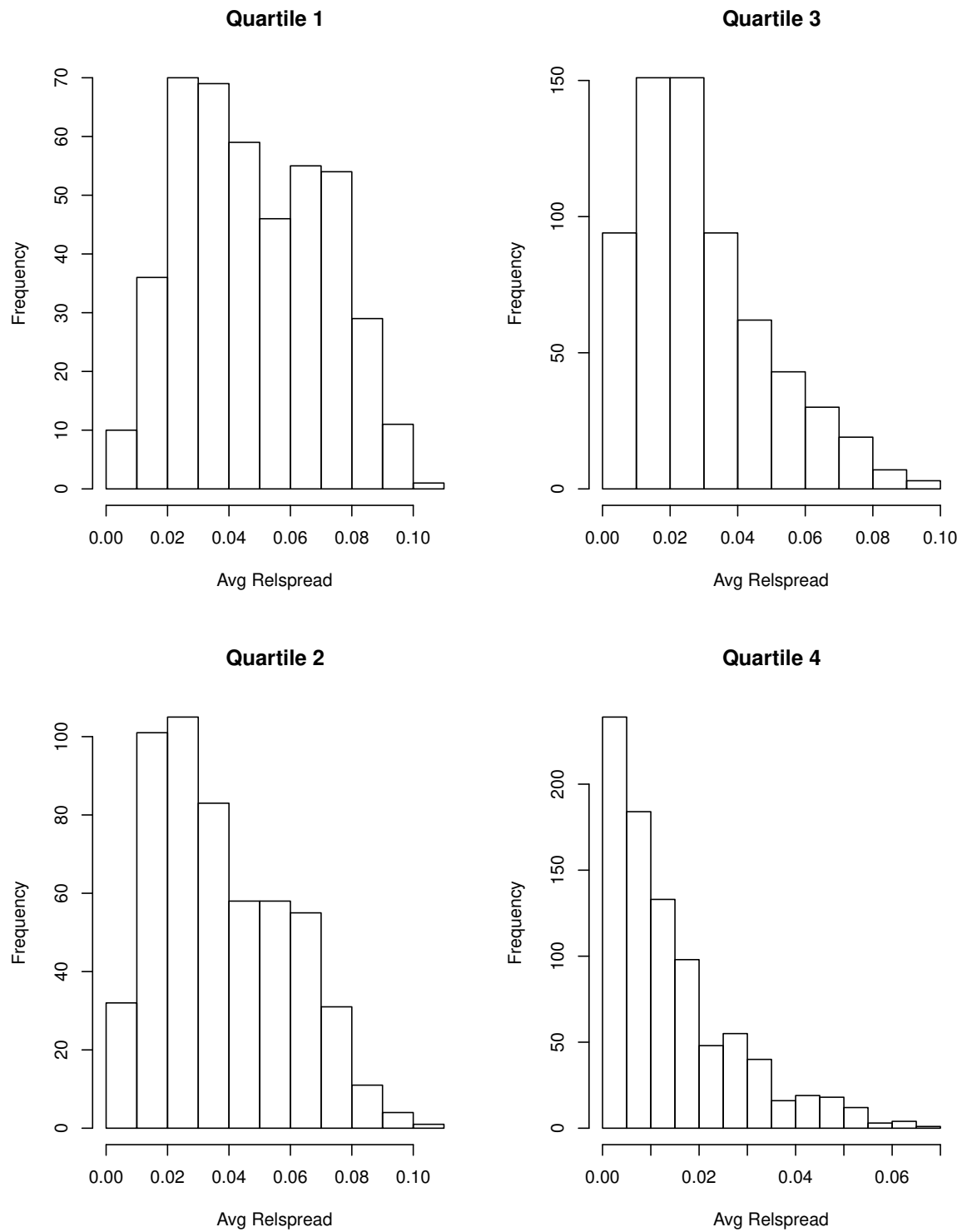
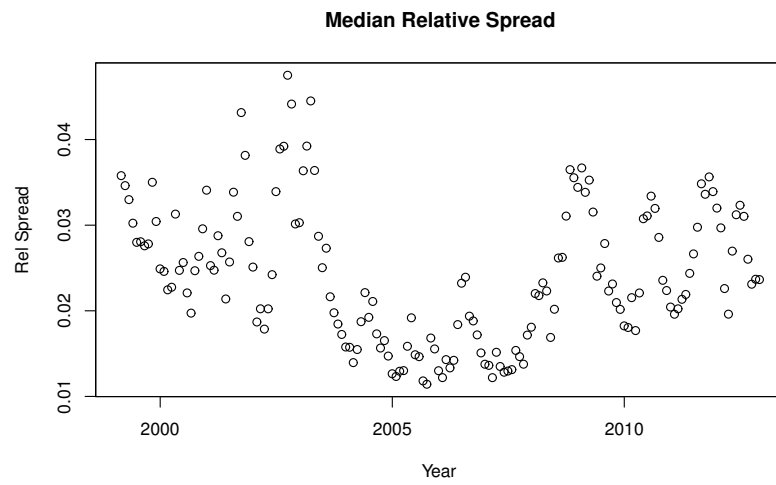
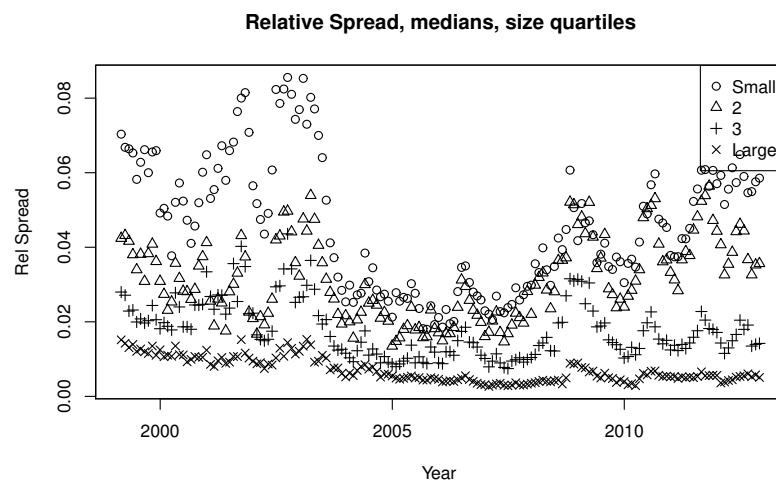


Figure 24 Time Series, Relative Spread, Monthly Median

Panel A: Crossectional Median



Panel B: Crossectional Median, Size sorted portfolios.



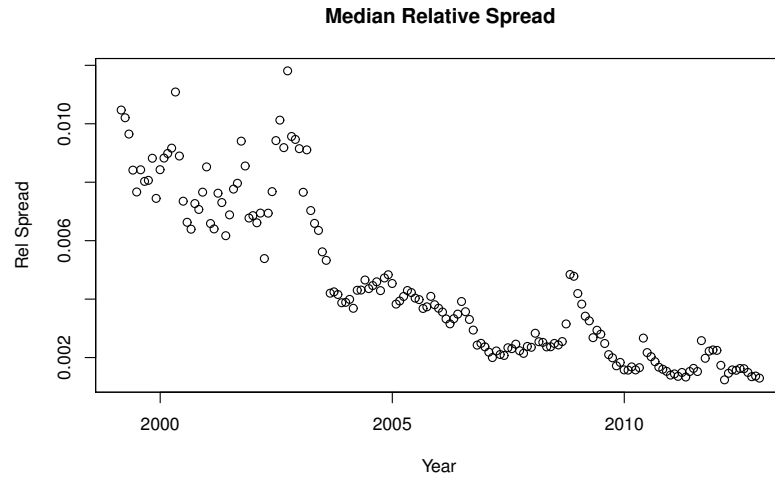
Time series plot of median relative spread across stocks on the Oslo Stock Exchange.

3.3.1 OBX constituents and others

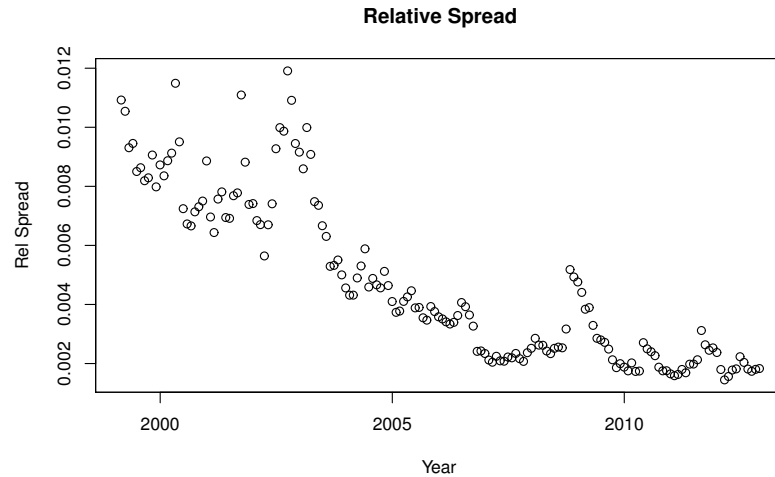
We now differentiate between firms in the OBX index, and others (still maintaining restriction that there is a minimum of 100 trading days)

Figure 25 Time Series, Relative Spread. OBX constituents

Panel A: Crossectional Median



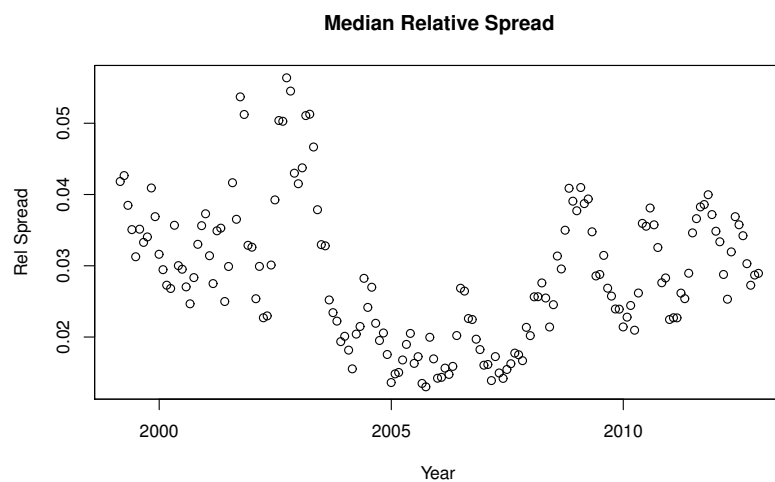
Panel B: Crossectional Mean



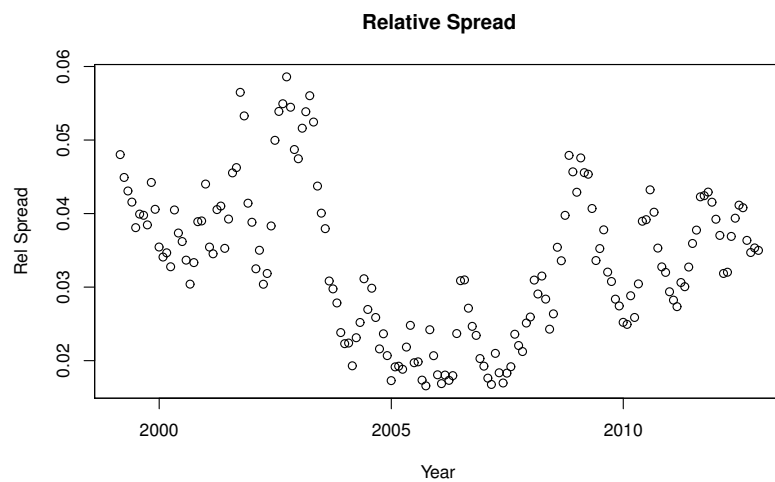
Time series plot of relative spread across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

Figure 26 Time Series, Relative Spread. Non-OBX constituents

Panel A: Crossectional Median



Panel B: Crossectional Mean

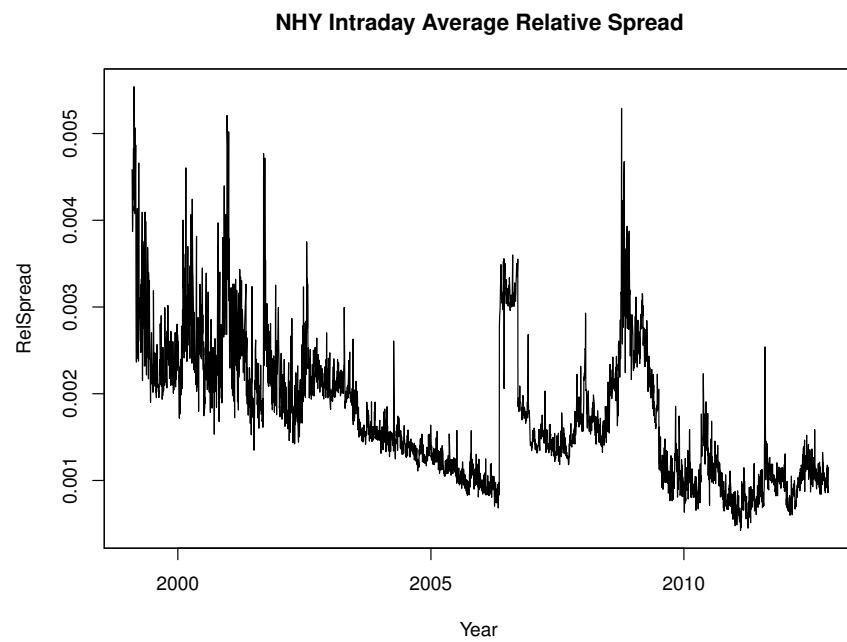


Time series plot of relative spread across stocks on the Oslo Stock Exchange.

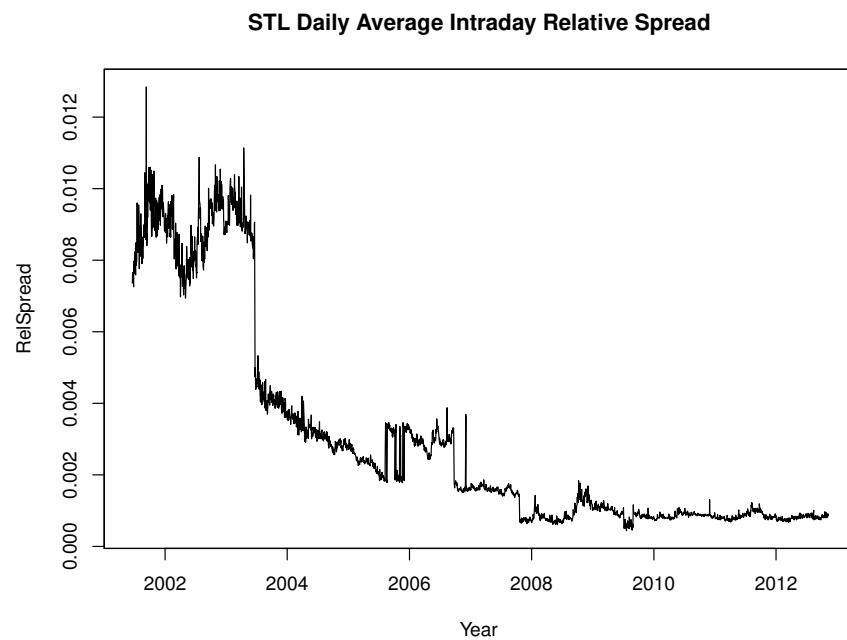
Figure 27 Example, Norsk Hydro intraday average relative spread

This figure illustrates the evolution of intraday relative spread for two stocks, Norsk Hydro and Statoil.

Panel A: Norsk Hydro



Panel B: Statoil



3.4 Effective Spread

The effective spread relates transaction prices to the spread when the order is submitted. In the following we calculate the effective proportional half-spread as

$$ehalfspread_{jt} = q_{jt}(p_{jt} - m_{jt})/m_{jt}$$

where q_{jt} is an indicator variable that equals +1 for buyer-initiated trades and -1 for seller-initiated trades, p_{jt} is the trade price, and m_{jt} is the quote midpoint prevailing at the time of the trade.

To determine whether the order is buyer or seller initiated we compare the price to the midpoint. If the price is above the midpoint, we classify it as buyer initiated. Otherwise, we classify it as seller initiated.

Table 6 Descriptive, Effective Spread

Period	min	Q1	med	mean	(std)	Q3	max	n	Size quartiles(means)			
									1(small)	2	3	4
1999-2012	0.0004	0.0043	0.0081	0.0093	(0.0062)	0.0129	0.0340	1534	0.0150	0.0115	0.0092	0.0050
1999	0.0010	0.0058	0.0099	0.0112	(0.0066)	0.0156	0.0296	171	0.0192	0.0127	0.0096	0.0066
2000	0.0011	0.0056	0.0088	0.0103	(0.0058)	0.0147	0.0268	169	0.0152	0.0101	0.0089	0.0065
2001	0.0010	0.0062	0.0103	0.0113	(0.0065)	0.0169	0.0266	156	0.0206	0.0151	0.0118	0.0078
2002	0.0009	0.0058	0.0099	0.0120	(0.0074)	0.0175	0.0340	139	0.0195	0.0164	0.0123	0.0069
2003	0.0008	0.0050	0.0086	0.0096	(0.0058)	0.0127	0.0287	134	0.0141	0.0125	0.0098	0.0055
2004	0.0006	0.0037	0.0068	0.0072	(0.0042)	0.0094	0.0194	151	0.0095	0.0096	0.0061	0.0040
2005	0.0006	0.0035	0.0058	0.0062	(0.0036)	0.0081	0.0212	182	0.0095	0.0083	0.0070	0.0040
2006	0.0010	0.0034	0.0065	0.0068	(0.0042)	0.0091	0.0239	202	0.0117	0.0083	0.0075	0.0042
2007	0.0007	0.0031	0.0059	0.0064	(0.0040)	0.0091	0.0189	212	0.0108	0.0080	0.0073	0.0030
2008	0.0005	0.0048	0.0096	0.0100	(0.0062)	0.0144	0.0281	200	0.0149	0.0134	0.0113	0.0049
2009	0.0005	0.0051	0.0096	0.0099	(0.0059)	0.0138	0.0259	179	0.0154	0.0129	0.0102	0.0044
2010	0.0005	0.0043	0.0088	0.0098	(0.0066)	0.0142	0.0256	204	0.0136	0.0108	0.0060	0.0031
2011	0.0005	0.0040	0.0095	0.0101	(0.0067)	0.0151	0.0256	208	0.0166	0.0146	0.0126	0.0045
2012	0.0004	0.0037	0.0090	0.0100	(0.0074)	0.0155	0.0296	196	0.0202	0.0142	0.0095	0.0037

Figure 28 Histogram Effective Relative Spread - Subperiod

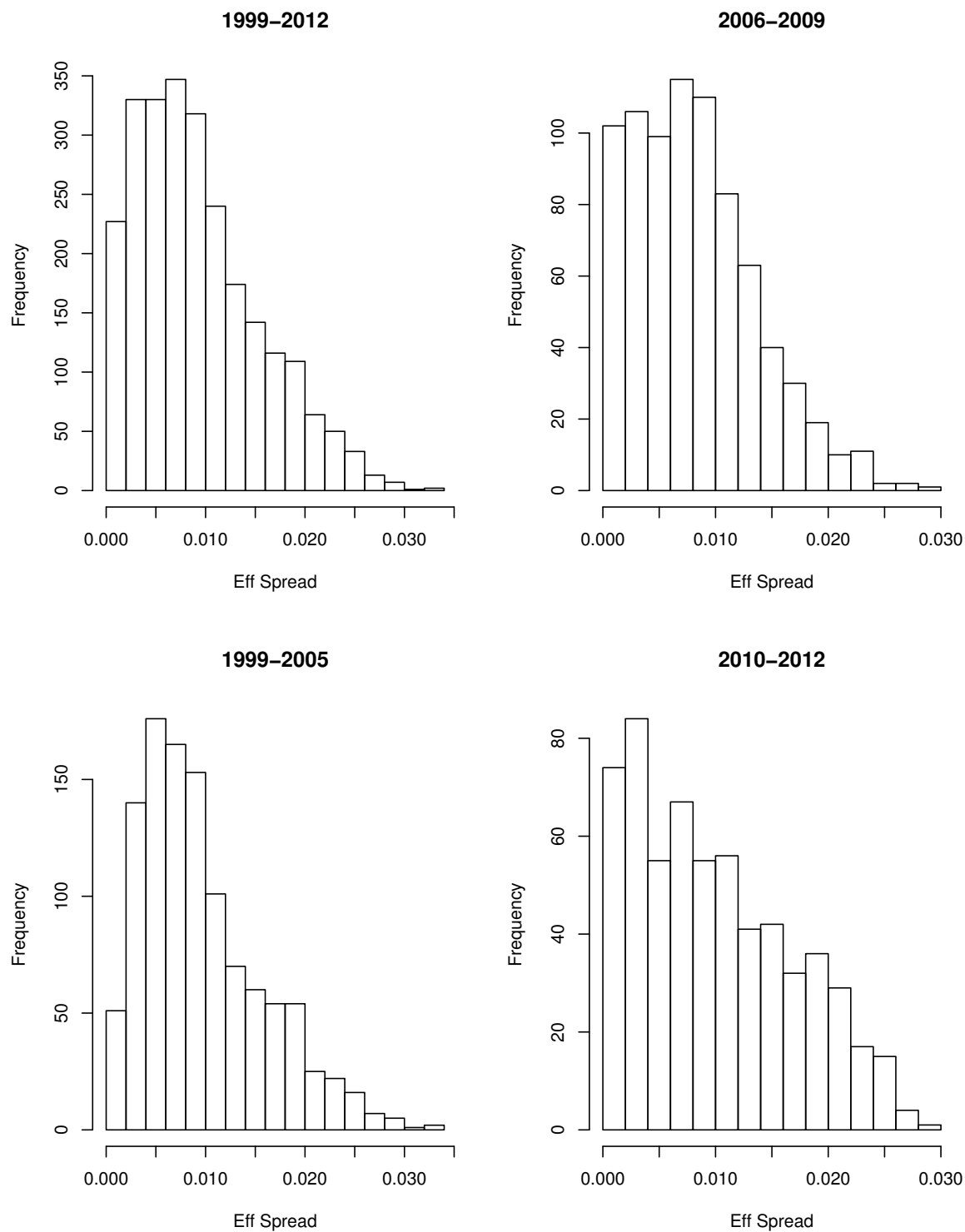


Figure 29 Histogram Effective Relative Spread - By Size

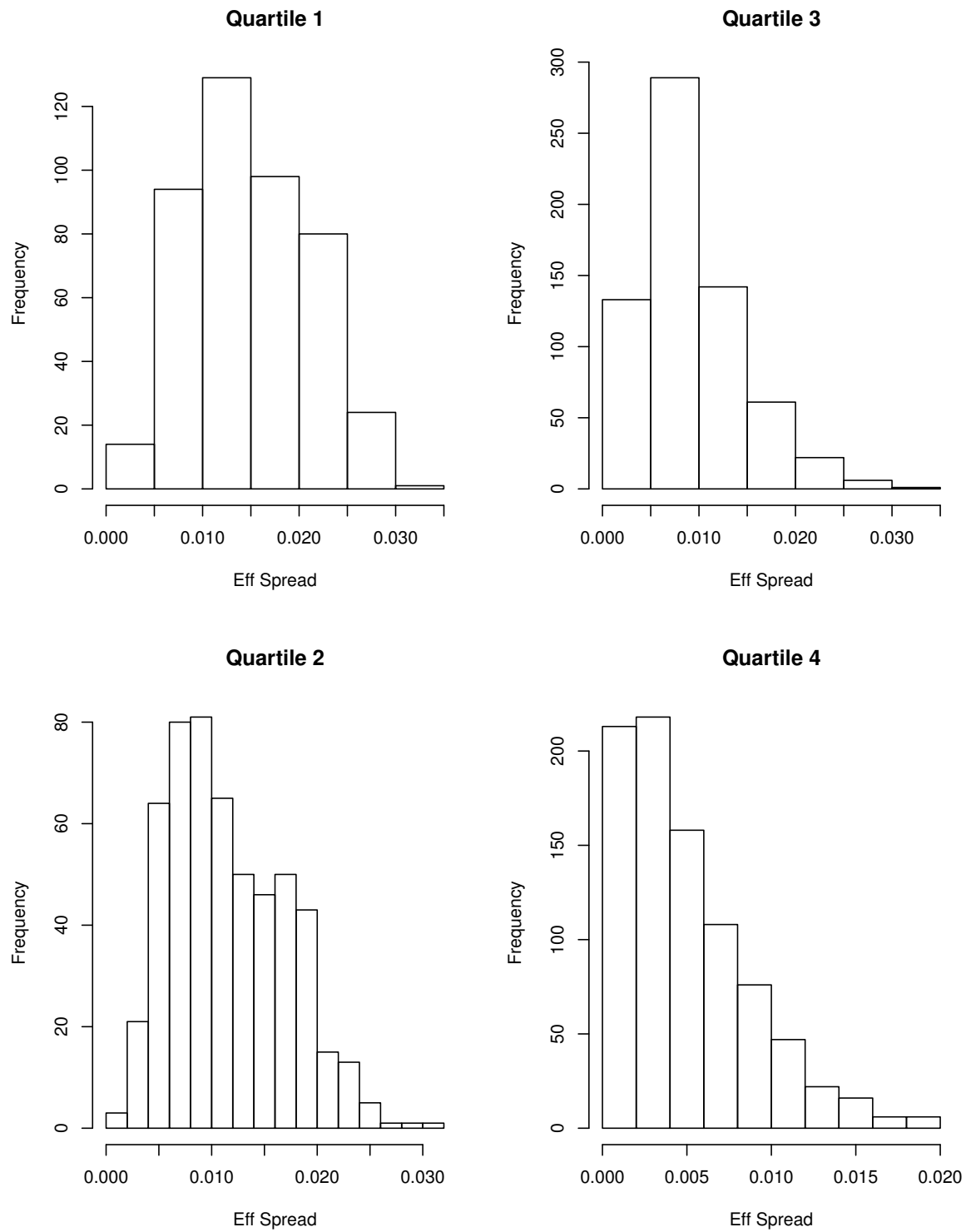
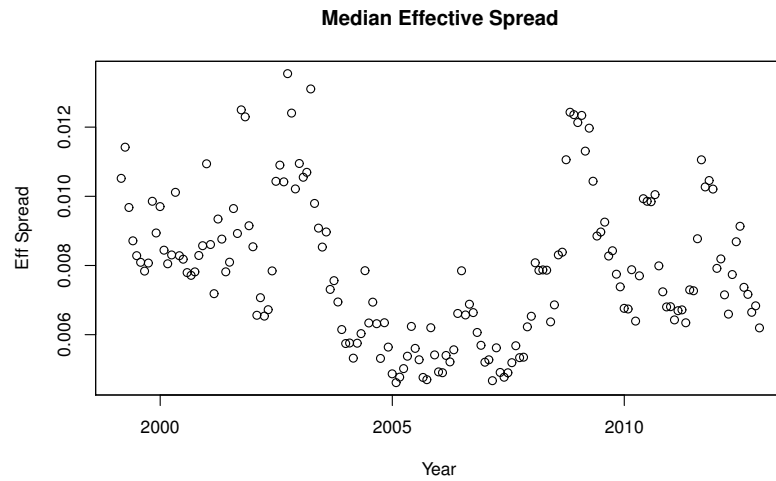


Figure 30 Time Series, Effective Spread, Monthly Median

Time series plot of median effective spread across stocks on the Oslo Stock Exchange.

Panel A: Crossectional Median



Panel B: Crossectional Median, Size sorted portfolios.

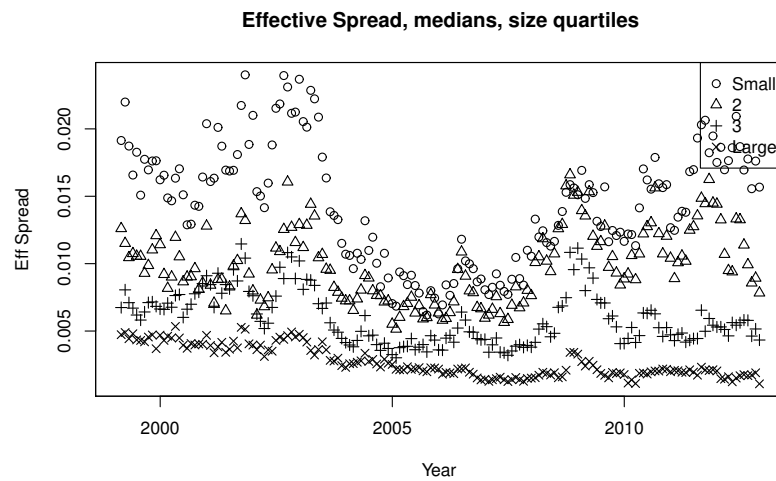
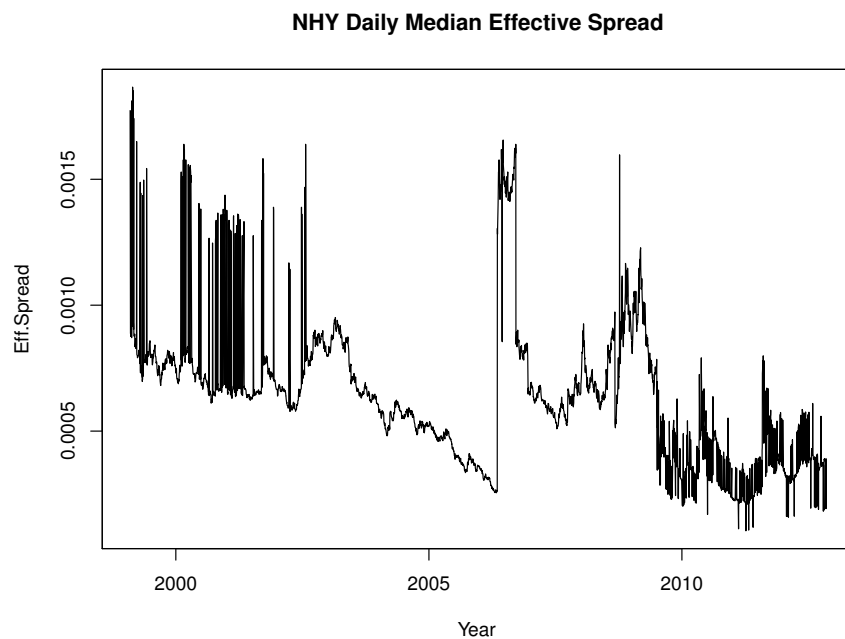
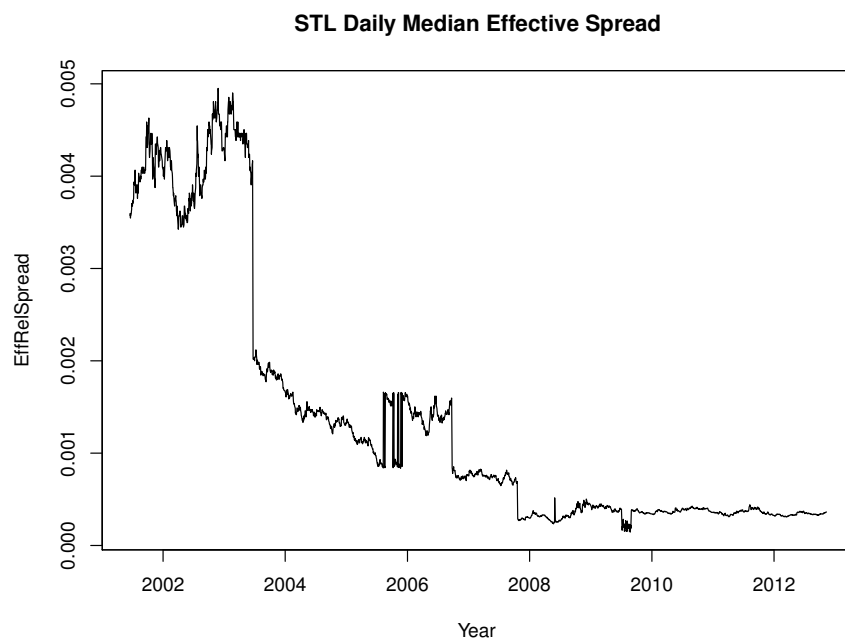


Figure 31 Example, Norsk Hydro effective spread

This figure illustrates the evolution of effective spread for two stocks, Norsk Hydro and Statoil. Median effective spread during the day.
Panel A: Norsk Hydro



Panel B: Statoil



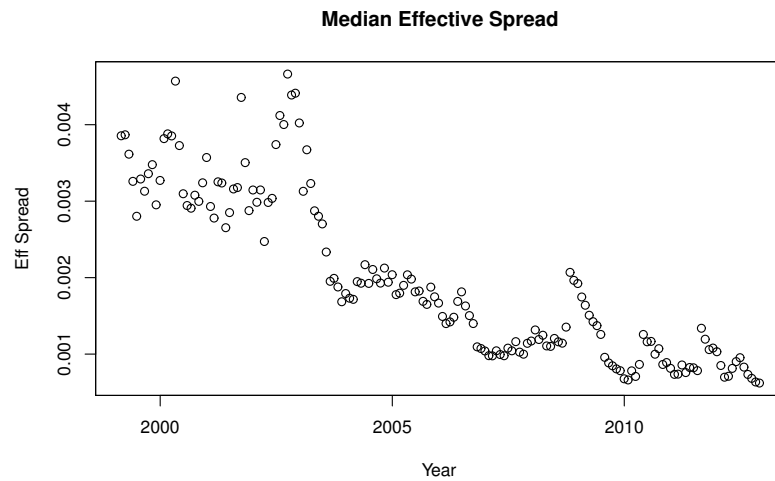
3.4.1 OBX constituents and others

We now differentiate between firms in the OBX index, and others (still maintaining restriction that there is a minimum of 100 trading days)

Figure 32 Time Series, Effective Spread, Monthly Median. OBX Constituents

Time series plot of median effective spread across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

Panel A: Crossectional Median



Panel B: Crossectional Mean

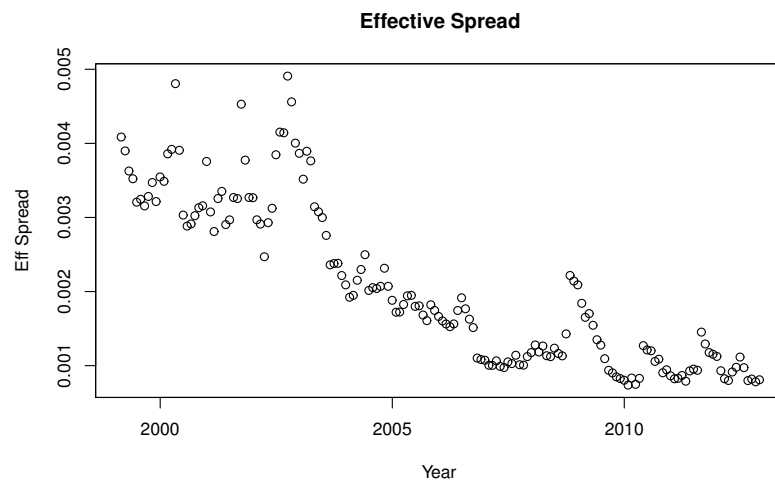
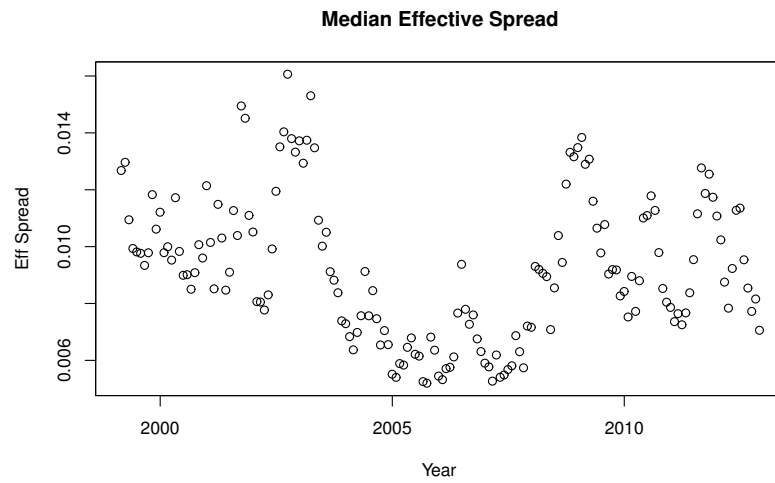


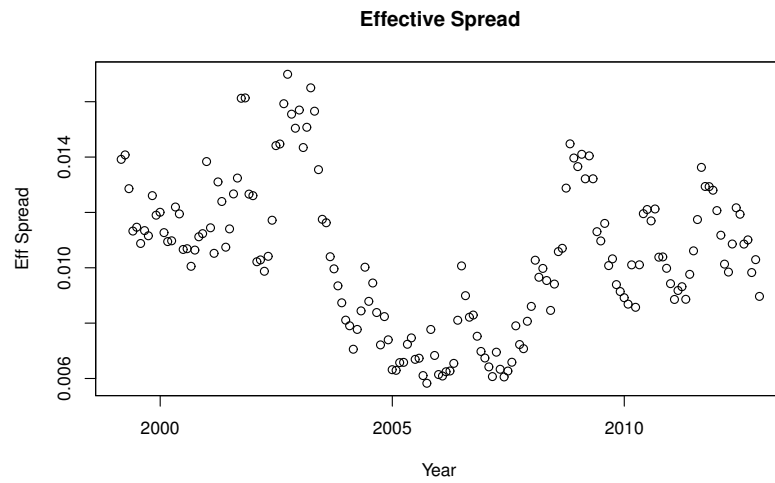
Figure 33 Time Series, Effective Spread, Monthly Median. Non-OBX Constituents

Time series plot of median effective spread across stocks on the Oslo Stock Exchange.

Panel A: Crossectional Median



Panel B: Crossectional Mean



4 Realized Spread

The proportional five-minute realized spread for the t 'th transaction in stock j is defined as

$$rspread_{jt} = q_{jt}(p_{jt} - m_{j,t+5\min})/m_{jt}$$

where p_{jt} is the trade price, q_{jt} is the buy-sell indicator (+1 for buys, -1 for sells), m_{jt} is the midpoint prevailing at the time of the t 'th trade, and $m_{j,t+5\min}$ is the quote midpoint 5 minutes after the t 'th trade.

Table 7 Descriptive, Realized Spread

Period	min	Q1	med	mean	(std)	Q3	max	n	Size quartiles(means)			
									1(small)	2	3	4
1999-2012	-0.0496	0.0012	0.0024	0.0038	(0.0056)	0.0044	0.0947	987	0.0072	0.0047	0.0036	0.0018
1999	-0.0496	0.0018	0.0029	0.0042	(0.0076)	0.0052	0.0387	161	0.0077	0.0054	0.0022	0.0026
2000	-0.0084	0.0018	0.0029	0.0042	(0.0048)	0.0049	0.0355	166	0.0073	0.0038	0.0030	0.0025
2001	-0.0312	0.0018	0.0032	0.0043	(0.0058)	0.0058	0.0385	150	0.0087	0.0052	0.0049	0.0027
2002	0.0002	0.0019	0.0036	0.0066	(0.0089)	0.0067	0.0564	134	0.0145	0.0107	0.0056	0.0024
2003	-0.0057	0.0015	0.0026	0.0036	(0.0042)	0.0044	0.0321	132	0.0064	0.0052	0.0032	0.0016
2004	-0.0045	0.0011	0.0019	0.0022	(0.0018)	0.0028	0.0109	146	0.0034	0.0022	0.0020	0.0012
2005	-0.0016	0.0009	0.0017	0.0024	(0.0040)	0.0025	0.0373	175	0.0034	0.0038	0.0026	0.0013
2006	-0.0038	0.0009	0.0019	0.0023	(0.0021)	0.0028	0.0150	197	0.0039	0.0023	0.0026	0.0017
2007	-0.0014	0.0009	0.0017	0.0022	(0.0020)	0.0030	0.0117	209	0.0036	0.0025	0.0025	0.0011
2008	-0.0041	0.0008	0.0026	0.0037	(0.0044)	0.0050	0.0289	192	0.0053	0.0052	0.0048	0.0015
2009	-0.0030	0.0010	0.0029	0.0044	(0.0081)	0.0054	0.0947	175	0.0100	0.0053	0.0043	0.0013
2010	-0.0264	0.0014	0.0029	0.0039	(0.0052)	0.0054	0.0318	196	0.0059	0.0031	0.0023	0.0015
2011	-0.0113	0.0011	0.0029	0.0043	(0.0053)	0.0069	0.0249	196	0.0075	0.0066	0.0057	0.0017
2012	-0.0029	0.0010	0.0030	0.0056	(0.0075)	0.0067	0.0410	183	0.0164	0.0081	0.0038	0.0016

Figure 34 Histogram Realized Spread - Subperiod

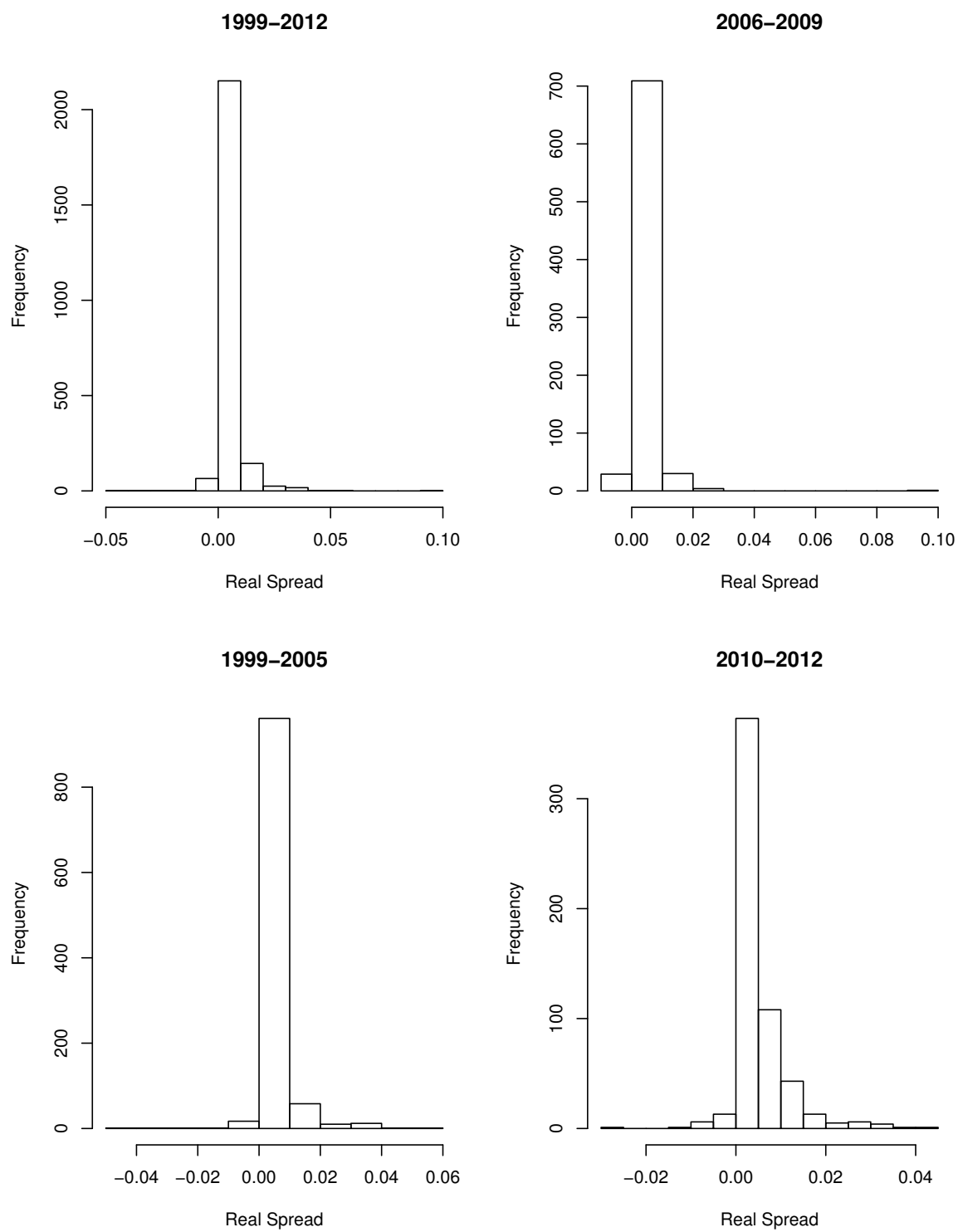


Figure 35 Histogram Realized Relative Spread - By Size

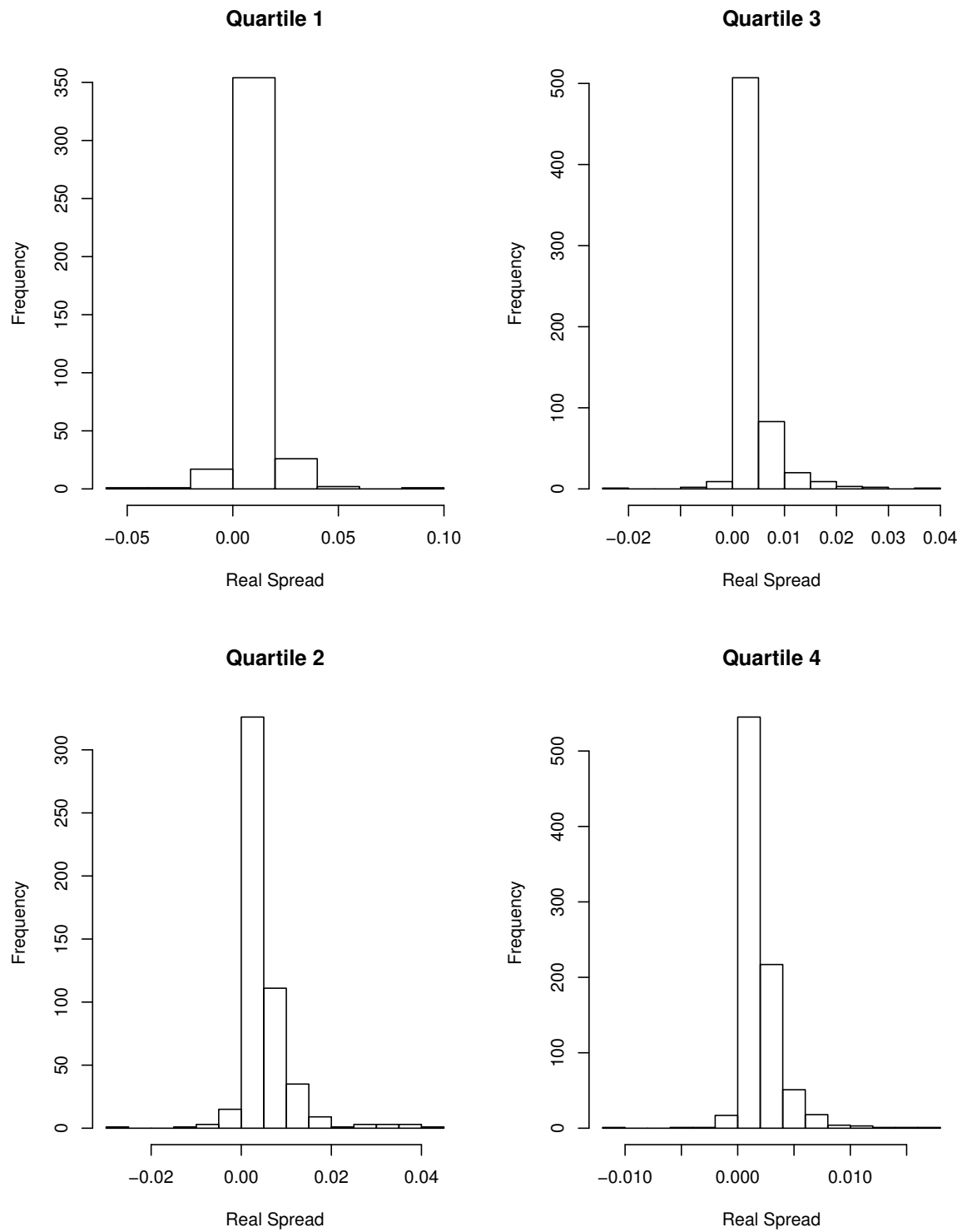
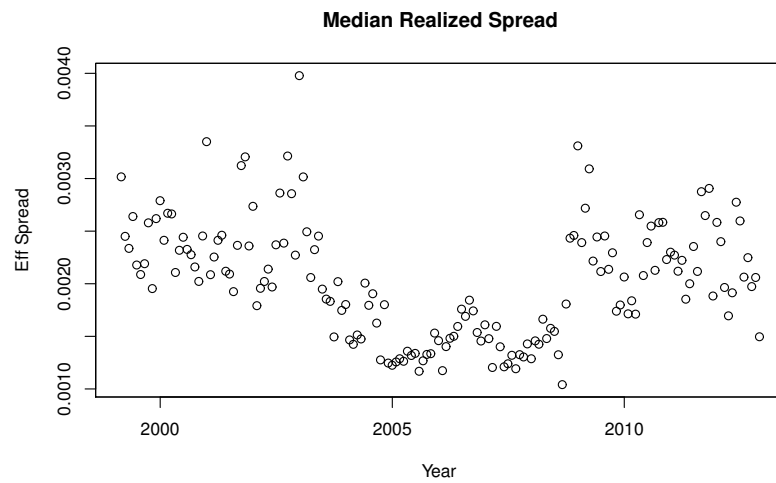


Figure 36 Time Series, Realized Spread, Monthly Median

Time series plot of median realized spread across stocks on the Oslo Stock Exchange.

Panel A: Crossectional Median



Panel B: Crossectional Median, Size sorted portfolios.

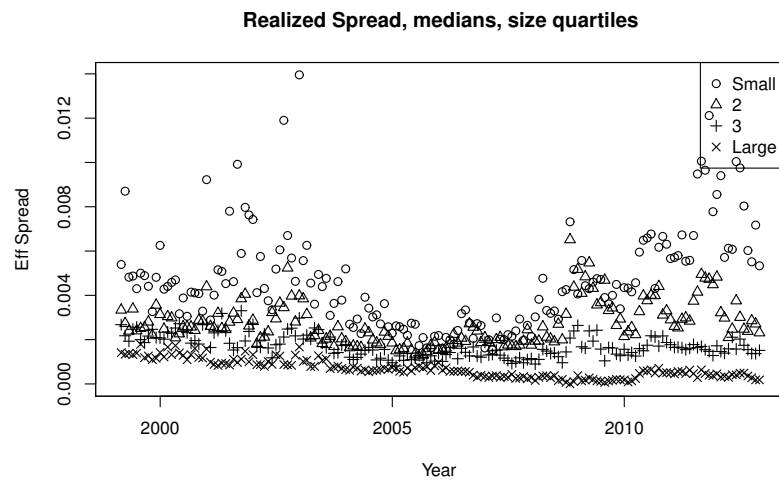
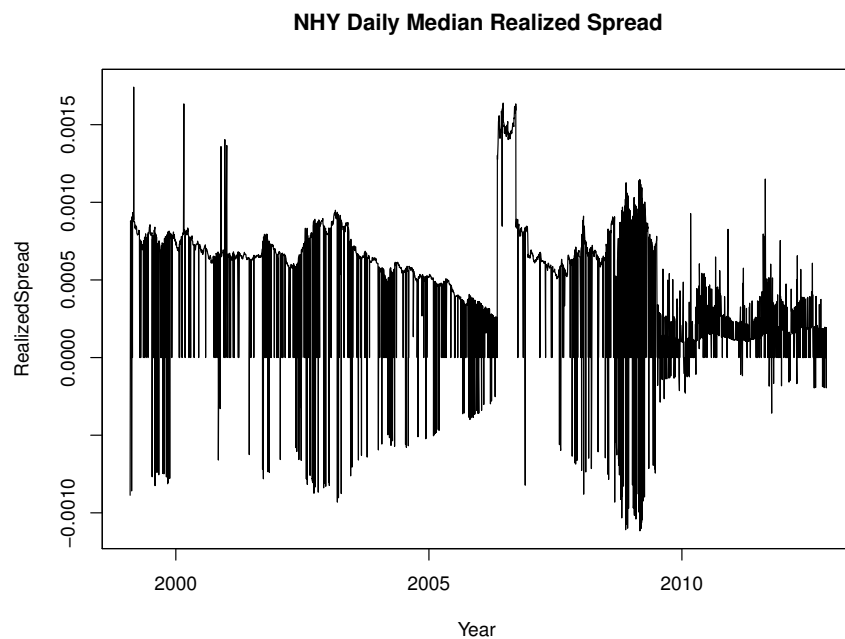


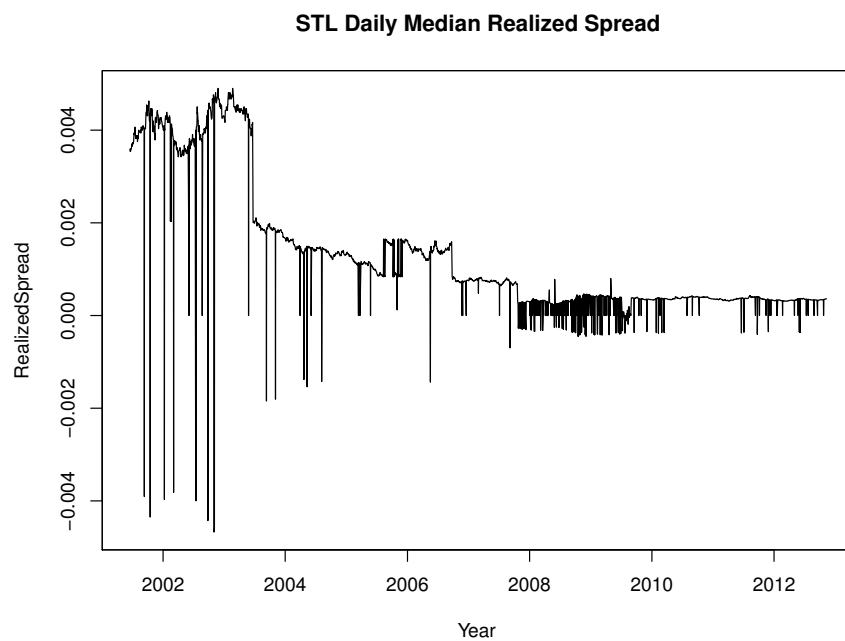
Figure 37 Example, Norsk Hydro and Statoil realized spread

This figure illustrates the evolution of realized spread for two stocks, Norsk Hydro and Statoil. Median realized spread during the day.

Panel A: Norsk Hydro



Panel B: Statoil



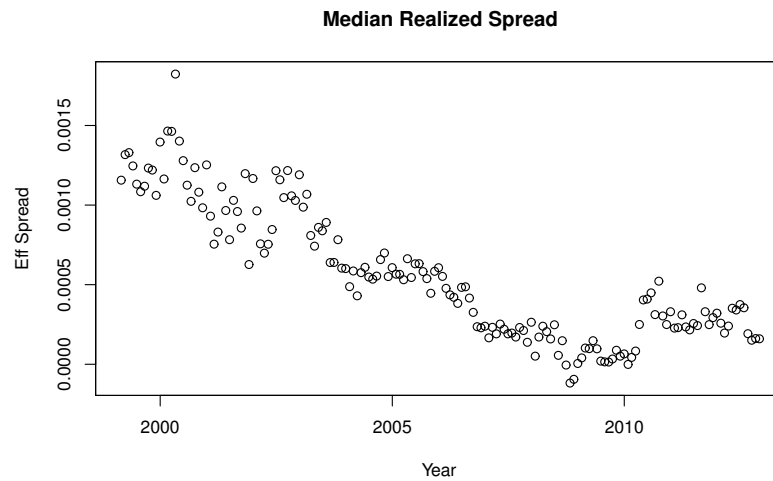
4.0.1 OBX constituents and others

We now differentiate between firms in the OBX index, and others (still maintaining restriction that there is a minimum of 100 trading days)

Figure 38 Time Series, Realized Spread, Monthly Median. OBX Constituents

Time series plot of median realized spread across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

Panel A: Crossectional Median



Panel B: Crossectional Average

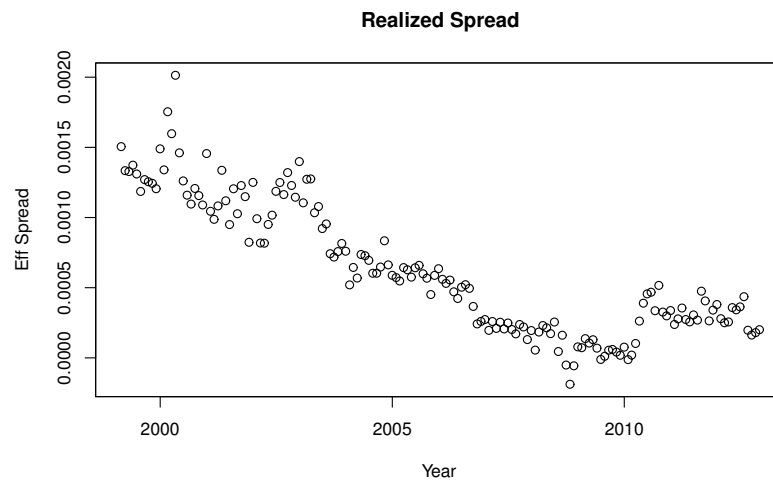
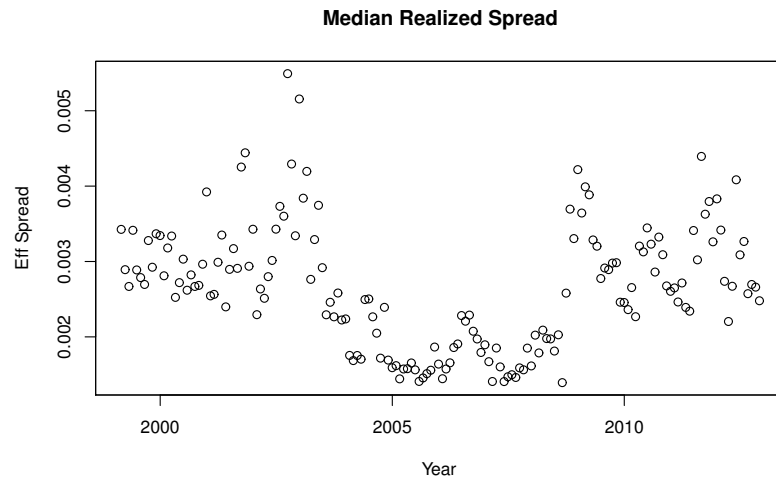


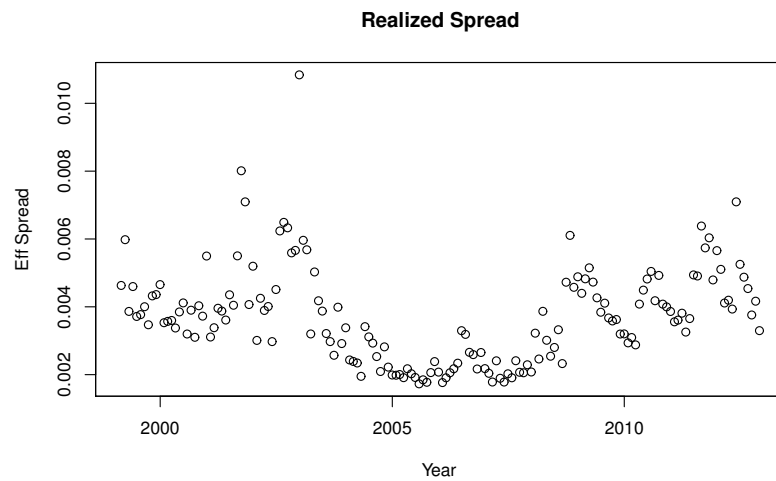
Figure 39 Time Series, Realized Spread, Monthly Median. Non-OBX Constituents

Time series plot of median realized spread across stocks on the Oslo Stock Exchange.

Panel A: Crossectional Median



Panel B: Crossectional Average



5 The trading process

In this section we look closer at what is going on on the OSE in terms of the trade process.

We look at the following

- Trade Size.

It has been a worldwide trend that trades are getting smaller, as a result of the automation of trading. We look at the average trade size (in NOK).

- Depth. A measure of liquidity in a limit order market is the amount of trading interest at the best bid and ask prices.
- Message to Trade Ratios.

5.1 Trade Size

The trade size is the size (in NOK) of the average trade. The following figures illustrate that trade sizes have fallen markedly.

Table 8 Descriptive, Trade Size

Numbers in thousands.

Period	min	Q1	med	mean	(std)	Q3	max	n	Size quartiles(means)			
									1(small)	2	3	4
1999-2012	0	28	61	212	(5455)	149	862056	2243	47	104	186	379
1999	2	51	126	361	(3770)	250	143390	171	65	203	525	536
2000	6	44	116	480	(5745)	228	173056	169	52	206	306	1283
2001	2	29	88	245	(962)	222	21586	156	21	73	172	383
2002	2	25	73	166	(343)	207	9398	139	29	61	167	259
2003	2	29	71	189	(1390)	171	52475	134	52	75	134	356
2004	5	40	83	156	(468)	172	17552	151	55	119	140	321
2005	5	51	106	176	(386)	193	11813	182	56	70	152	273
2006	2	54	123	199	(487)	215	14821	202	55	141	174	289
2007	9	49	96	231	(1704)	184	67450	212	83	97	399	230
2008	2	28	55	516	(17805)	103	862056	200	36	126	212	1209
2009	3	21	38	78	(468)	70	20532	179	36	103	72	86
2010	0	19	38	74	(324)	69	12655	204	51	77	100	99
2011	1	16	29	57	(289)	51	12962	208	24	38	83	59
2012	1	14	26	59	(363)	47	15653	196	18	50	67	75

Figure 40 Histogram Trade Size - Subperiod

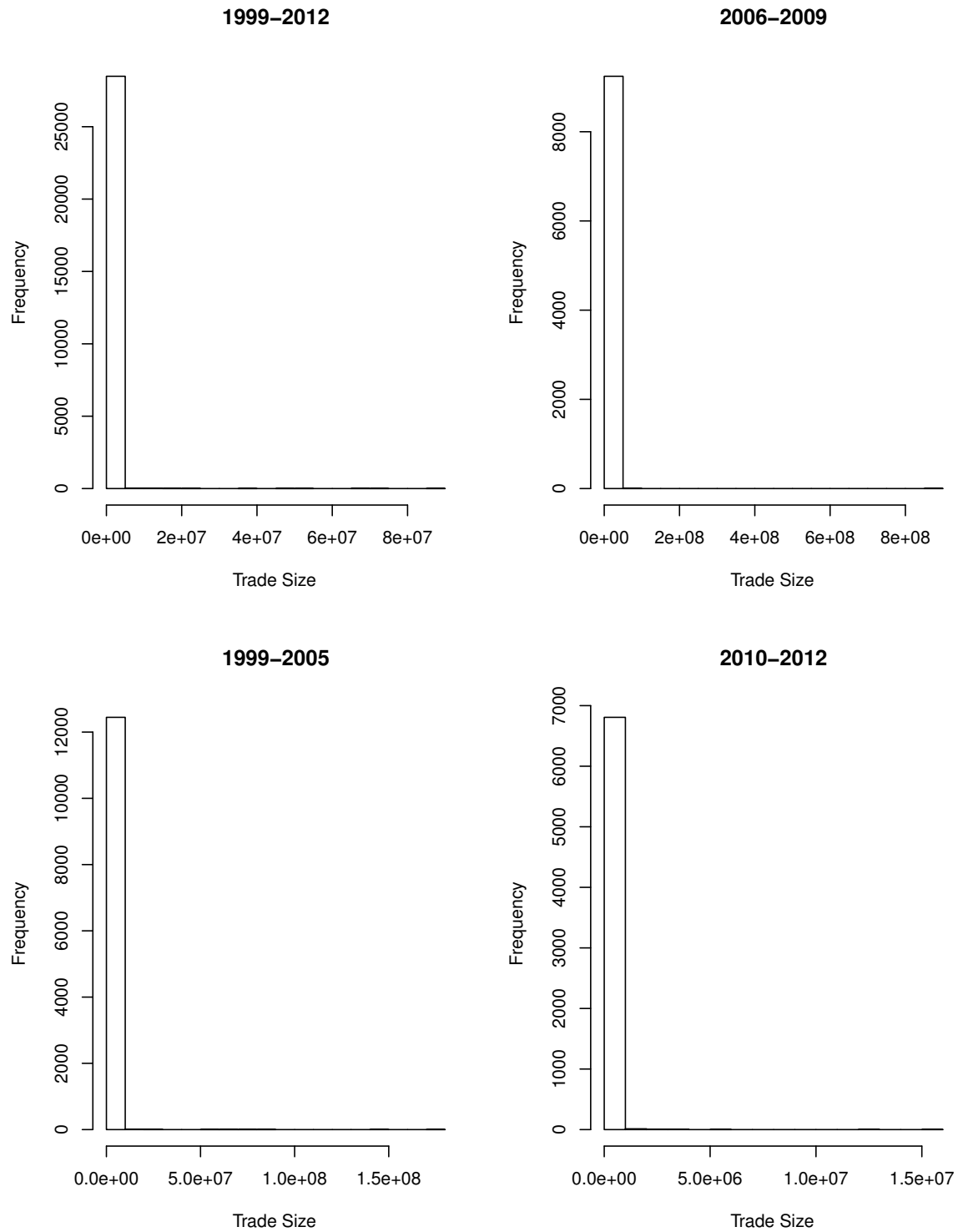


Figure 41 Histogram Trade Size - By Size

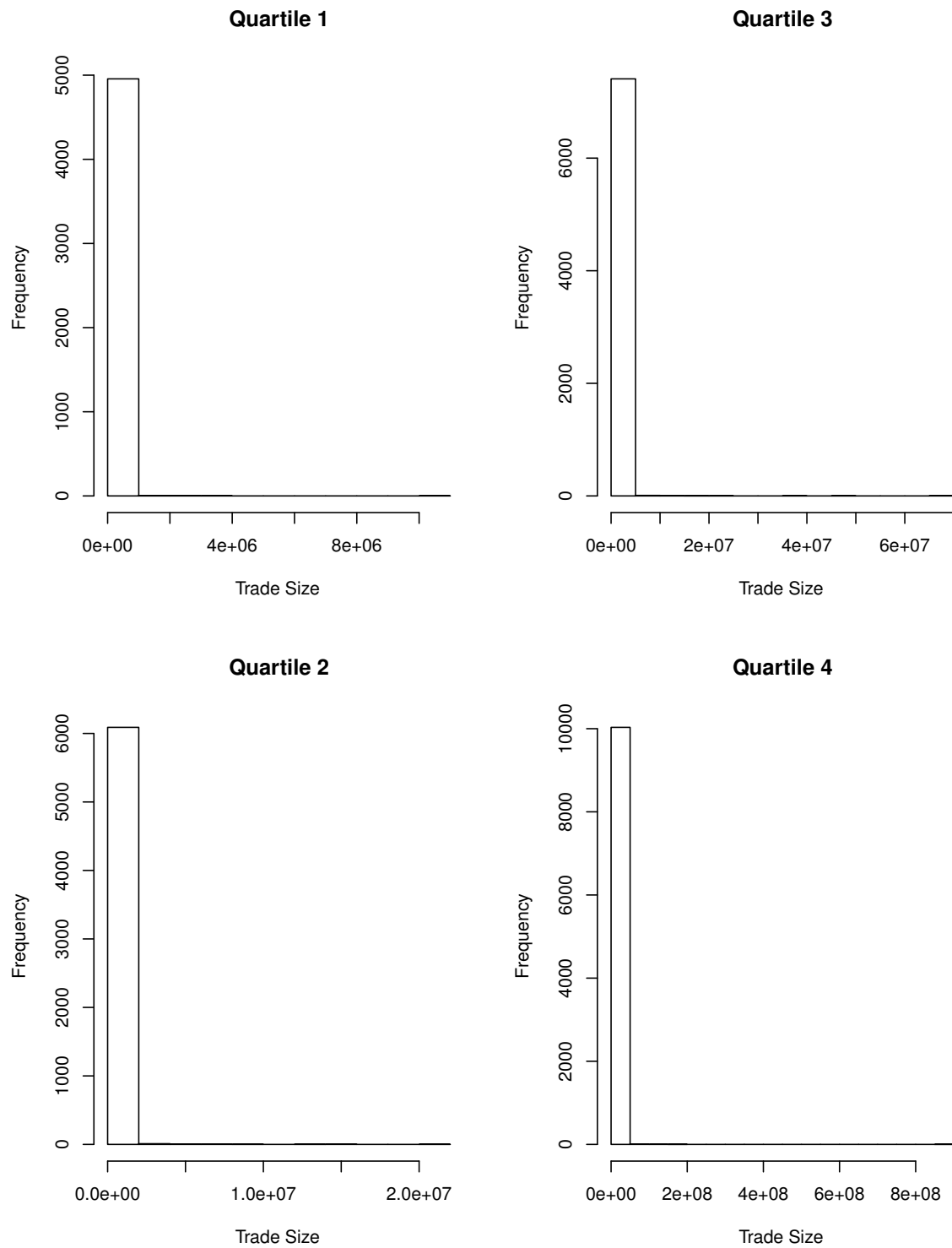
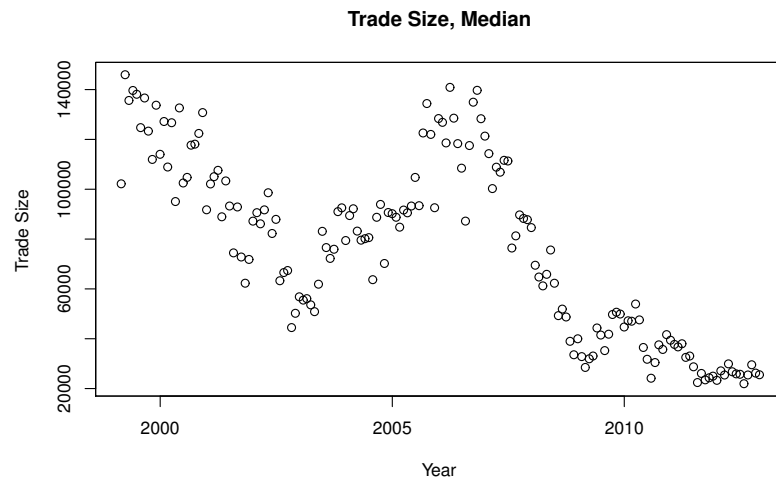
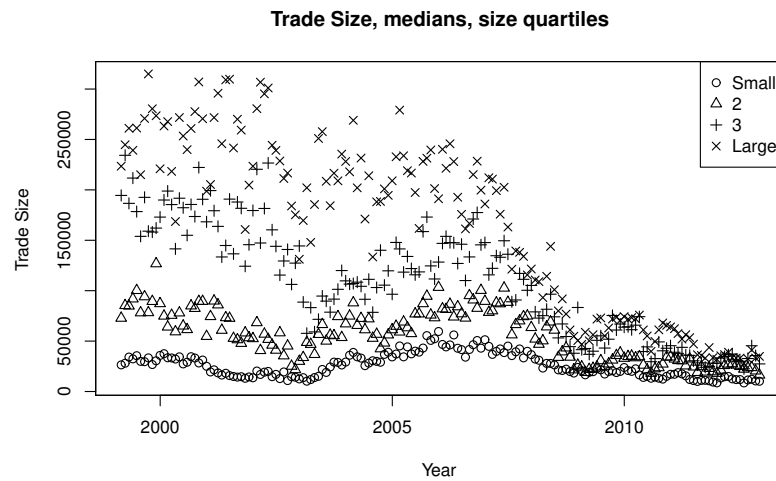


Figure 42 Time Series, Trade Size, Monthly Median

Panel A: Crossectional Median



Panel B: Crossectional Median, Size sorted portfolios.

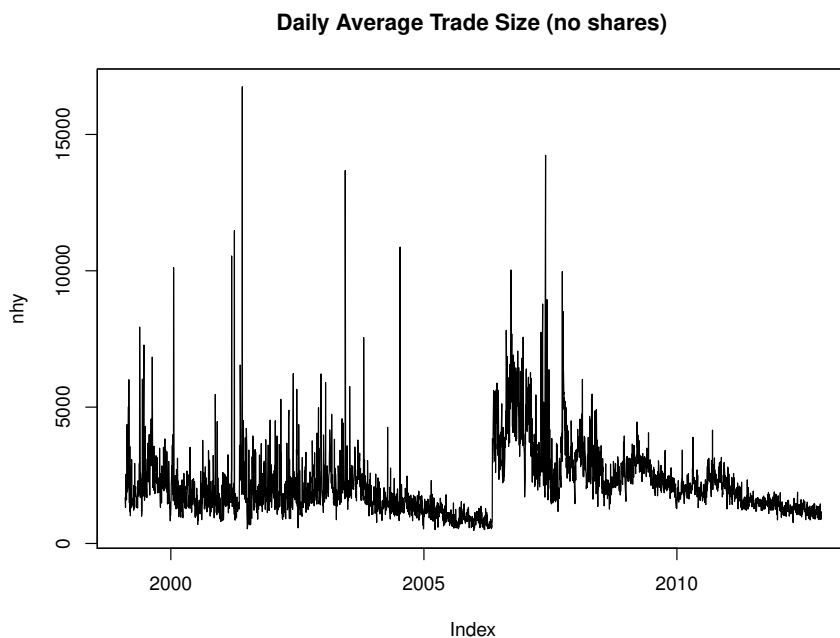


Time series plot of median trade size (in NOK) across stocks on the Oslo Stock Exchange.

Figure 43 Example, Single companies

This figure illustrates the evolution of the typical trade size for two example stocks, Norsk Hydro and Statoil. For each of the stocks we have removed one day with a single, extremely large transaction.

Panel A: Norsk Hydro



Panel B: Statoil

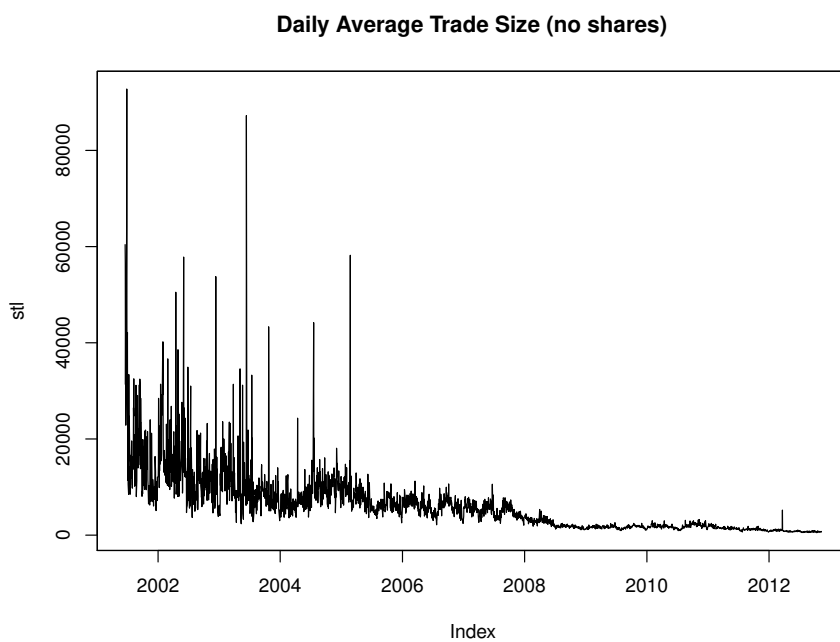
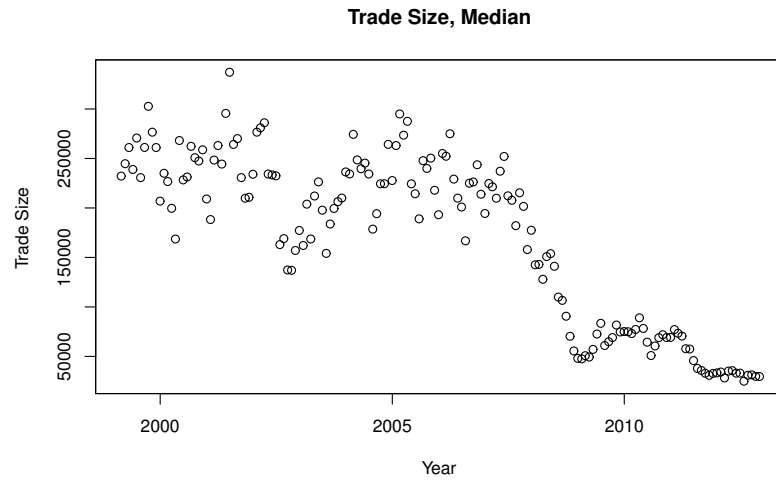
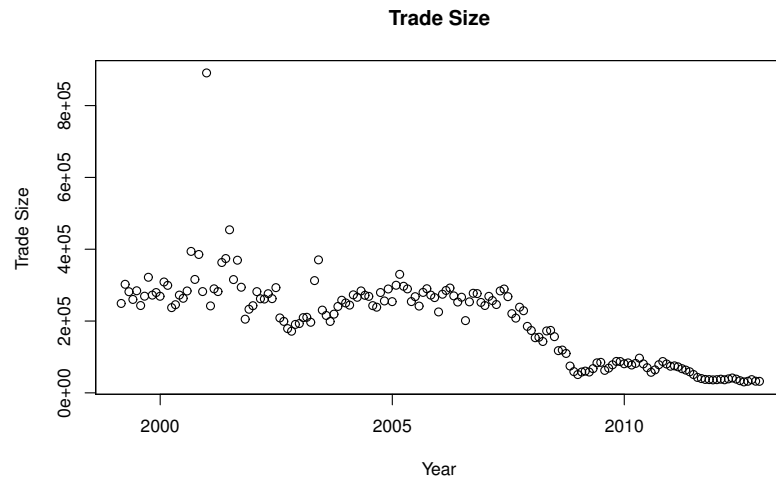


Figure 44 Time Series, Trade Size, Monthly, OBX Constituents

Panel A: Crossectional Median



Panel B: Crossectional Average(trimmed)



Time series plot of median trade size (in NOK) across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

5.2 Message to Trade Ratio

Modern Equity Markets involve Massive Amounts of order submission, withdrawals of orders, and order modification. A common measure that informs us about this is the Message to Trade ratio, also called the Order to Trade Ratio. This counts the numbers of messages submitted to the limit order book per consummated transaction.

Table 9 Descriptive, MTR

Period									Size quartiles(means)			
	min	Q1	med	mean	(std)	Q3	max	n	1(small)	2	3	4
1999-2012	1	4	7	12	(41)	10	1634	2182	10	9	9	17
1999	1	3	3	4	(1)	4	9	171	4	4	3	3
2000	1	3	4	4	(2)	5	20	169	5	4	4	3
2001	2	3	4	5	(2)	6	19	156	8	6	5	4
2002	2	4	6	6	(3)	8	21	138	9	8	7	4
2003	2	5	7	7	(3)	9	36	131	9	9	7	5
2004	2	4	6	7	(4)	9	41	143	8	7	6	6
2005	2	4	6	7	(4)	8	60	182	8	8	7	6
2006	2	4	7	8	(7)	9	132	202	10	8	8	7
2007	2	5	7	9	(16)	10	429	212	10	9	9	9
2008	2	6	8	15	(56)	12	1634	200	12	13	13	19
2009	3	7	9	14	(40)	11	830	179	13	9	9	23
2010	3	8	10	19	(48)	14	1117	204	13	16	21	37
2011	3	9	12	29	(90)	17	1250	208	11	14	12	51
2012	3	9	12	24	(65)	18	1028	196	11	12	14	44

Figure 45 Histogram Message to Trade Ratio - Subperiod

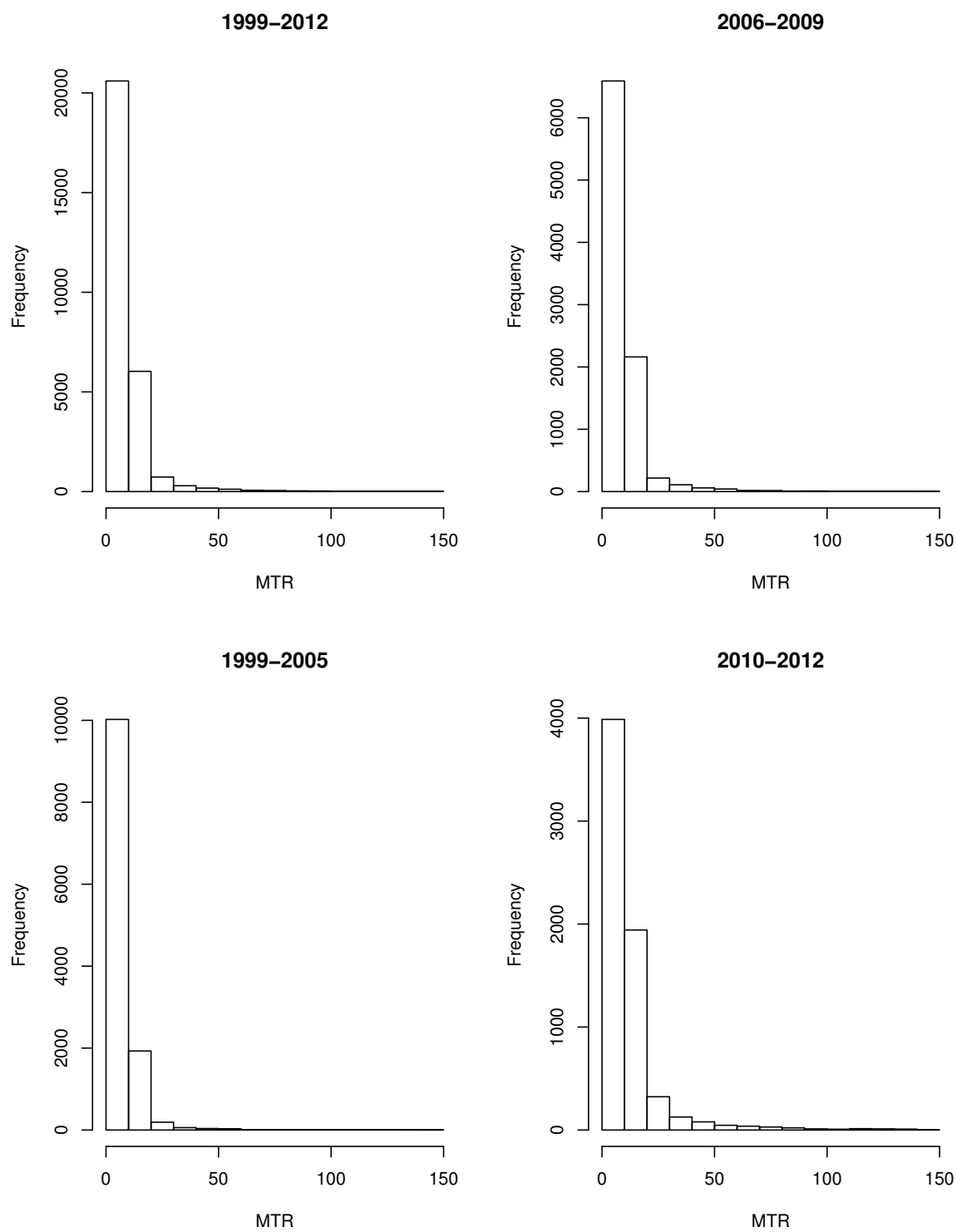


Figure 46 Histogram Message to Trade Ratio - By Size

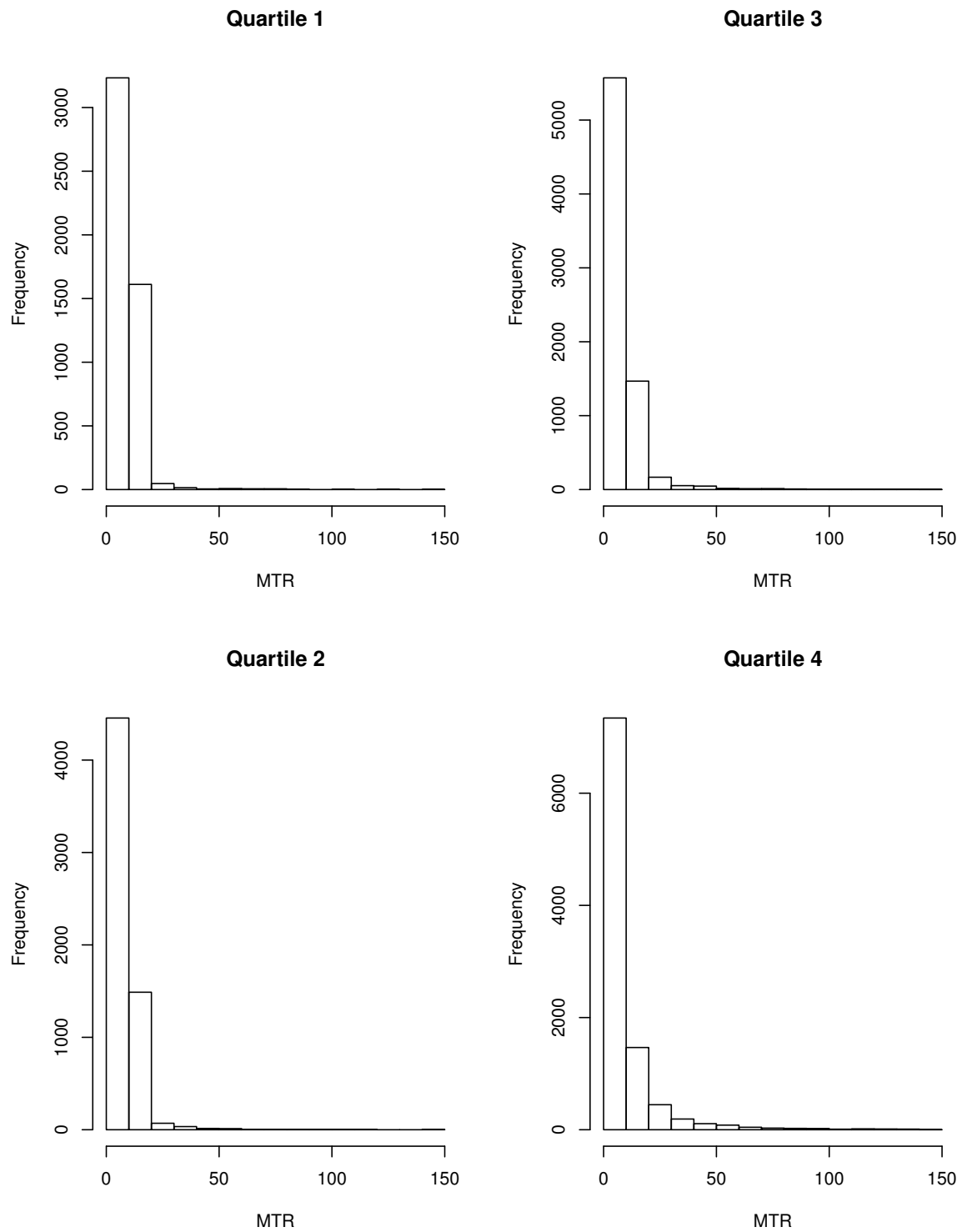
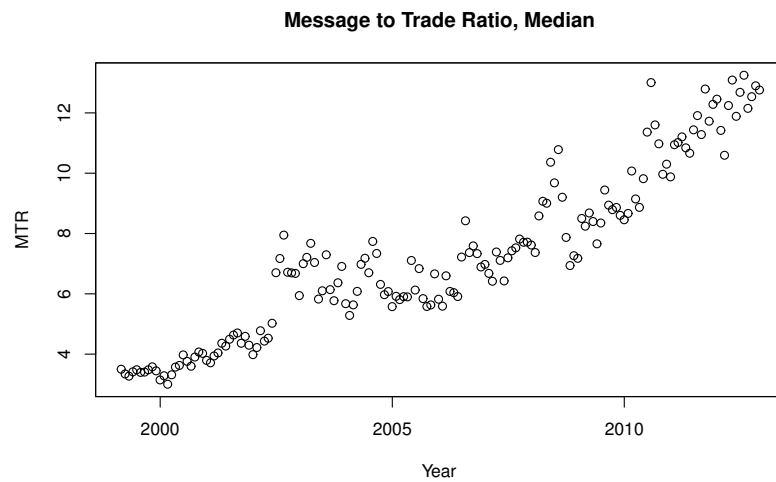
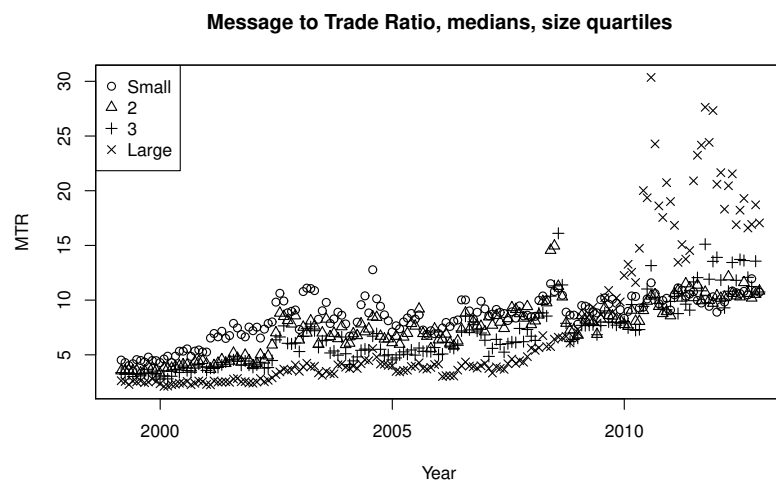


Figure 47 Time Series, Message to Trade Ratio, Monthly Median

Panel A: Crossectional Median



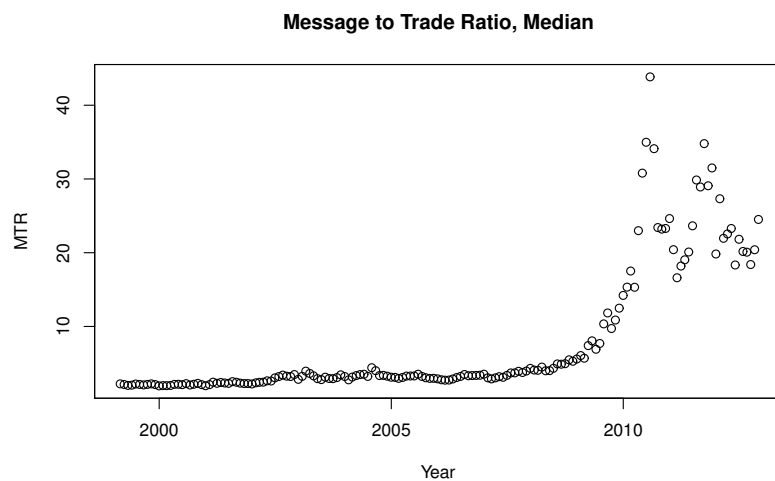
Panel B: Crossectional Median, Size sorted portfolios.



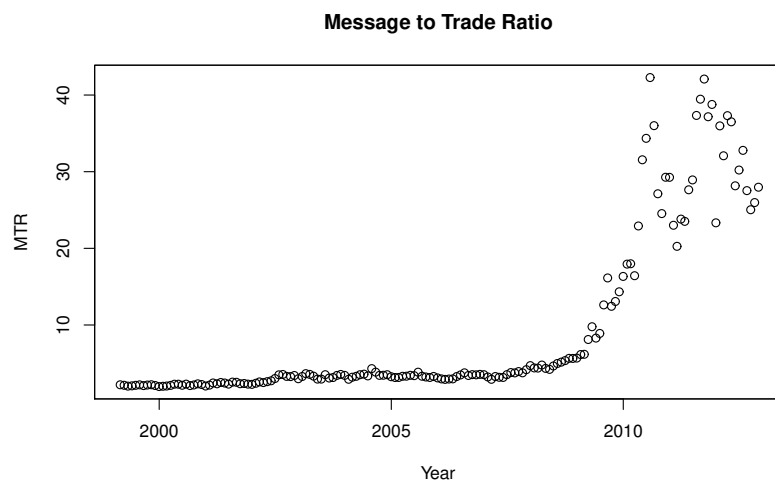
Time series plot of Message to Trade Ratios across stocks on the Oslo Stock Exchange.

Figure 48 Time Series, Message to Trade Ratio, Monthly, OBX Constituents

Panel A: Crossectional Median



Panel B: Crossectional Mean (trimmed)

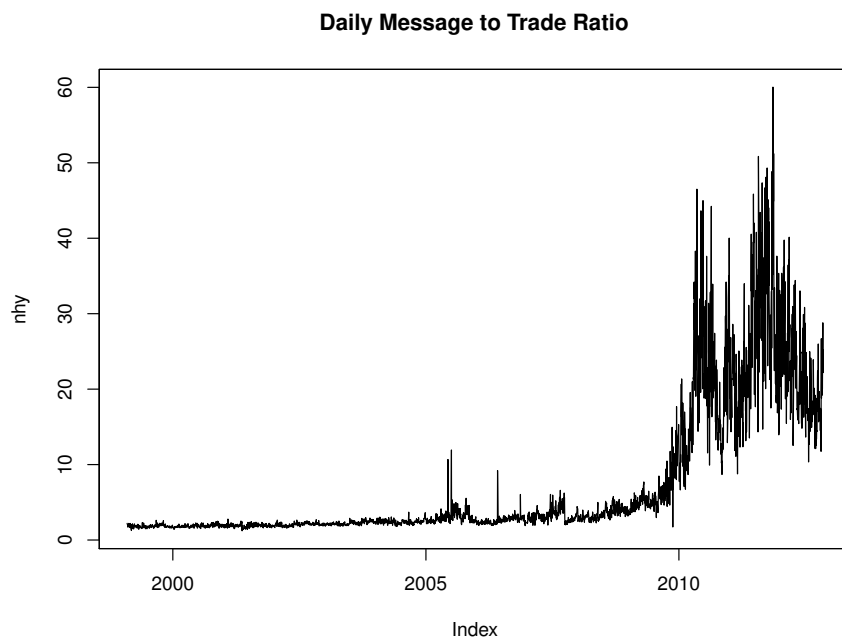


Time series plot of Message to Trade Ratios across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

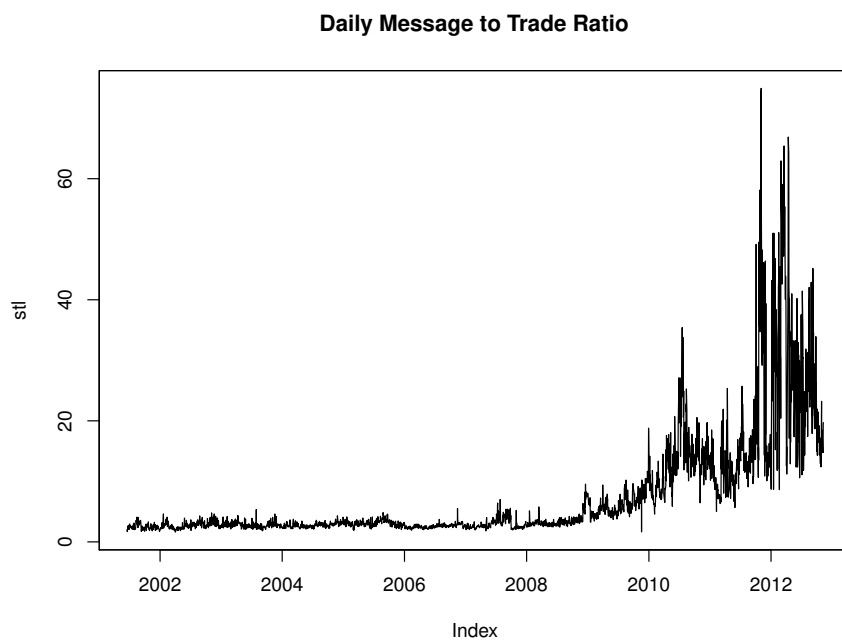
Figure 49 Example, Single companies

This figure illustrates the evolution of the message to trade ratio for two example stocks, Norsk Hydro and Statoil.

Panel A: Norsk Hydro



Panel B: Statoil



5.3 Depth

Depth measures the amount of order interest available at the best bid and best ask. One can calculate depth either as the number of shares available at the best bid or ask, or the kroner volume available at the best bid and ask. To make comparisons across stocks one typically uses the latter definition.

Table 10 Descriptive, Depth in NOK

Numbers in thousands.

Period									Size quartiles(means)			
	min	Q1	med	mean	(std)	Q3	max	n	1(small)	2	3	4
1999-2012	6	153	343	1438	(17278)	775	1355805	2983	215	445	726	3182
1999	28	255	499	2114	(22543)	958	731018	171	238	403	1476	4266
2000	28	232	507	1189	(7576)	974	264983	169	274	822	916	2668
2001	15	147	410	1607	(14385)	883	388886	156	74	1346	374	2638
2002	7	122	306	1058	(3359)	750	51883	139	123	191	730	1944
2003	6	140	346	1674	(13955)	742	365694	134	314	264	475	3939
2004	27	214	414	1160	(3328)	877	45157	151	339	377	705	3257
2005	31	272	567	1539	(4792)	1150	95591	182	410	410	637	3054
2006	39	278	607	1726	(5143)	1364	89163	202	285	648	1121	3293
2007	46	277	601	5251	(51312)	1410	1355805	212	228	435	1813	13142
2008	19	139	297	748	(2498)	659	77591	200	141	268	607	1382
2009	12	111	221	361	(516)	431	11396	179	119	251	294	627
2010	6	98	197	418	(1990)	426	86946	204	214	582	417	787
2011	9	98	182	485	(4647)	383	204046	208	108	215	261	858
2012	6	91	189	471	(5276)	347	239558	196	100	212	385	835

Figure 50 Histogram Depth in NOK - Subperiod

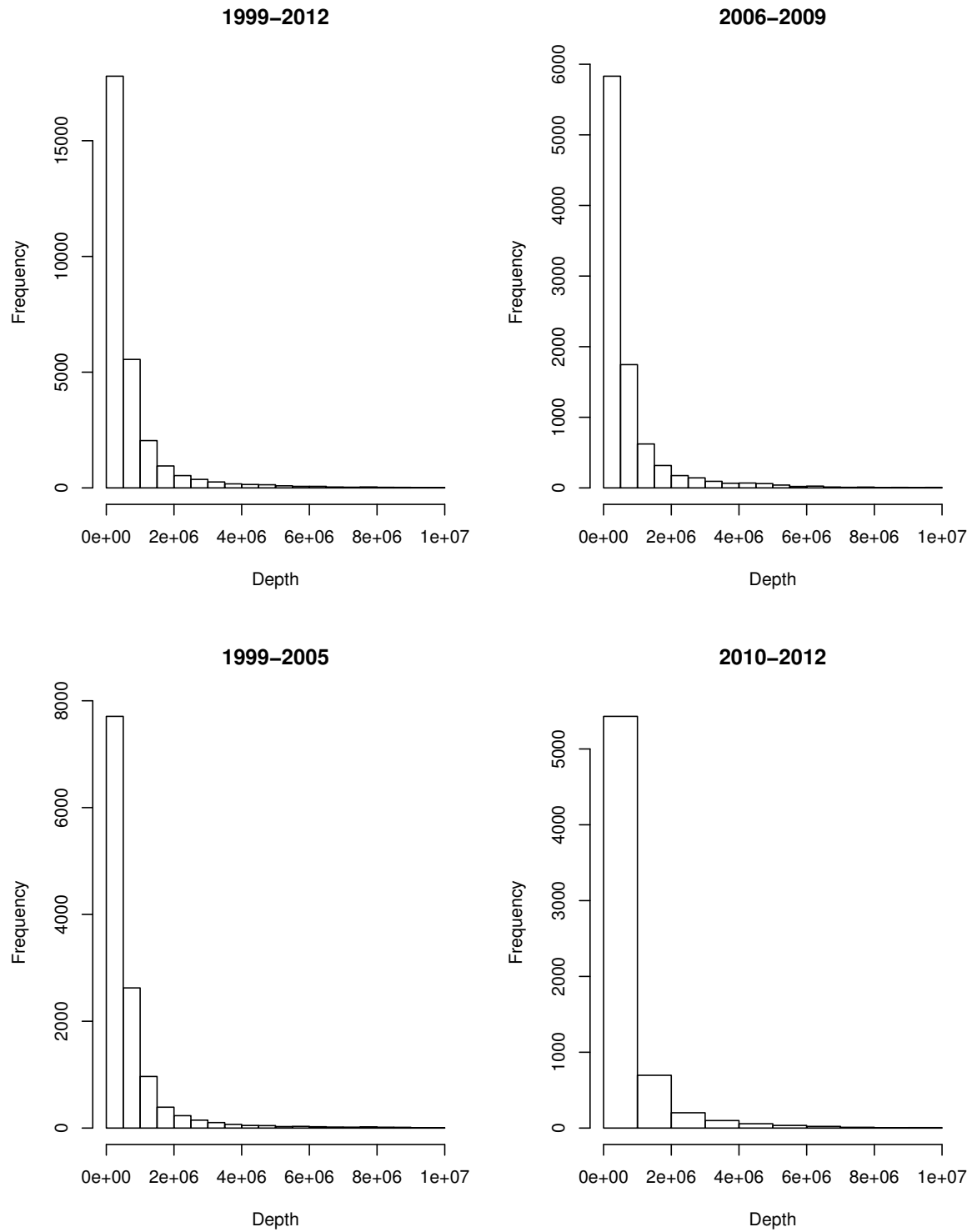


Figure 51 Histogram Depth in NOK - By Size

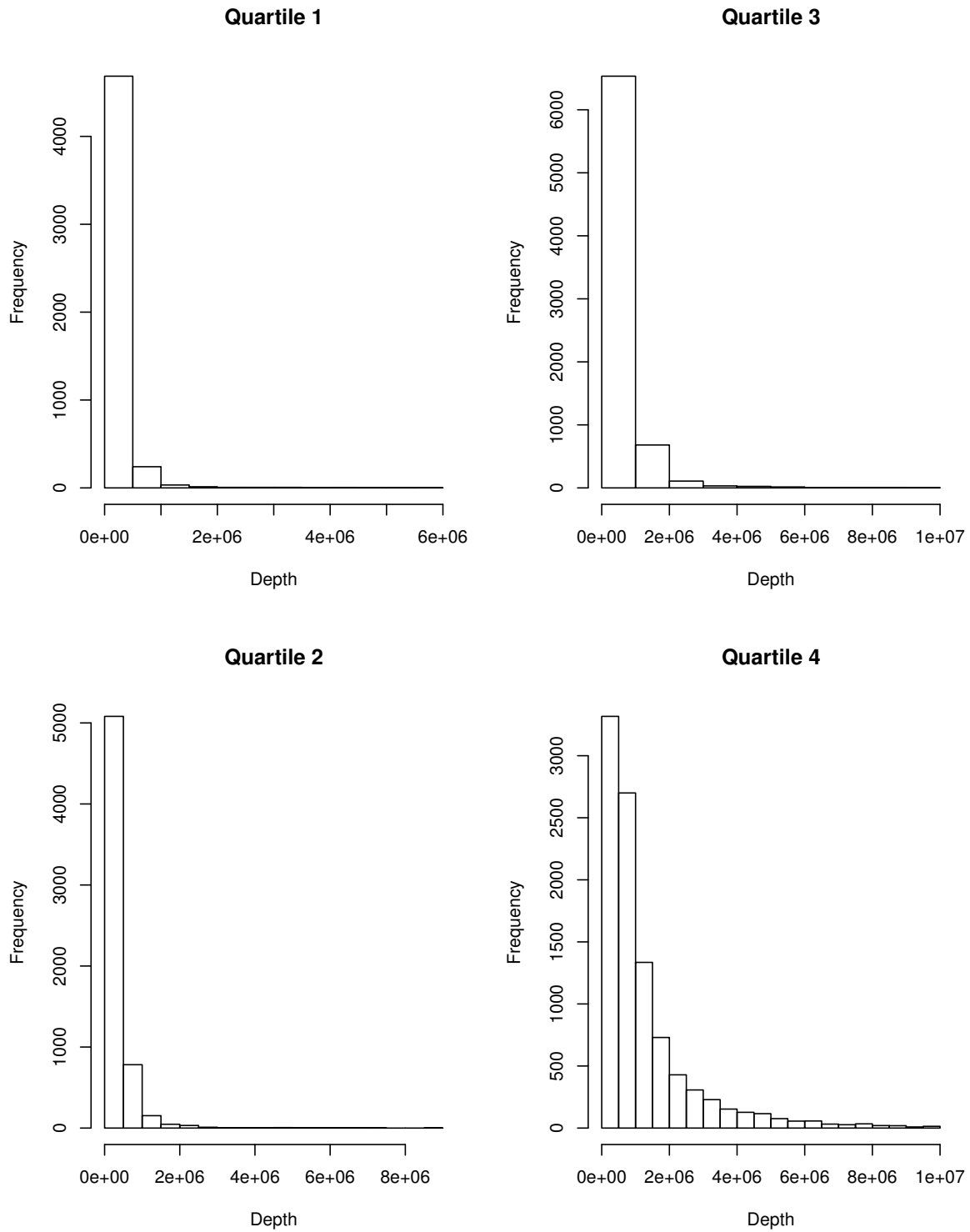
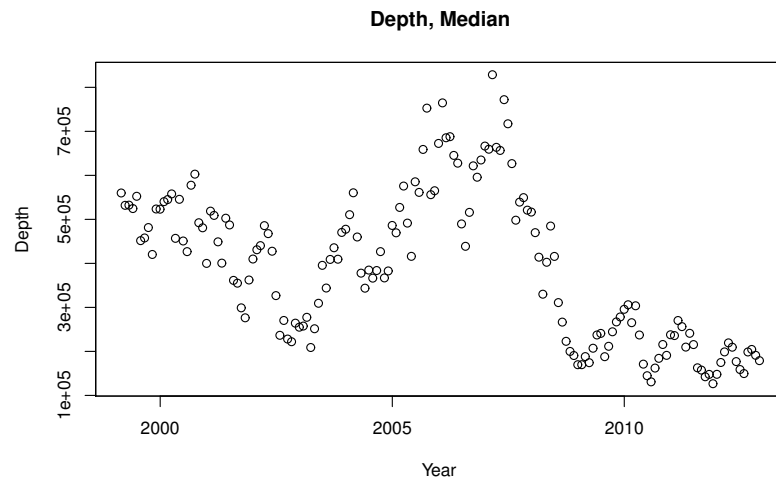
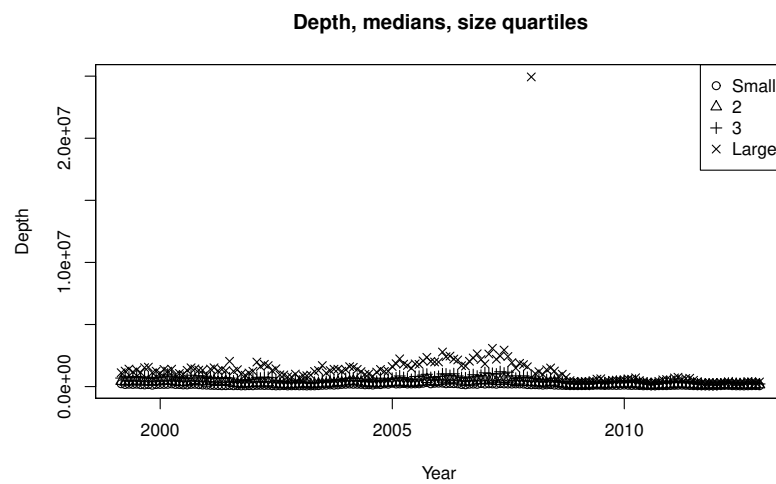


Figure 52 Time Series, Depth in NOK, Monthly Median

Panel A: Crossectional Median



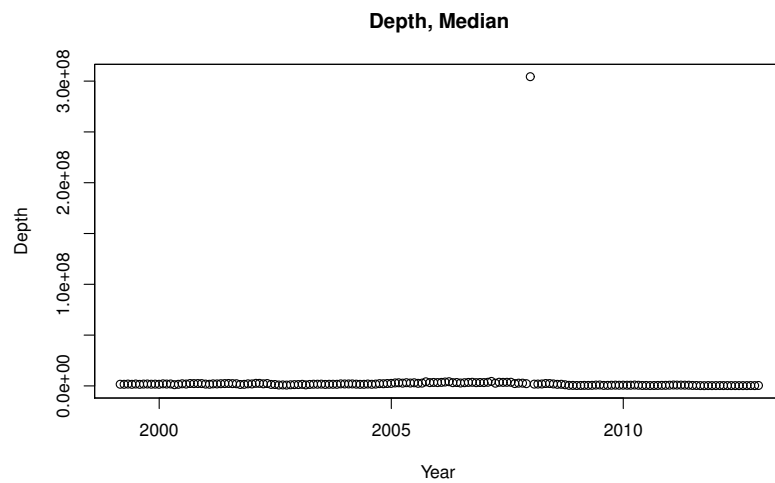
Panel B: Crossectional Median, Size sorted portfolios.



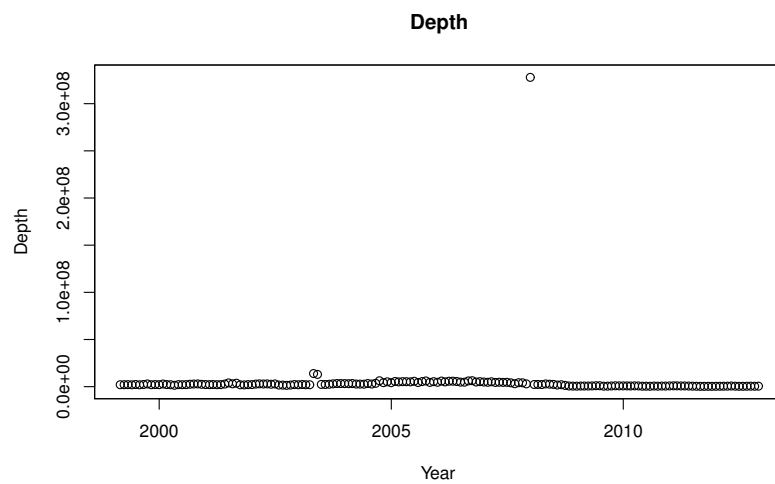
Time series plot of depth (in NOK) across stocks on the Oslo Stock Exchange.

Figure 53 Time Series, Depth, Monthly, OBX Constituents

Panel A: Crossectional Median



Panel B: Crossectional Mean (trimmed)



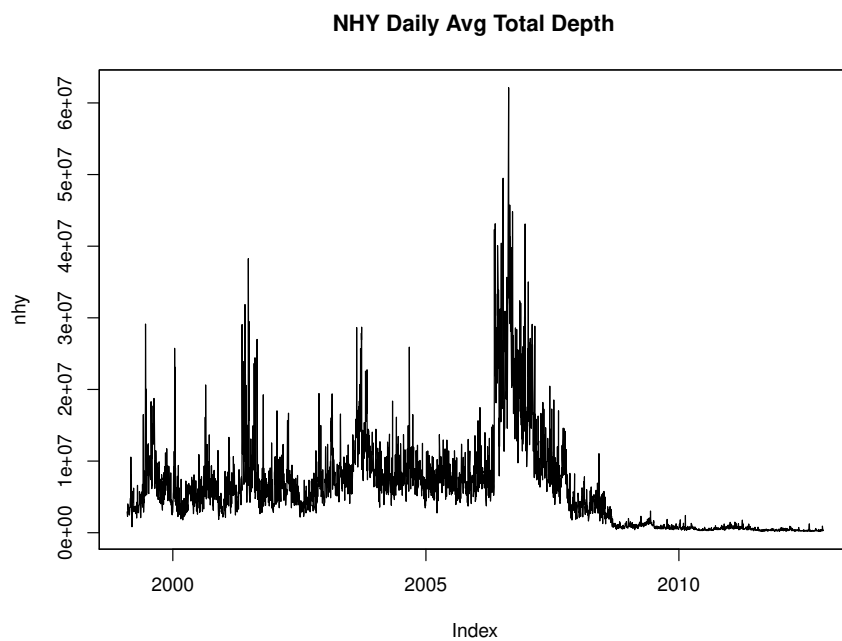
Time series plot of depth (in NOK) across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

For a given company one usually looks at the depth in number of shares. We show this for the usual two usual example companies in figure 54.

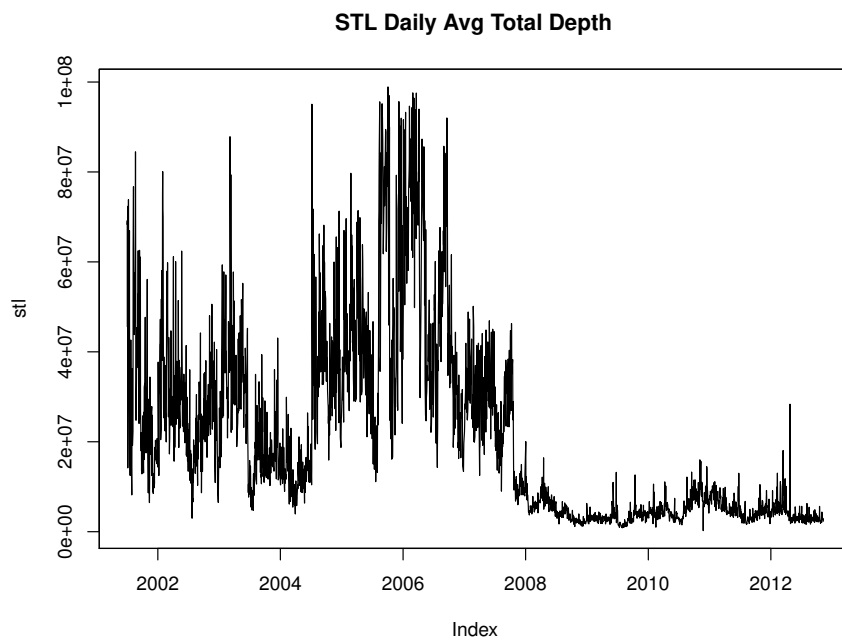
Figure 54 Example, Single companies

This figure illustrates the evolution of the depth (in number of shares) for two example stocks, Norsk Hydro and Statoil.

Panel A: Norsk Hydro



Panel B: Statoil



6 The LOT cost measure

Typical estimates of actual transaction costs of trading are calculated from microstructure data on actual trades. The goal of Lesmond, Ogden, and Trzcinka (1999) (LOT) is to find a measure of transaction costs that can be calculated using lower frequency data, such as daily returns. The idea of the model is to estimate a *threshold* where transaction costs are higher than the cost of *not* updating the price (by trading).

To understand the LOT measure, let us start by assuming returns are generated according the usual “market model”

$$\tilde{R}_{jt} = a_j + b_j \tilde{R}_{mt} + \tilde{\varepsilon}_{jt}$$

where $\tilde{R}_{jt}$ is the return on stock j at time t , $\tilde{R}_{mt}$ is the corresponding return on the market portfolio, a_j and b_j are (stock specific) constants, and $\tilde{\varepsilon}_{jt}$ an error term.

For any change in the market return $\tilde{R}_{mt}$ we should expect a corresponding change in the return $\tilde{R}_{jt}$ of stock j . If we now posit a (constant) transaction cost we would only expect a change in $\tilde{R}_{jt}$ when the change in R_{mt} is large enough to outweigh the transaction cost. Lesmond et al. (1999) propose a limited dependent variable model where observed returns $\tilde{R}_{jt}^*$ are related to the “true” returns $\tilde{R}_{jt}$ as follows

$$R_{jt}^* = \beta_j R_{mt} + \varepsilon_{jt}$$

where

$$\begin{aligned} R_{jt} &= R_{jt}^* - \alpha_{1j} & \text{if } R_{jt}^* < \alpha_{1j} \\ R_{jt} &= 0 & \text{if } \alpha_{2j} \geq R_{jt}^* \geq \alpha_{1j} \\ R_{jt} &= R_{jt}^* - \alpha_{2j} & \text{if } R_{jt}^* > \alpha_{2j} \end{aligned}$$

The transaction costs are represented by the constants α_{1j} and α_{2j} for each stock j . The LOT measure of trading costs are found by estimating the thresholds α_{1j} and α_{2j} . These are found by a maximum likelihood formulation by assuming Gaussian errors. From this one gets estimates $\hat{\alpha}_{1j}$ and $\hat{\alpha}_{2j}$. The difference

$$\widehat{LOT}_j = \hat{\alpha}_{2j} - \hat{\alpha}_{1j}$$

is the estimate of the round trip transaction cost for this stock.

The maximum likelihood estimation uses the following likelihood function

$$\begin{aligned} \ln \ell_T &= \sum_1 \ln \frac{1}{\sqrt{2\pi\sigma_j^2}} - \sum_1 \frac{1}{2\sigma_j^2} (R_{jt} + \alpha_{1j} - \beta_j R_{mt})^2 \\ &+ \sum_2 \ln \frac{1}{\sqrt{2\pi\sigma_j^2}} - \sum_2 \frac{1}{2\sigma_j^2} (R_{jt} + \alpha_{2j} - \beta_j R_{mt})^2 \\ &+ \sum_0 \ln (\Phi_{j2} - \Phi_{j1}) \end{aligned}$$

Here the summation are over three different sets. 0 is the set of cases where the return of the stock is zero. 1 where the market return is positive, and 2 where the market return is negative. Φ_{j2} and Φ_{j1} are the cumulative normal functions.

To see where this is coming from, let us for comparison look at the similar likelihood function for maximum likelihood estimation of a univariate regression, ie. the market model.

$$\ln \ell_T = \sum \ln \frac{1}{\sqrt{2\pi\sigma_j^2}} + \sum \frac{1}{2\sigma_j^2} (R_{jt} - \alpha_j - \beta_j R_{mt})^2$$

which simplifies to

$$\ln \ell_T = -T \frac{1}{2} \ln(2\pi) - T \ln(\sigma_j) - \frac{1}{2} \sum \frac{(R_{jt} - \alpha_j - \beta_j R_{mt})^2}{\sigma_j^2}$$

So much of the likelihood function for LOT estimation is the same as the one for estimating the ordinary (least squares) regression, the special case is what allows estimation of α_{1j} and α_{2j} .

6.1 Quarterly estimates of LOT

Table 11 Descriptive, LOT

Period	min	Q1	med	mean	(std)	Q3	max	n	Size quartiles(means)			
									1(small)	2	3	4
1980-2016	0.000	0.020	0.030	0.038	(0.031)	0.046	0.500	843	0.056	0.045	0.038	0.027
1980	0.009	0.019	0.026	0.029	(0.013)	0.034	0.081	14				0.029
1981	0.011	0.024	0.033	0.038	(0.021)	0.045	0.157	22				0.038
1982	0.011	0.024	0.032	0.035	(0.017)	0.042	0.103	24			0.052	0.032
1983	0.012	0.028	0.041	0.045	(0.024)	0.055	0.206	54	0.081	0.063	0.051	0.037
1984	0.009	0.026	0.036	0.040	(0.019)	0.047	0.143	86	0.062	0.050	0.036	0.032
1985	0.008	0.021	0.031	0.035	(0.020)	0.042	0.188	114	0.069	0.047	0.037	0.028
1986	0.006	0.021	0.030	0.035	(0.021)	0.044	0.182	113	0.059	0.044	0.035	0.028
1987	0.003	0.023	0.034	0.042	(0.030)	0.051	0.289	106	0.060	0.069	0.038	0.032
1988	0.006	0.028	0.042	0.049	(0.036)	0.060	0.369	74	0.069	0.056	0.047	0.042
1989	0.004	0.025	0.038	0.044	(0.029)	0.056	0.232	100	0.067	0.058	0.042	0.033
1990	0.008	0.027	0.039	0.045	(0.032)	0.057	0.477	102	0.059	0.072	0.043	0.035
1991	0.005	0.025	0.038	0.047	(0.039)	0.055	0.427	95		0.075	0.053	0.040
1992	0.008	0.028	0.045	0.066	(0.064)	0.077	0.426	83	0.066	0.104	0.080	0.043
1993	0.006	0.025	0.038	0.052	(0.049)	0.061	0.423	100	0.075	0.084	0.048	0.027
1994	0.008	0.021	0.028	0.035	(0.023)	0.042	0.242	111	0.047	0.047	0.035	0.026
1995	0.006	0.021	0.028	0.035	(0.028)	0.039	0.334	122	0.116	0.046	0.034	0.027
1996	0.003	0.020	0.027	0.032	(0.020)	0.037	0.227	143	0.044	0.036	0.032	0.022
1997	0.000	0.021	0.030	0.034	(0.018)	0.042	0.151	168	0.048	0.038	0.036	0.024
1998	0.006	0.028	0.042	0.049	(0.034)	0.060	0.391	170	0.073	0.060	0.052	0.034
1999	0.004	0.027	0.041	0.049	(0.035)	0.061	0.408	172	0.074	0.057	0.046	0.032
2000	0.002	0.027	0.040	0.045	(0.028)	0.056	0.306	172	0.059	0.047	0.040	0.033
2001	0.000	0.027	0.040	0.048	(0.030)	0.061	0.202	163	0.075	0.059	0.049	0.038
2002	0.000	0.024	0.038	0.046	(0.032)	0.058	0.213	139	0.069	0.057	0.047	0.032
2003	0.002	0.025	0.036	0.046	(0.035)	0.054	0.370	134	0.064	0.056	0.045	0.031
2004	0.002	0.020	0.026	0.031	(0.019)	0.038	0.220	151	0.042	0.034	0.028	0.021
2005	0.000	0.019	0.026	0.030	(0.016)	0.036	0.122	184	0.048	0.037	0.031	0.022
2006	0.000	0.018	0.025	0.027	(0.015)	0.033	0.171	205	0.036	0.031	0.030	0.021
2007	0.000	0.017	0.023	0.025	(0.013)	0.032	0.105	223	0.033	0.029	0.028	0.018
2008	0.000	0.022	0.032	0.041	(0.035)	0.049	0.500	223	0.056	0.043	0.044	0.027
2009	0.000	0.022	0.034	0.045	(0.042)	0.054	0.448	198	0.070	0.055	0.044	0.023
2010	0.000	0.017	0.027	0.034	(0.029)	0.042	0.286	207	0.045	0.035	0.022	0.014
2011	0.000	0.018	0.027	0.036	(0.035)	0.042	0.457	210	0.056	0.050	0.040	0.022
2012	0.000	0.018	0.027	0.036	(0.037)	0.041	0.399	198	0.079	0.043	0.031	0.018
2013	0.000	0.015	0.023	0.032	(0.033)	0.037	0.337	197	0.054	0.031	0.027	0.012
2014	0.000	0.014	0.021	0.029	(0.027)	0.033	0.247	202	0.044	0.034	0.028	0.014
2015	0.000	0.016	0.025	0.032	(0.026)	0.038	0.211	205	0.051	0.037	0.033	0.018
2016	0.000	0.016	0.025	0.033	(0.031)	0.041	0.495	207	0.049	0.039	0.031	0.018

Figure 55 Histogram LOT - Subperiod

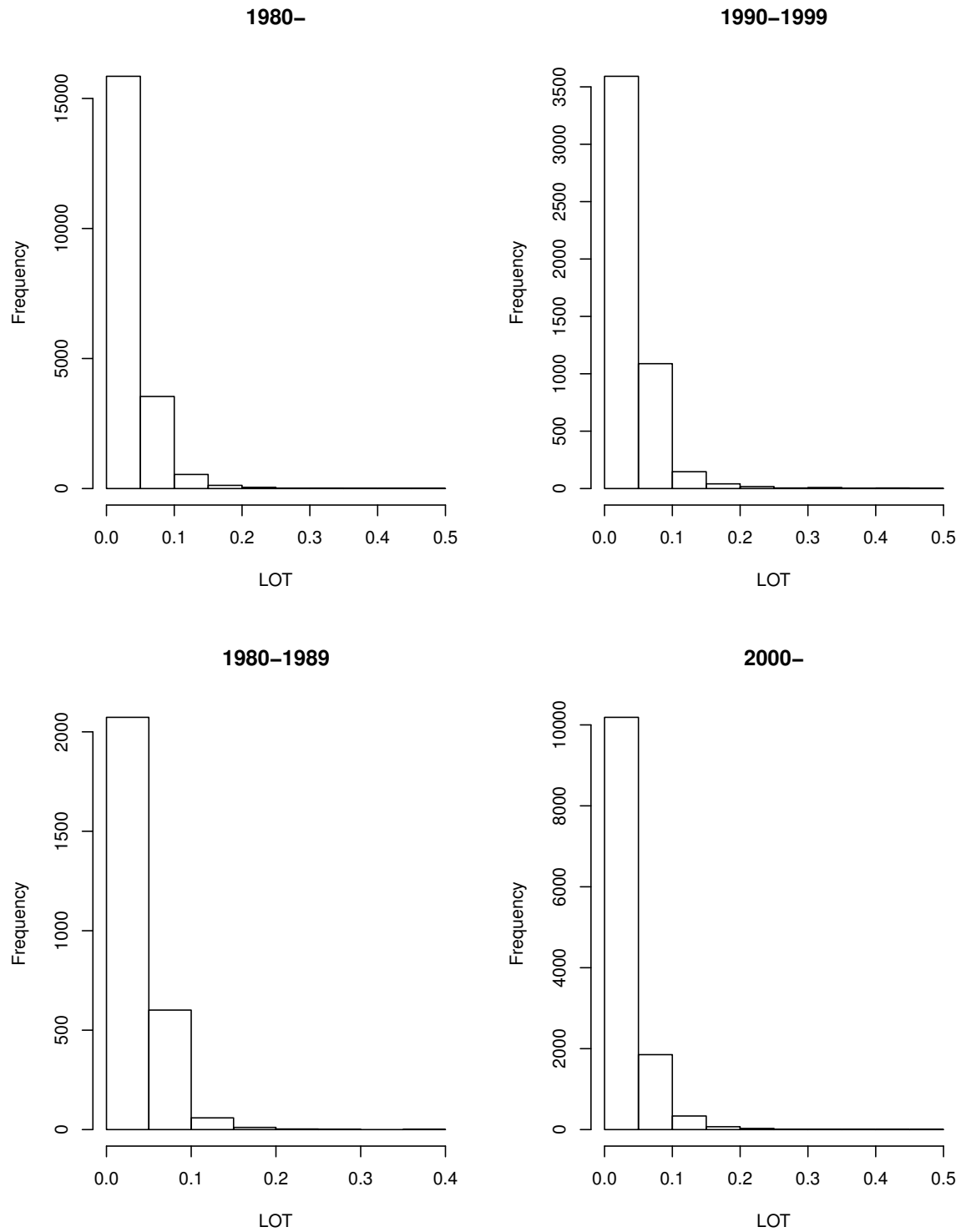


Figure 56 Histogram LOT - By Size

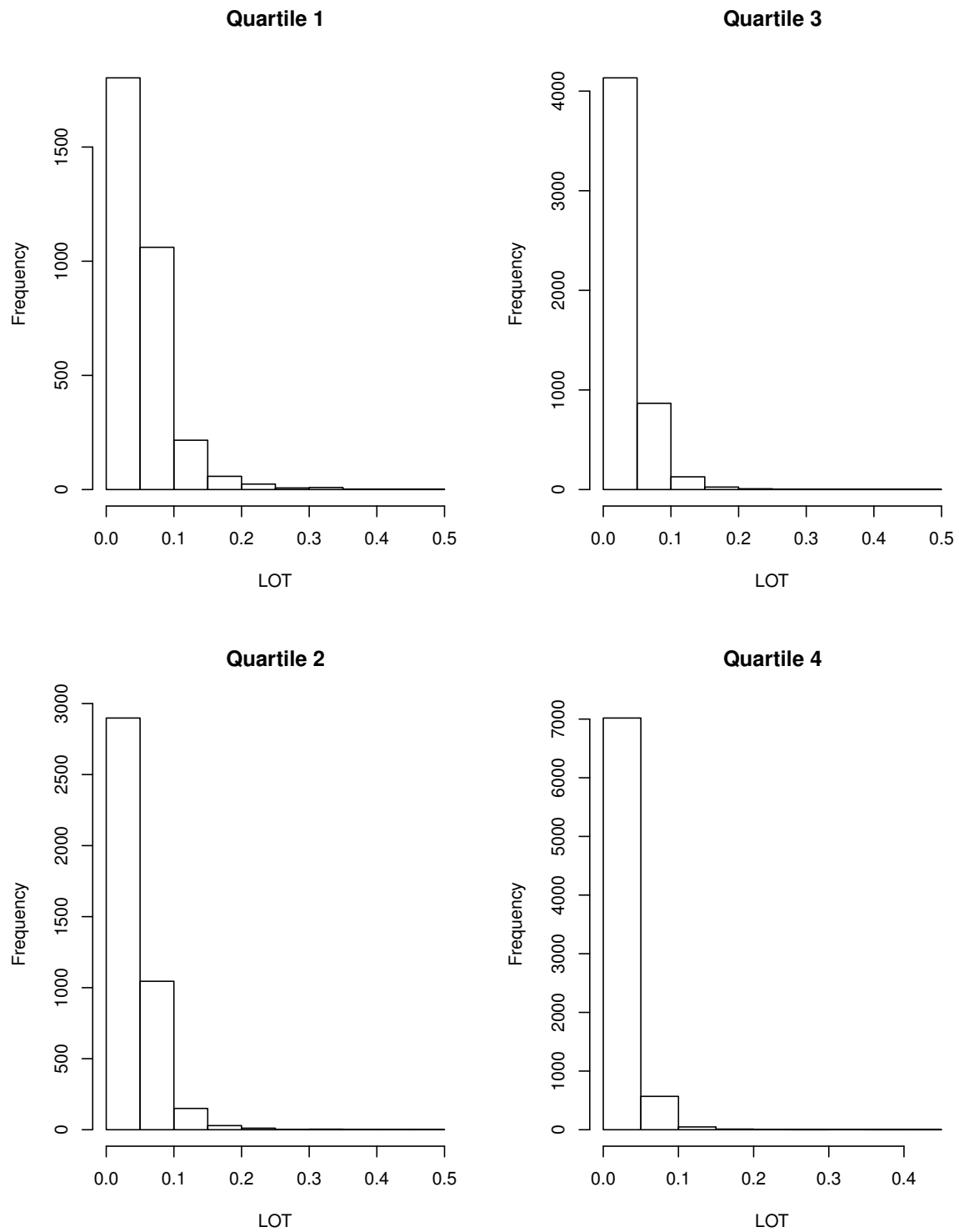
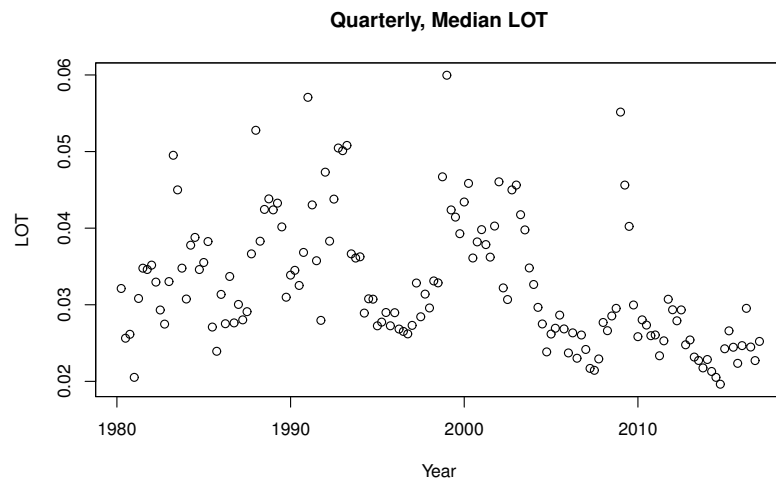
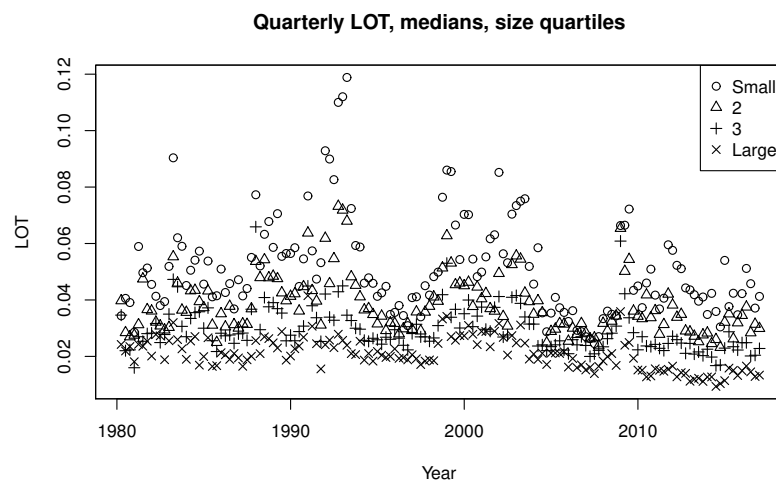


Figure 57 Time Series, LOT, Quarterly Median

Panel A: Crossectional Median



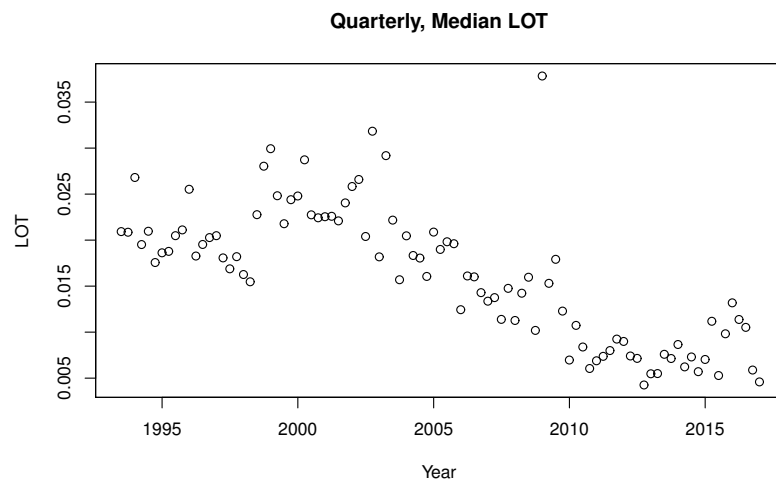
Panel B: Crossectional Median, Size sorted portfolios.



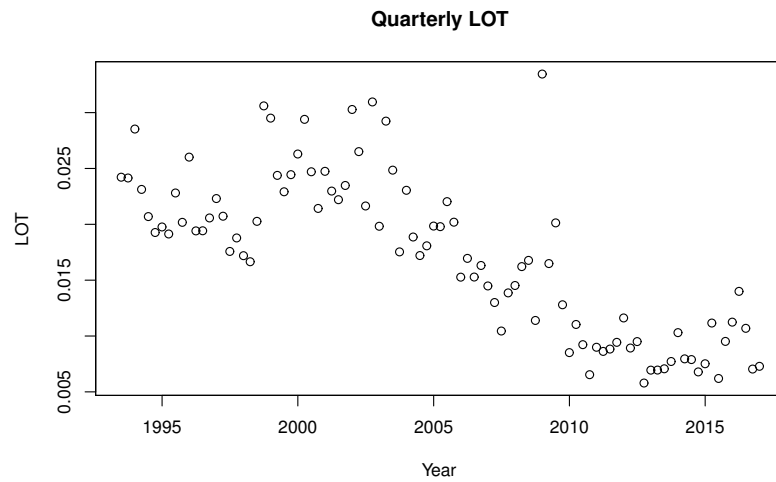
Time series plot of median lot across stocks on the Oslo Stock Exchange.

Figure 58 Time Series, LOT, Quarterly Median, OBX Constituents

Panel A: Crossectional Median



Panel B: Crossectional Mean



Time series plot of median lot across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

7 The Roll Liquidity Measure

We calculate the Roll estimate of spread using daily (closing) returns, and intraday.

7.1 Quarterly estimates of Roll, using daily returns

Table 12 Descriptive, Roll

Period	min	Q1	med	mean	(std)	Q3	max	n	Size quartiles(means)			
									1(small)	2	3	4
1980-2016	0.000	0.013	0.021	0.028	(0.028)	0.035	0.619	759	0.046	0.033	0.026	0.018
1980	0.000	0.004	0.008	0.012	(0.011)	0.014	0.053	13				0.012
1981	0.003	0.008	0.018	0.019	(0.013)	0.026	0.056	16				0.019
1982	0.005	0.008	0.016	0.019	(0.014)	0.026	0.070	19			0.032	0.016
1983	0.002	0.011	0.017	0.024	(0.020)	0.034	0.122	32		0.031	0.027	0.020
1984	0.001	0.010	0.018	0.023	(0.021)	0.028	0.156	60	0.051	0.024	0.022	0.017
1985	0.002	0.009	0.015	0.018	(0.013)	0.024	0.073	90	0.032	0.022	0.020	0.013
1986	0.002	0.012	0.018	0.022	(0.016)	0.026	0.115	93	0.036	0.024	0.022	0.018
1987	0.001	0.012	0.020	0.031	(0.030)	0.037	0.200	87	0.033	0.050	0.026	0.028
1988	0.001	0.015	0.025	0.030	(0.022)	0.041	0.126	73	0.046	0.036	0.031	0.023
1989	0.001	0.016	0.025	0.029	(0.019)	0.036	0.107	98	0.043	0.034	0.024	0.023
1990	0.000	0.014	0.022	0.026	(0.018)	0.032	0.094	95	0.040	0.036	0.023	0.019
1991	0.003	0.014	0.024	0.030	(0.022)	0.040	0.118	92		0.036	0.037	0.025
1992	0.001	0.017	0.028	0.041	(0.037)	0.053	0.238	80	0.040	0.062	0.044	0.031
1993	0.002	0.012	0.021	0.029	(0.030)	0.035	0.256	86	0.037	0.043	0.024	0.018
1994	0.002	0.012	0.017	0.022	(0.021)	0.026	0.258	109	0.031	0.026	0.019	0.017
1995	0.001	0.009	0.016	0.018	(0.013)	0.023	0.113	112	0.038	0.026	0.019	0.015
1996	0.000	0.010	0.014	0.017	(0.013)	0.020	0.099	138	0.020	0.020	0.019	0.011
1997	0.001	0.011	0.018	0.022	(0.017)	0.028	0.115	157	0.033	0.022	0.024	0.015
1998	0.002	0.015	0.025	0.031	(0.023)	0.040	0.160	153	0.049	0.032	0.030	0.022
1999	0.002	0.014	0.025	0.034	(0.032)	0.042	0.312	163	0.058	0.033	0.027	0.019
2000	0.001	0.018	0.028	0.034	(0.025)	0.043	0.180	167	0.049	0.036	0.026	0.022
2001	0.002	0.017	0.026	0.038	(0.032)	0.049	0.213	160	0.075	0.051	0.035	0.024
2002	0.003	0.016	0.030	0.043	(0.043)	0.052	0.428	137	0.075	0.055	0.036	0.024
2003	0.002	0.014	0.022	0.033	(0.038)	0.038	0.426	130	0.061	0.039	0.031	0.019
2004	0.001	0.011	0.018	0.021	(0.015)	0.027	0.127	143	0.029	0.023	0.017	0.014
2005	0.001	0.010	0.016	0.020	(0.014)	0.024	0.100	171	0.031	0.026	0.020	0.014
2006	0.001	0.012	0.019	0.022	(0.015)	0.029	0.135	205	0.038	0.024	0.022	0.018
2007	0.001	0.011	0.017	0.019	(0.012)	0.024	0.123	219	0.028	0.022	0.019	0.014
2008	0.001	0.018	0.026	0.035	(0.031)	0.044	0.484	211	0.045	0.036	0.036	0.024
2009	0.001	0.017	0.026	0.033	(0.026)	0.039	0.193	192	0.048	0.037	0.028	0.023
2010	0.001	0.015	0.024	0.032	(0.037)	0.037	0.619	205	0.041	0.033	0.022	0.014
2011	0.001	0.015	0.025	0.033	(0.026)	0.042	0.165	202	0.046	0.041	0.036	0.019
2012	0.001	0.014	0.022	0.031	(0.028)	0.038	0.211	195	0.062	0.040	0.025	0.016
2013	0.001	0.011	0.018	0.028	(0.034)	0.033	0.373	192	0.049	0.029	0.018	0.010
2014	0.001	0.011	0.019	0.028	(0.032)	0.034	0.472	198	0.047	0.032	0.022	0.011
2015	0.001	0.013	0.020	0.029	(0.033)	0.036	0.411	199	0.053	0.034	0.024	0.017
2016	0.001	0.013	0.020	0.029	(0.027)	0.036	0.339	204	0.042	0.033	0.025	0.016

Figure 59 Histogram Roll - Subperiod

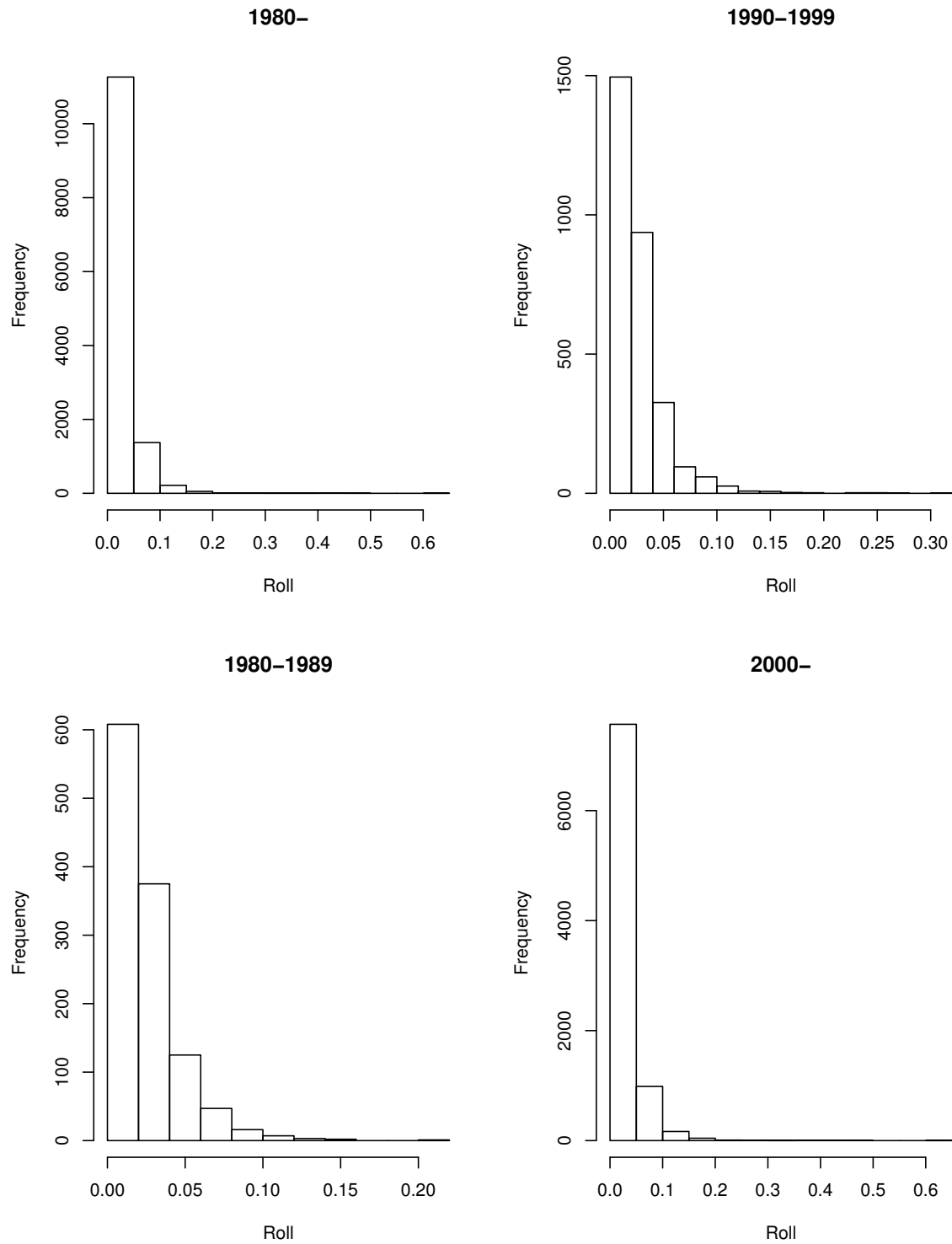


Figure 60 Histogram Roll - By Size

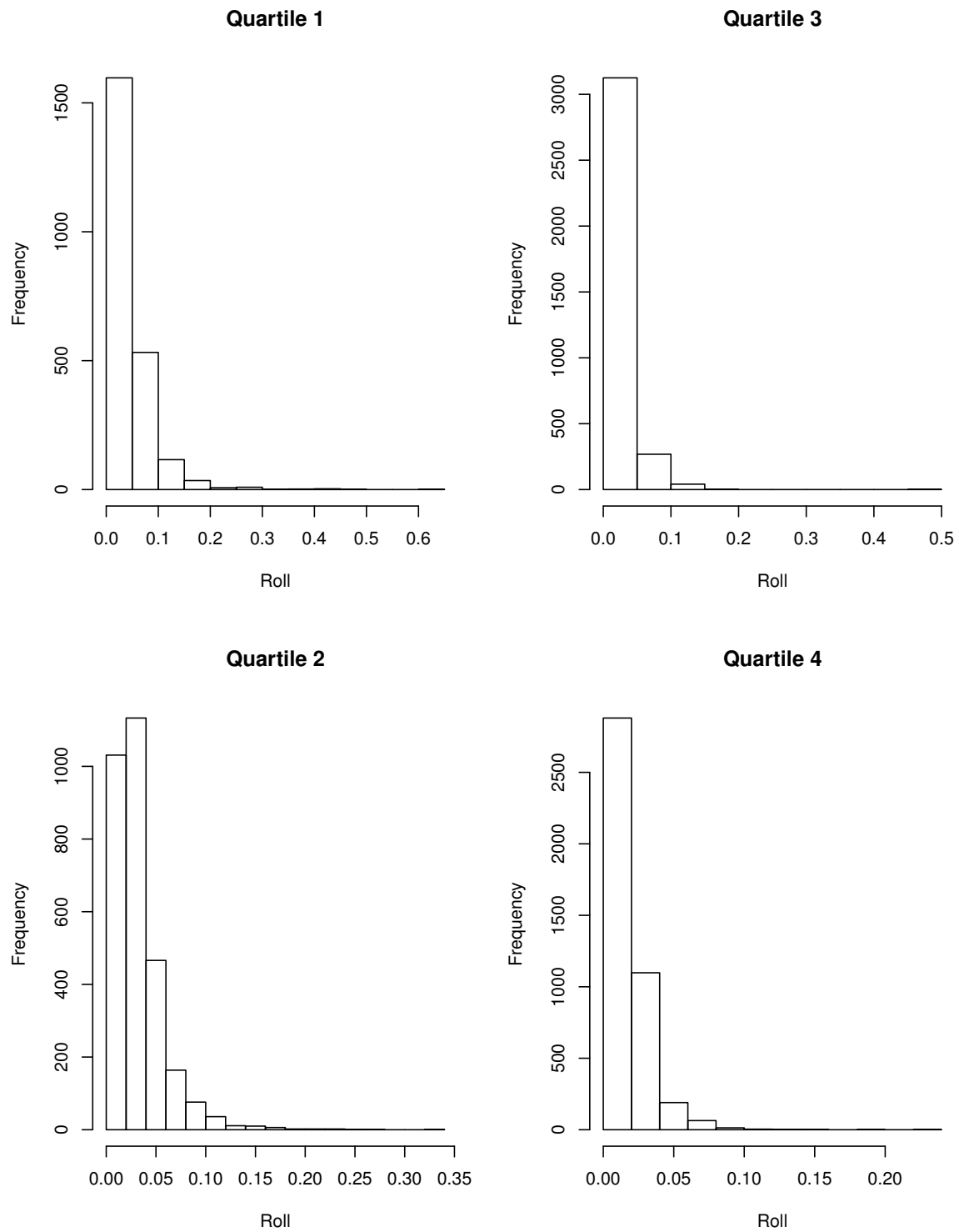
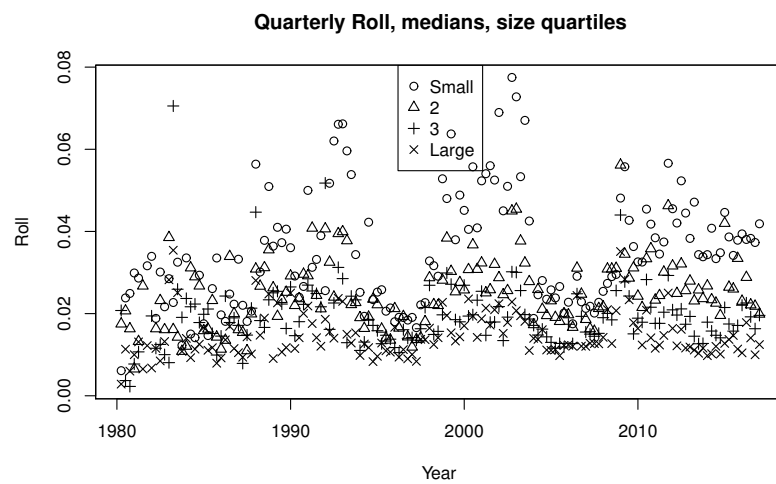


Figure 61 Time Series, Roll, Monthly Median

Panel A: Crossectional Median



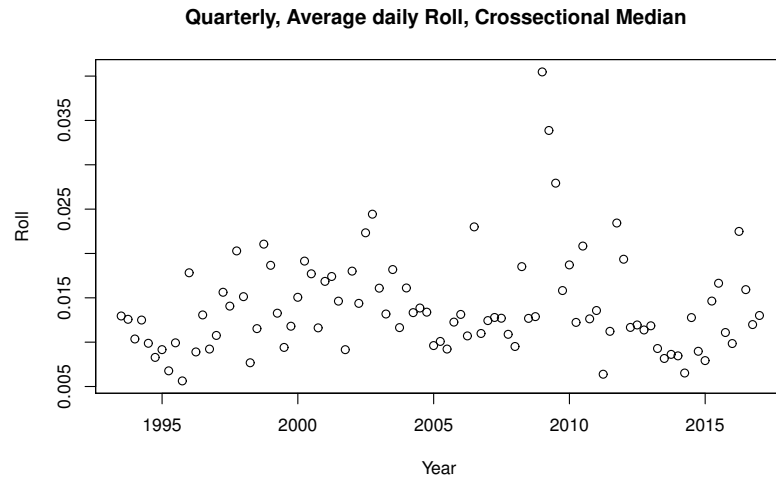
Panel B: Crossectional Median, Size sorted portfolios.



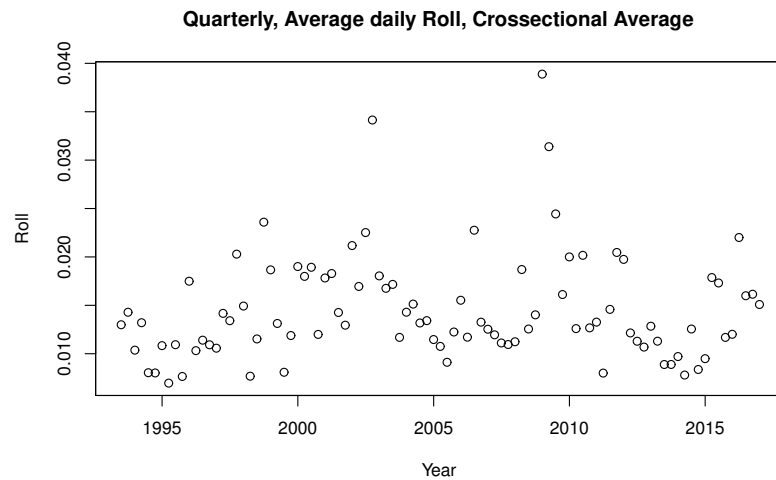
Time series plot of median Roll across stocks on the Oslo Stock Exchange.

Figure 62 Time Series, Roll, Monthly Median, OBX Constituents

Panel A: Crossectional Median



Panel B: Crossectional Average



The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

7.2 Monthly estimates of Roll, using intraday returns

Table 13 Descriptive, Roll

Period	min	Q1	med	mean	(std)	Q3	max	n	Size quartiles(means)			
									1(small)	2	3	4
1999-2012	0.000	0.003	0.005	0.008	(0.010)	0.009	0.174	588	0.014	0.011	0.009	0.006
1999	0.001	0.005	0.008	0.012	(0.014)	0.013	0.156	122	0.023	0.015	0.014	0.010
2000	0.001	0.006	0.009	0.012	(0.011)	0.013	0.157	136	0.016	0.013	0.012	0.009
2001	0.000	0.005	0.007	0.010	(0.011)	0.012	0.147	114	0.022	0.015	0.013	0.008
2002	0.000	0.005	0.007	0.011	(0.014)	0.011	0.171	102	0.024	0.015	0.012	0.008
2003	0.000	0.004	0.006	0.009	(0.010)	0.010	0.174	114	0.013	0.010	0.010	0.006
2004	0.000	0.003	0.005	0.007	(0.007)	0.008	0.125	133	0.010	0.007	0.006	0.005
2005	0.000	0.003	0.005	0.006	(0.006)	0.007	0.077	155	0.009	0.007	0.007	0.005
2006	0.000	0.003	0.005	0.006	(0.006)	0.007	0.103	175	0.009	0.007	0.007	0.005
2007	0.000	0.003	0.004	0.006	(0.005)	0.007	0.083	184	0.009	0.007	0.006	0.005
2008	0.000	0.004	0.006	0.008	(0.009)	0.010	0.115	155	0.015	0.013	0.009	0.006
2009	0.000	0.003	0.006	0.007	(0.007)	0.010	0.069	154	0.014	0.011	0.009	0.004
2010	0.000	0.003	0.005	0.008	(0.010)	0.010	0.144	170	0.014	0.009	0.005	0.003
2011	0.000	0.003	0.005	0.009	(0.011)	0.011	0.110	178	0.020	0.019	0.013	0.005
2012	0.000	0.002	0.004	0.008	(0.011)	0.009	0.142	159	0.031	0.014	0.008	0.004

Figure 63 Histogram Roll - Subperiod

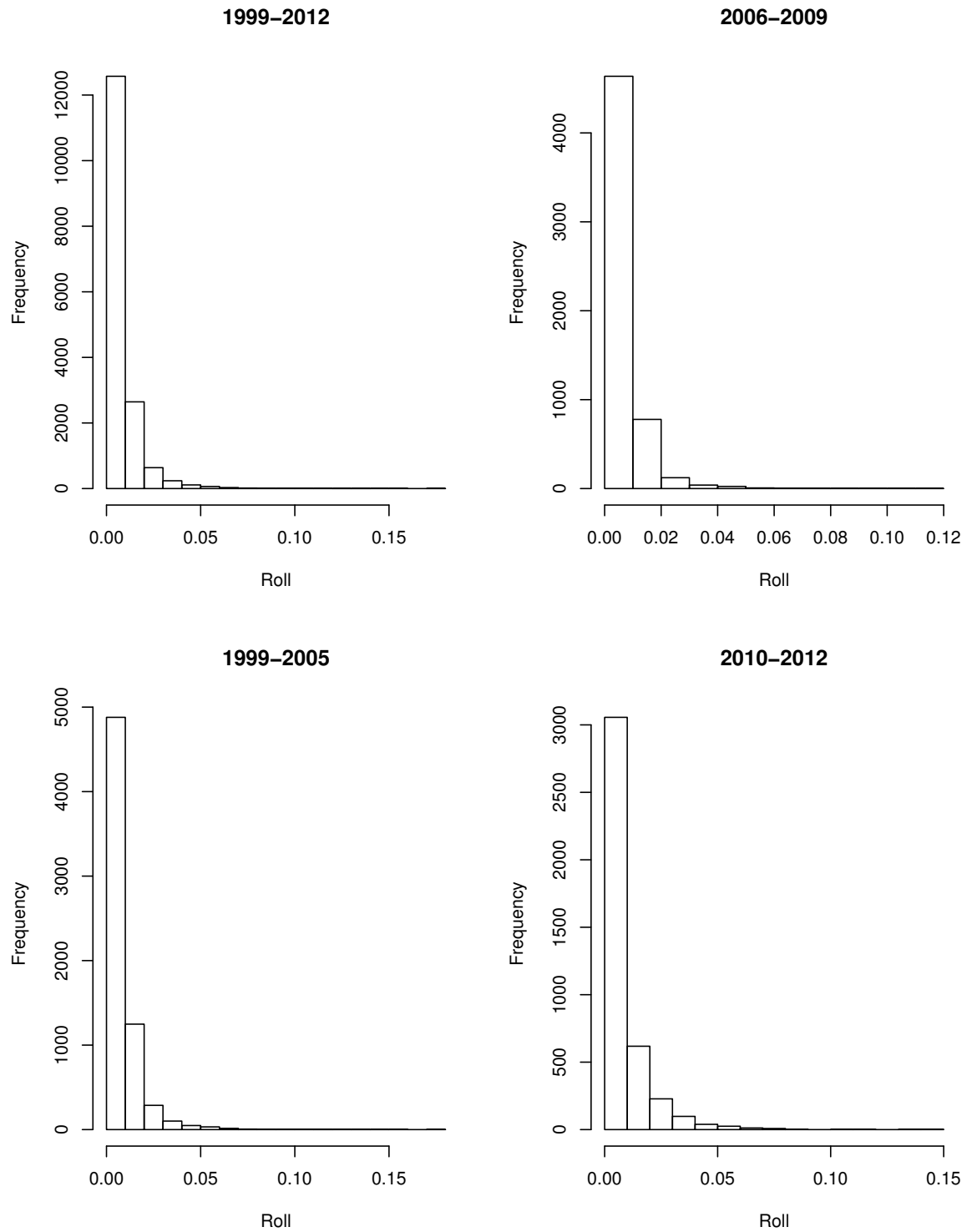


Figure 64 Histogram Roll - By Size

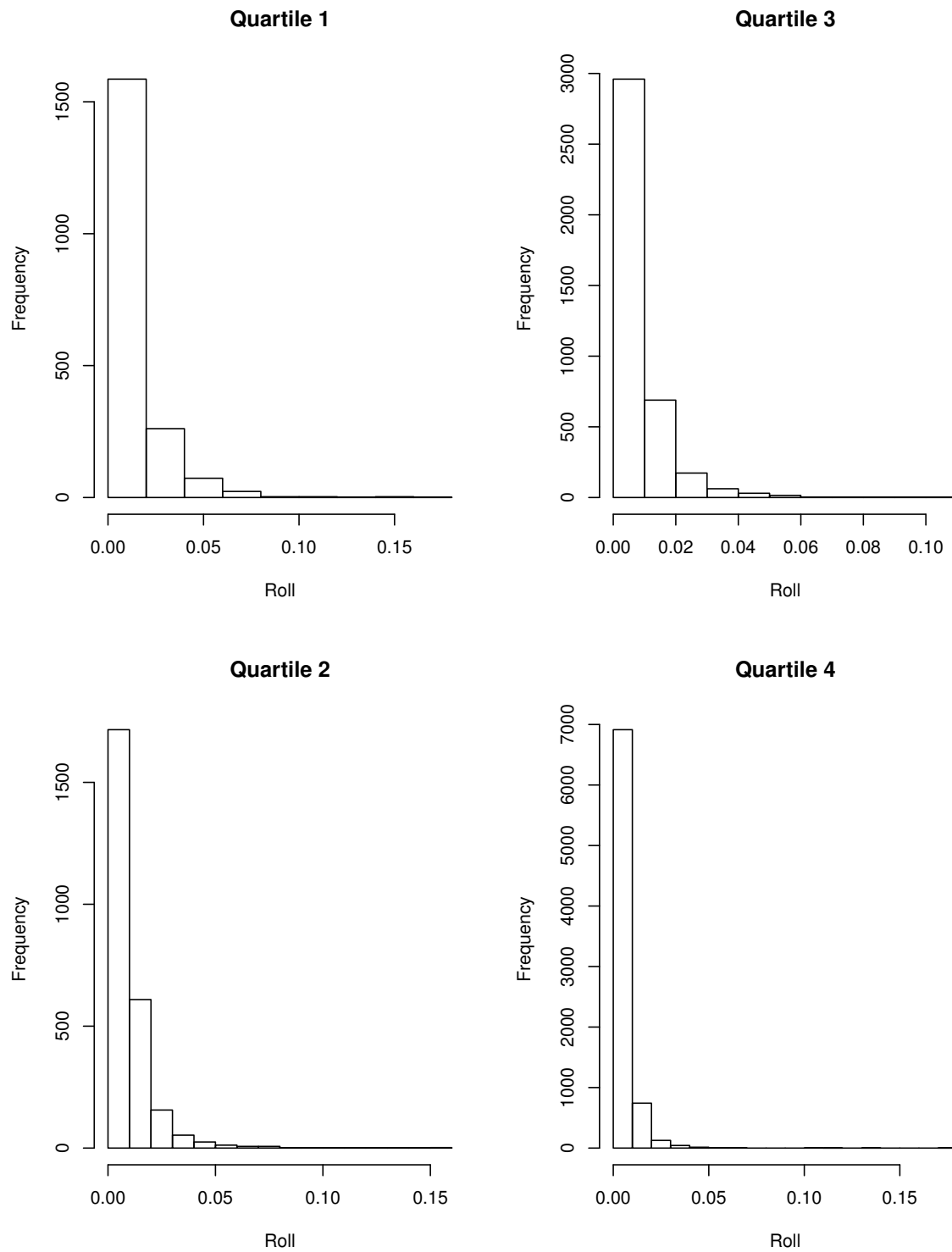
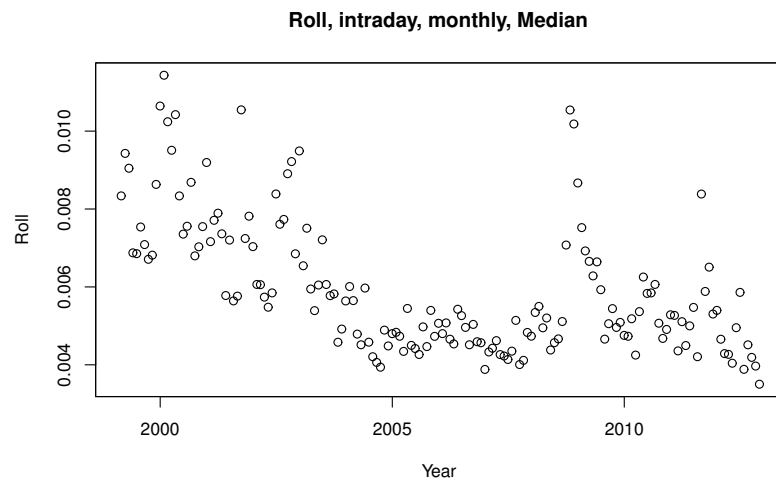
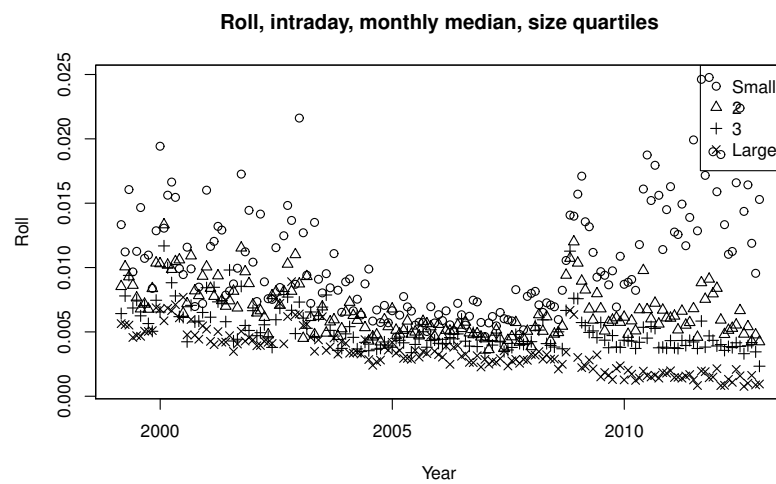


Figure 65 Time Series, Roll, Monthly Median

Panel A: Crossectional Median



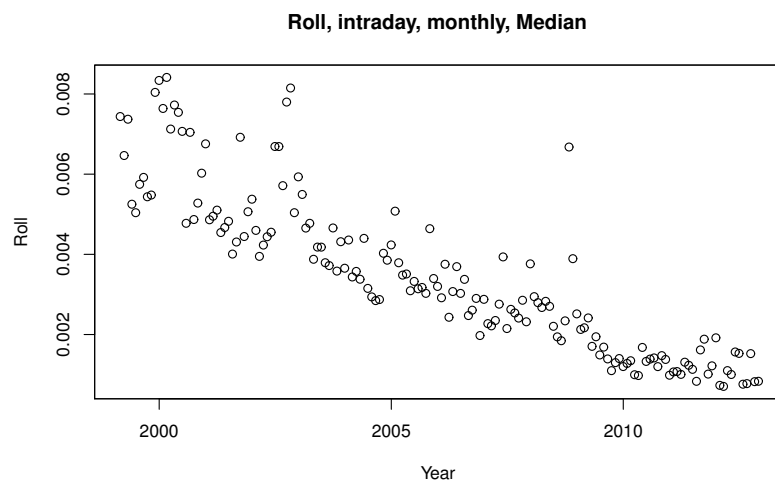
Panel B: Crossectional Median, Size sorted portfolios.



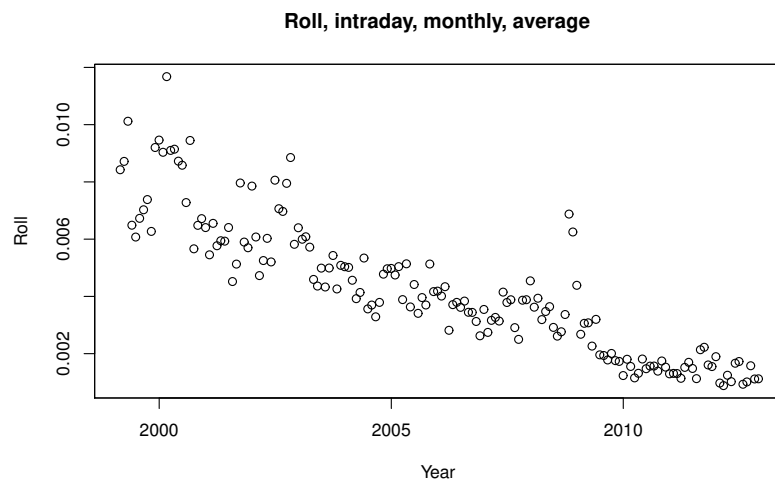
Time series plot of median Roll across stocks on the Oslo Stock Exchange.

Figure 66 Time Series, Roll, Monthly Median, OBX Constituents

Panel A: Crossectional Median



Panel B: Crossectional Average



Time series plot of median Roll across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

8 The Amihud measure

8.1 Quarterly estimates of Amihud

Table 14 Descriptive, Amihud

Period	min	Q1	med	mean	(std)	Q3	max	n	Size quartiles(means)			
									1(small)	2	3	4
1980-2016	0.000	0.010	0.092	1.018	(7.212)	0.455	561.307	864	3.370	1.292	0.632	0.152
1980	0.013	0.091	0.320	0.447	(0.617)	0.507	4.019	14				0.447
1981	0.038	0.225	0.459	0.638	(0.592)	0.850	2.979	22				0.638
1982	0.065	0.229	0.382	0.637	(0.732)	0.789	5.278	24			1.452	0.474
1983	0.006	0.101	0.357	0.727	(1.273)	0.841	12.206	54	2.957	1.217	1.120	0.420
1984	0.003	0.063	0.237	0.386	(0.504)	0.459	3.580	86	1.017	0.627	0.262	0.187
1985	0.003	0.061	0.224	0.483	(1.080)	0.448	14.010	114	2.741	0.824	0.640	0.153
1986	0.000	0.104	0.302	0.599	(0.979)	0.610	8.759	113	2.268	0.910	0.586	0.256
1987	0.001	0.071	0.214	0.677	(1.776)	0.590	23.804	106	1.177	2.137	0.406	0.234
1988	0.001	0.035	0.234	0.799	(1.777)	0.851	20.069	74	2.241	1.359	0.719	0.304
1989	0.000	0.019	0.135	0.622	(1.378)	0.597	15.375	100	1.908	0.877	0.579	0.199
1990	0.000	0.036	0.145	0.512	(1.581)	0.446	26.847	102	0.839	1.418	0.364	0.218
1991	0.000	0.047	0.218	0.871	(2.388)	0.654	28.843	95		2.565	1.022	0.497
1992	0.000	0.046	0.271	1.948	(6.068)	1.035	61.887	83	1.418	3.723	2.756	0.777
1993	0.000	0.014	0.075	0.534	(2.353)	0.276	40.971	100	0.706	0.957	0.721	0.057
1994	0.000	0.016	0.072	0.242	(0.541)	0.231	6.511	111	0.502	0.328	0.182	0.132
1995	0.000	0.012	0.057	0.216	(0.630)	0.173	9.915	122	1.654	0.430	0.162	0.106
1996	0.000	0.006	0.037	0.130	(0.295)	0.124	4.020	143	0.201	0.176	0.168	0.032
1997	0.000	0.006	0.031	0.139	(0.322)	0.128	3.796	168	0.370	0.169	0.130	0.039
1998	0.000	0.012	0.098	0.373	(0.792)	0.371	9.261	170	1.096	0.414	0.386	0.114
1999	0.000	0.013	0.114	0.482	(1.014)	0.519	15.351	172	1.376	0.473	0.248	0.080
2000	0.000	0.008	0.072	0.345	(0.750)	0.378	9.711	172	0.631	0.355	0.273	0.083
2001	0.000	0.013	0.129	0.643	(1.312)	0.626	9.464	163	2.853	0.807	0.487	0.138
2002	0.000	0.018	0.124	0.908	(2.217)	0.739	20.726	139	2.759	1.379	0.686	0.098
2003	0.000	0.012	0.103	0.873	(3.239)	0.514	44.990	134	3.072	0.789	0.459	0.080
2004	0.000	0.005	0.037	0.161	(0.374)	0.170	4.458	151	0.254	0.266	0.085	0.049
2005	0.000	0.003	0.019	0.114	(0.297)	0.094	3.801	184	0.281	0.218	0.110	0.033
2006	0.000	0.002	0.030	0.141	(0.281)	0.147	2.183	205	0.356	0.194	0.157	0.048
2007	0.000	0.002	0.026	0.125	(0.242)	0.145	2.393	223	0.354	0.157	0.122	0.026
2008	0.000	0.009	0.138	0.471	(0.910)	0.535	9.579	223	0.951	0.722	0.455	0.091
2009	0.000	0.012	0.139	0.578	(1.529)	0.516	15.622	198	1.431	0.874	0.310	0.067
2010	0.000	0.007	0.097	1.792	(7.911)	0.709	120.850	207	3.041	1.607	0.394	0.127
2011	0.000	0.008	0.177	2.002	(5.593)	1.352	55.668	210	6.059	3.475	1.911	0.208
2012	0.000	0.007	0.164	3.036	(10.126)	1.886	133.916	198	10.558	4.453	1.721	0.187
2013	0.000	0.004	0.076	3.700	(26.362)	0.777	561.307	197	11.625	1.680	0.364	0.030
2014	0.000	0.003	0.057	2.342	(12.901)	0.782	316.969	202	7.060	2.544	0.632	0.050
2015	0.000	0.003	0.065	2.407	(12.777)	0.926	282.943	205	8.843	3.143	0.742	0.049
2016	0.000	0.003	0.056	2.108	(7.215)	0.773	82.166	207	4.924	2.181	1.957	0.008

Figure 67 Histogram Amihud - Subperiod

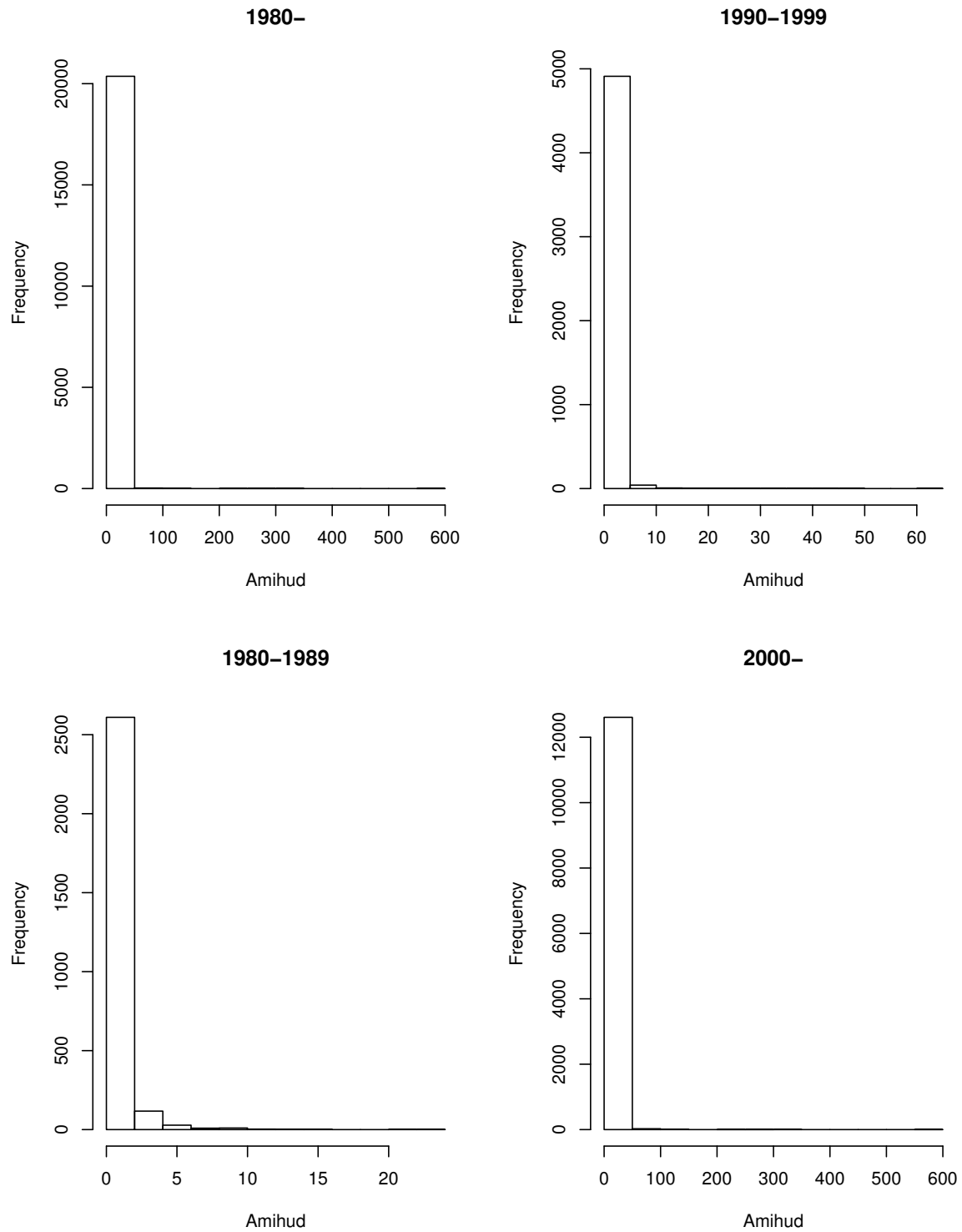


Figure 68 Histogram Amihud - By Size

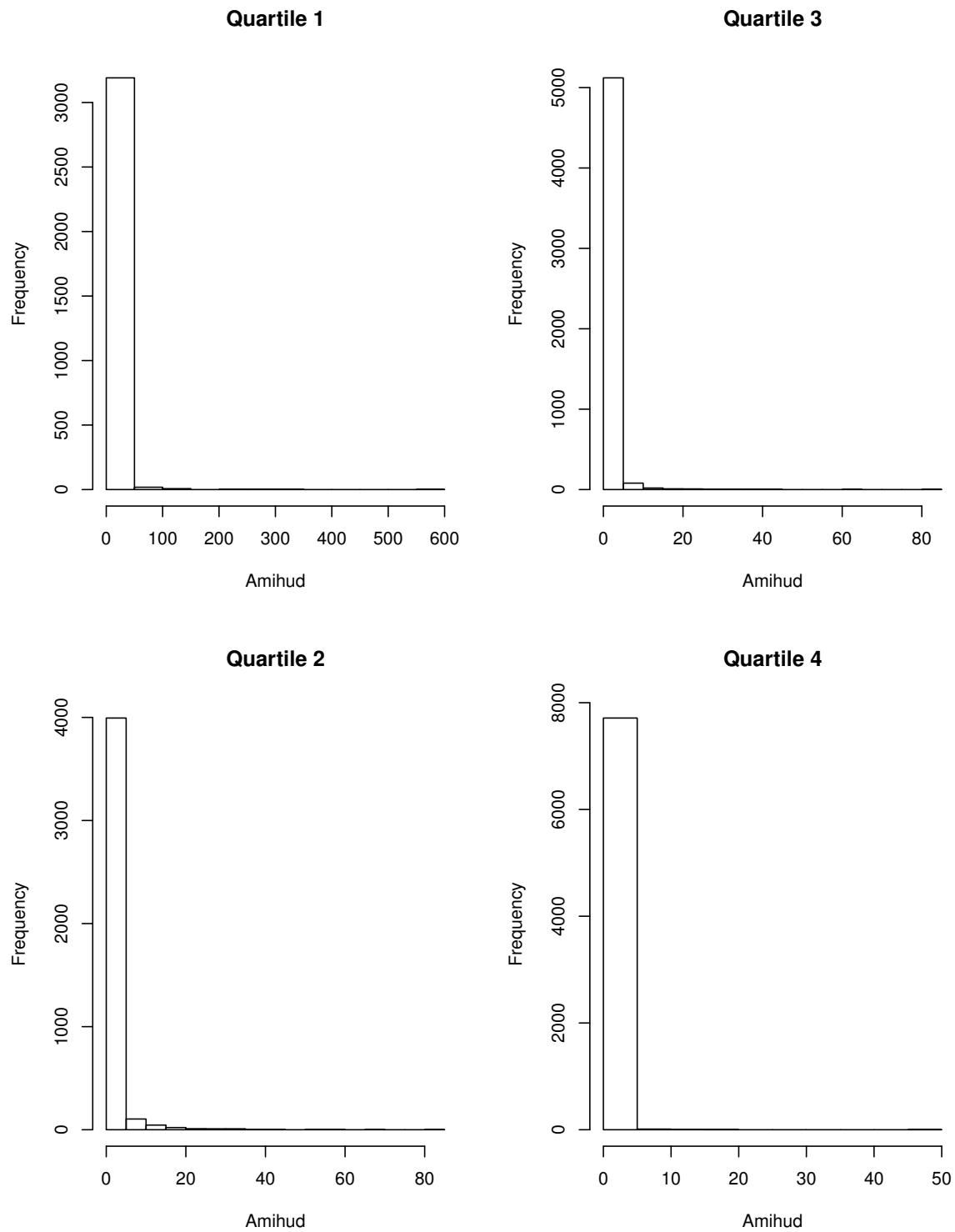
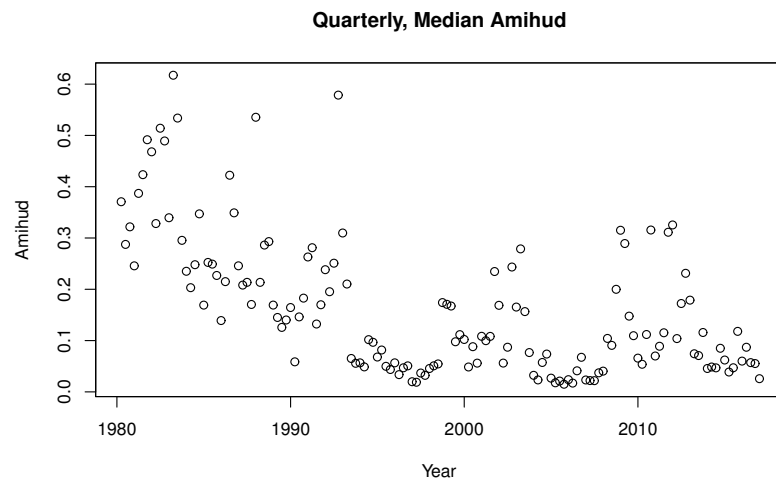
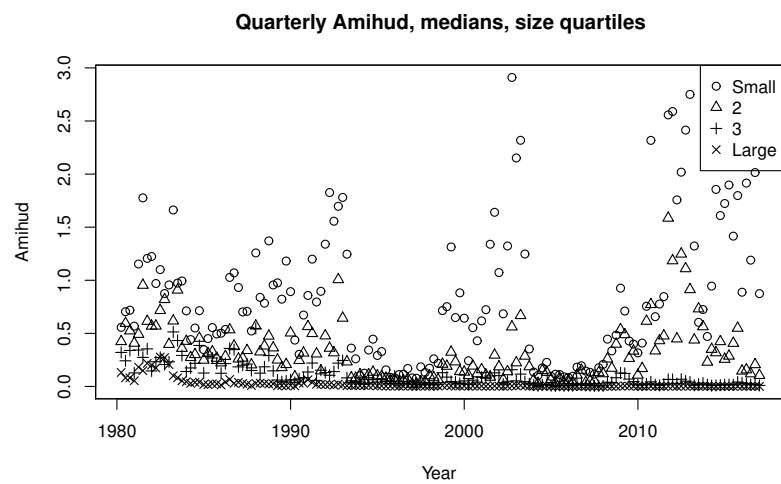


Figure 69 Time Series, Amihud, Monthly Median

Panel A: Crossectional Median



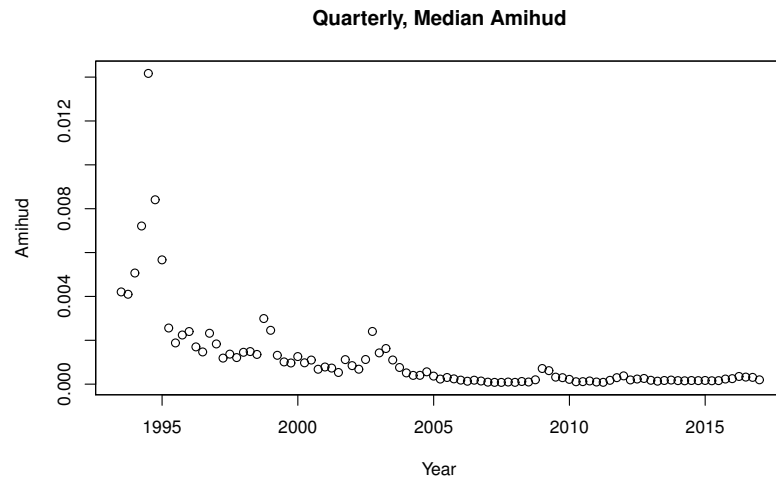
Panel B: Crossectional Median, Size sorted portfolios.



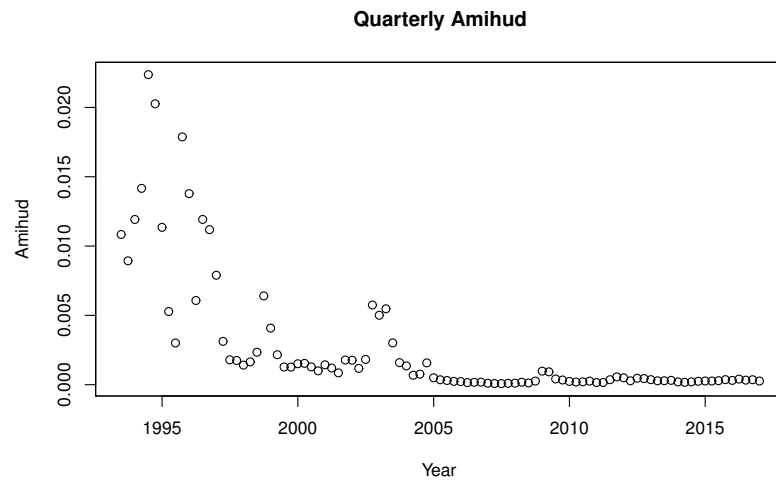
Time series plot of median Amihud across stocks on the Oslo Stock Exchange.

Figure 70 Time Series, Amihud, OBX Constituents

Panel A: Crossectional Median



Panel B: Crossectional Average



Time series plot of median Amihud across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

9 The High/Low Measure

In this section we describe the estimate of spread proposed by Corwin and Schultz (2012).

Table 15 Descriptive

Period	min	Q1	med	mean	(std)	Q3	max	n	Size quartiles(means)			
									1(small)	2	3	4
1980-2016	0.000	0.008	0.012	0.015	(0.011)	0.018	0.201	798	0.022	0.017	0.014	0.011
1980	0.002	0.004	0.005	0.006	(0.003)	0.008	0.014	12				0.006
1981	0.002	0.005	0.008	0.010	(0.007)	0.013	0.036	21				0.010
1982	0.001	0.005	0.007	0.008	(0.005)	0.010	0.027	24			0.012	0.008
1983	0.003	0.007	0.010	0.012	(0.010)	0.014	0.066	54	0.025	0.020	0.016	0.009
1984	0.002	0.007	0.009	0.010	(0.006)	0.012	0.038	86	0.018	0.012	0.009	0.008
1985	0.001	0.005	0.008	0.009	(0.006)	0.011	0.051	114	0.020	0.013	0.009	0.007
1986	0.001	0.005	0.007	0.009	(0.007)	0.012	0.054	113	0.020	0.013	0.009	0.007
1987	0.001	0.005	0.008	0.010	(0.010)	0.012	0.094	106	0.015	0.019	0.009	0.007
1988	0.003	0.006	0.009	0.012	(0.009)	0.014	0.066	74	0.020	0.013	0.011	0.010
1989	0.001	0.006	0.009	0.011	(0.008)	0.013	0.068	100	0.018	0.016	0.011	0.008
1990	0.000	0.007	0.010	0.011	(0.006)	0.014	0.047	102	0.016	0.015	0.011	0.009
1991	0.001	0.007	0.010	0.013	(0.010)	0.016	0.065	95		0.019	0.015	0.012
1992	0.001	0.008	0.012	0.018	(0.017)	0.022	0.187	83	0.022	0.029	0.020	0.013
1993	0.000	0.007	0.011	0.013	(0.009)	0.016	0.070	100	0.017	0.017	0.012	0.009
1994	0.003	0.006	0.008	0.010	(0.006)	0.012	0.070	111	0.014	0.013	0.010	0.008
1995	0.002	0.006	0.008	0.009	(0.005)	0.011	0.038	122	0.012	0.013	0.010	0.008
1996	0.001	0.006	0.008	0.010	(0.006)	0.011	0.058	143	0.013	0.011	0.009	0.007
1997	0.002	0.006	0.009	0.010	(0.006)	0.013	0.035	168	0.015	0.012	0.010	0.008
1998	0.002	0.008	0.012	0.015	(0.010)	0.018	0.077	170	0.023	0.016	0.015	0.011
1999	0.002	0.010	0.014	0.016	(0.010)	0.020	0.083	172	0.024	0.017	0.015	0.012
2000	0.002	0.011	0.017	0.018	(0.011)	0.022	0.120	172	0.023	0.018	0.015	0.015
2001	0.001	0.011	0.015	0.019	(0.013)	0.023	0.127	163	0.031	0.023	0.018	0.015
2002	0.000	0.010	0.015	0.020	(0.016)	0.023	0.159	139	0.030	0.025	0.016	0.015
2003	0.001	0.009	0.013	0.017	(0.013)	0.020	0.124	134	0.025	0.019	0.015	0.012
2004	0.002	0.007	0.010	0.012	(0.006)	0.015	0.060	151	0.016	0.012	0.010	0.008
2005	0.000	0.008	0.010	0.011	(0.005)	0.014	0.046	184	0.016	0.013	0.012	0.009
2006	0.001	0.009	0.011	0.012	(0.005)	0.015	0.052	205	0.015	0.012	0.012	0.011
2007	0.000	0.009	0.011	0.012	(0.005)	0.014	0.034	223	0.014	0.012	0.011	0.011
2008	0.000	0.012	0.017	0.020	(0.012)	0.025	0.098	223	0.022	0.018	0.019	0.021
2009	0.001	0.012	0.016	0.019	(0.012)	0.023	0.111	198	0.024	0.018	0.018	0.018
2010	0.000	0.011	0.015	0.018	(0.012)	0.022	0.160	207	0.022	0.017	0.014	0.013
2011	0.000	0.011	0.016	0.019	(0.013)	0.025	0.101	210	0.027	0.023	0.021	0.015
2012	0.000	0.010	0.015	0.018	(0.014)	0.023	0.121	198	0.034	0.022	0.016	0.012
2013	0.001	0.008	0.012	0.016	(0.014)	0.021	0.201	197	0.026	0.016	0.014	0.009
2014	0.001	0.009	0.012	0.016	(0.011)	0.019	0.096	202	0.021	0.017	0.016	0.010
2015	0.000	0.010	0.014	0.017	(0.012)	0.022	0.167	205	0.024	0.019	0.017	0.014
2016	0.000	0.010	0.015	0.018	(0.012)	0.023	0.086	207	0.022	0.020	0.017	0.014

Figure 71 Histogram - Subperiod

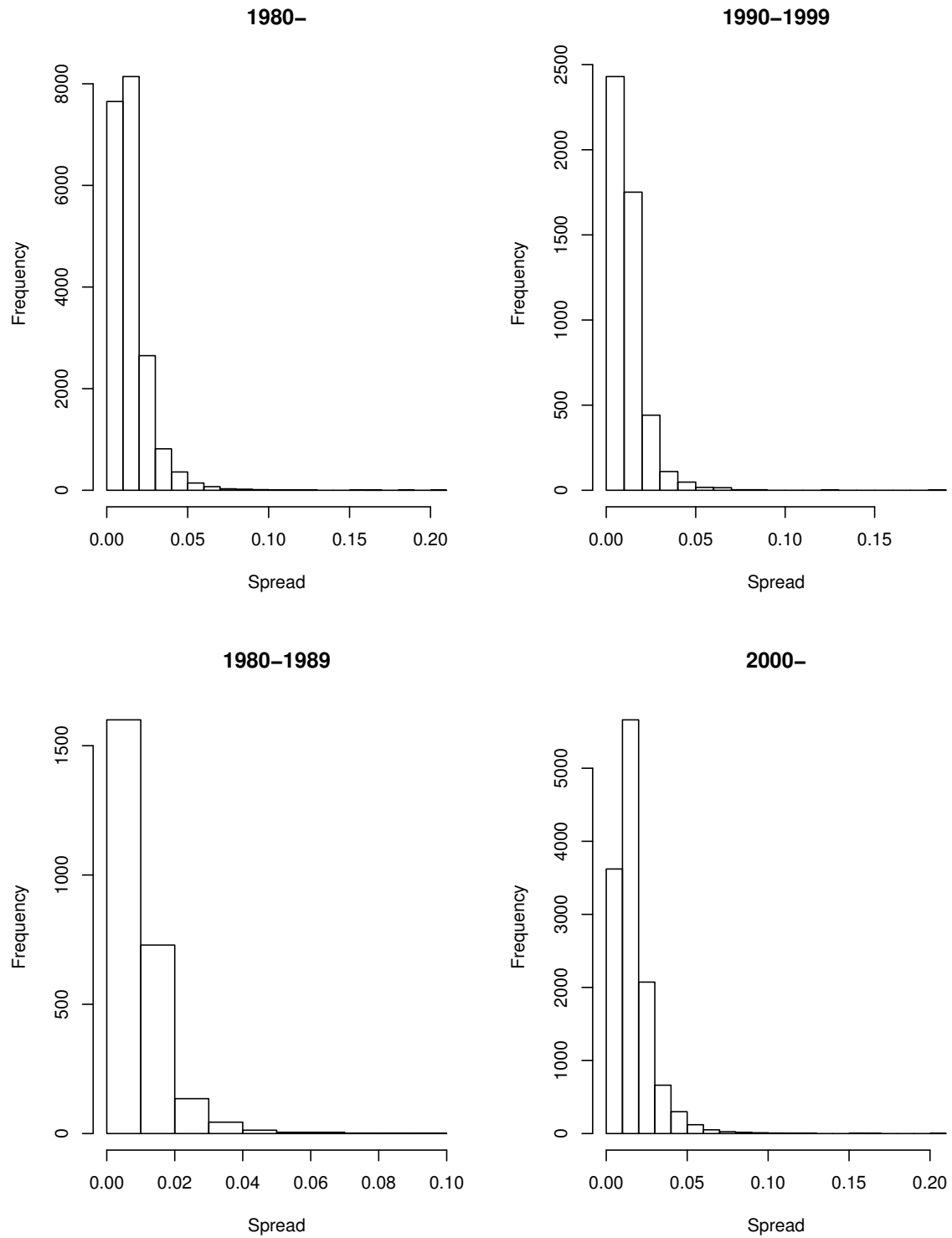


Figure 72 Histogram - By Size

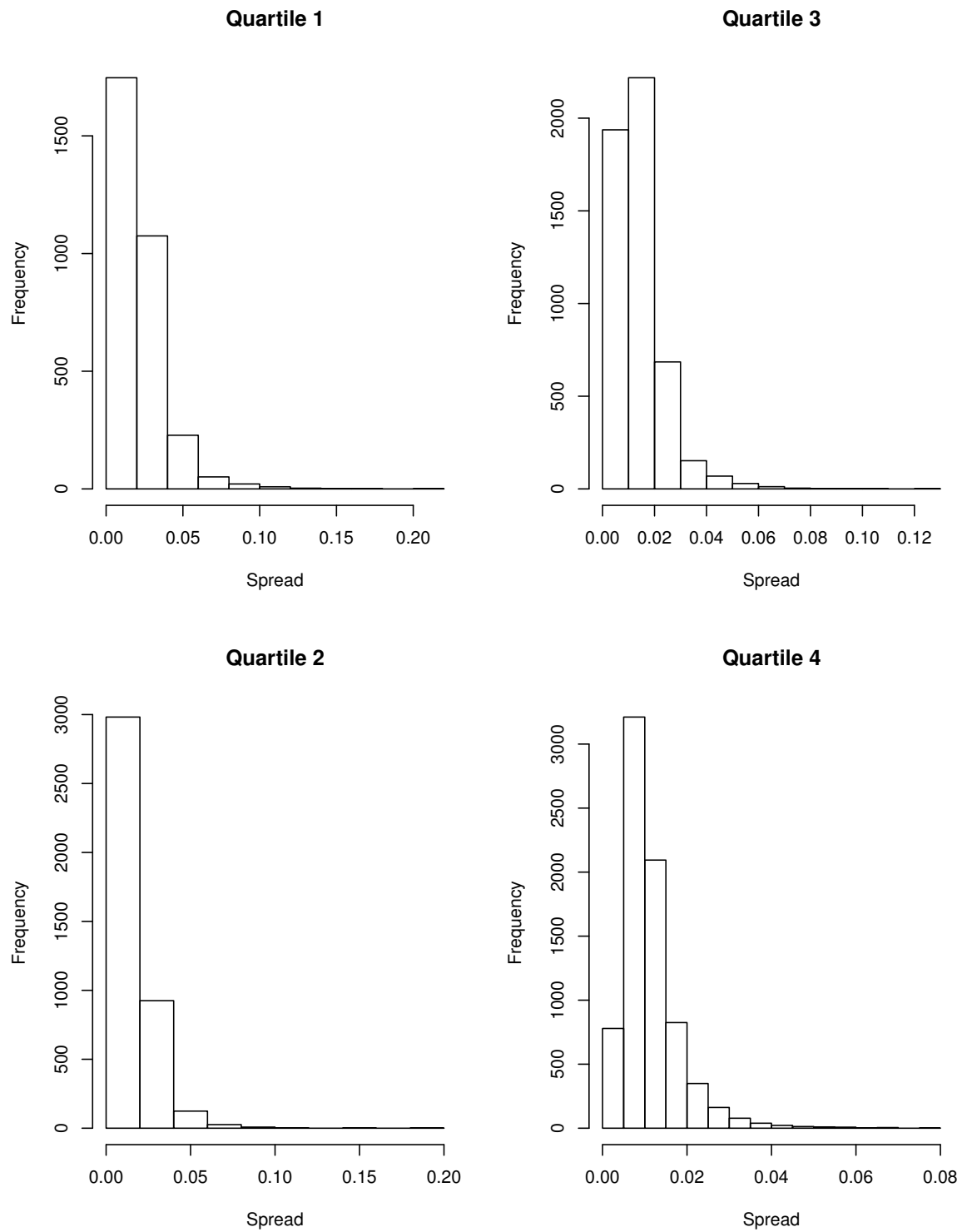
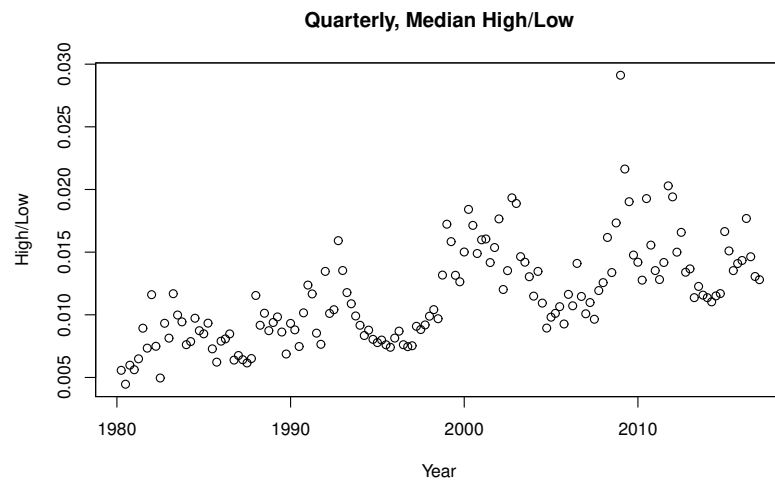
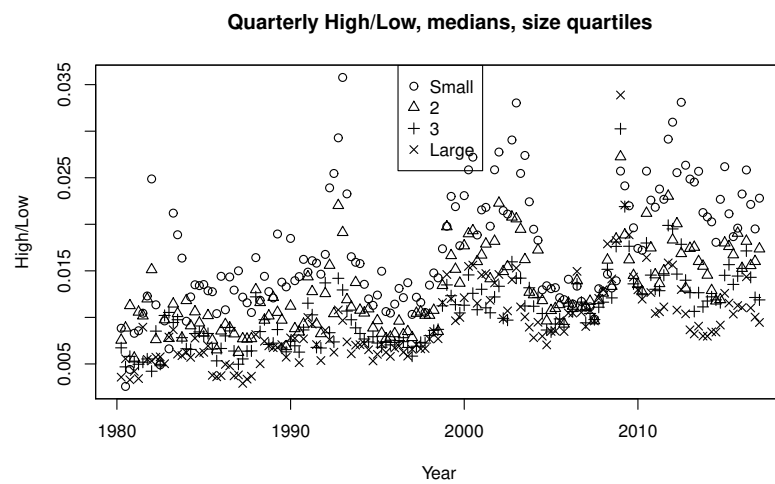


Figure 73 Time Series, Roll, Monthly Median

Panel A: Crossectional Median



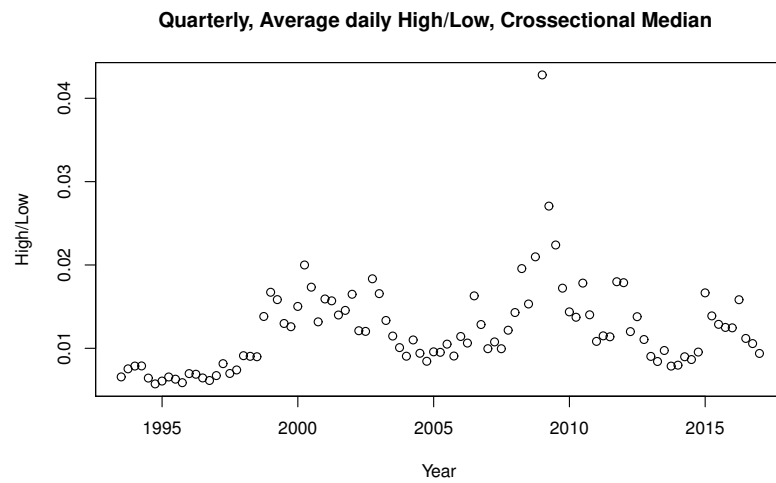
Panel B: Crossectional Median, Size sorted portfolios.



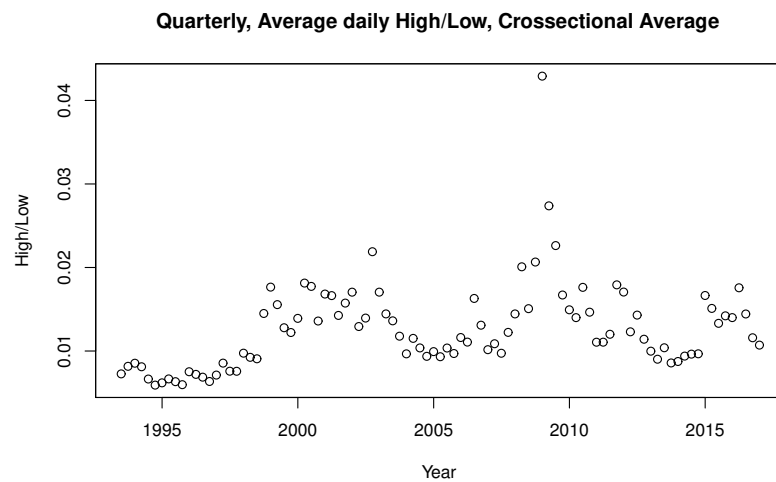
Time series plot of median Roll across stocks on the Oslo Stock Exchange.

Figure 74 Time Series, High Low, Monthly Median, OBX Constituents

Panel A: Crossectional Median



Panel B: Crossectional Average



The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

10 Volatility measures

We calculate various measures of volatility.

10.1 Quarterly estimates of Volatility

In this section we look at daily stock returns. For each stock, we estimate the standard deviation of daily returns on a quarterly basis.

Table 16 Descriptive, Volatility

Period	min	Q1	med	mean	(std)	Q3	max	n	Size quartiles(means)			
									1(small)	2	3	4
1980-2016	0.001	0.017	0.023	0.027	(0.016)	0.032	0.515	680	0.033	0.032	0.027	0.024
1980	0.002	0.008	0.015	0.016	(0.009)	0.022	0.052	14				0.016
1981	0.008	0.015	0.020	0.023	(0.014)	0.028	0.100	22				0.023
1982	0.008	0.014	0.019	0.021	(0.011)	0.025	0.080	24			0.030	0.019
1983	0.006	0.019	0.026	0.028	(0.014)	0.035	0.094	53	0.046	0.041	0.028	0.025
1984	0.009	0.019	0.025	0.028	(0.013)	0.033	0.110	86	0.045	0.035	0.025	0.023
1985	0.007	0.015	0.020	0.024	(0.013)	0.029	0.111	113	0.044	0.033	0.025	0.020
1986	0.004	0.017	0.022	0.026	(0.013)	0.030	0.110	108	0.033	0.032	0.025	0.023
1987	0.005	0.018	0.025	0.033	(0.021)	0.043	0.126	98	0.043	0.045	0.029	0.031
1988	0.007	0.021	0.030	0.032	(0.020)	0.038	0.285	68	0.044	0.038	0.032	0.028
1989	0.001	0.020	0.026	0.030	(0.017)	0.034	0.203	94	0.044	0.037	0.027	0.026
1990	0.011	0.021	0.028	0.032	(0.023)	0.037	0.407	96	0.040	0.046	0.029	0.028
1991	0.007	0.022	0.030	0.033	(0.016)	0.040	0.152	76		0.038	0.041	0.030
1992	0.004	0.023	0.033	0.037	(0.019)	0.045	0.162	64	0.047	0.046	0.040	0.033
1993	0.010	0.019	0.025	0.030	(0.018)	0.034	0.144	83	0.042	0.038	0.032	0.022
1994	0.009	0.016	0.020	0.023	(0.009)	0.028	0.056	99	0.026	0.029	0.026	0.020
1995	0.008	0.014	0.018	0.021	(0.010)	0.024	0.109	107	0.032	0.028	0.022	0.018
1996	0.003	0.014	0.018	0.020	(0.010)	0.024	0.099	129	0.023	0.024	0.020	0.016
1997	0.005	0.017	0.023	0.025	(0.010)	0.030	0.102	148	0.031	0.027	0.027	0.021
1998	0.008	0.022	0.030	0.032	(0.015)	0.040	0.128	127	0.039	0.038	0.032	0.030
1999	0.003	0.021	0.029	0.033	(0.027)	0.039	0.515	130	0.043	0.040	0.031	0.028
2000	0.004	0.024	0.033	0.036	(0.017)	0.045	0.118	132	0.045	0.044	0.032	0.031
2001	0.008	0.022	0.030	0.035	(0.019)	0.042	0.126	101	0.082	0.044	0.039	0.031
2002	0.008	0.019	0.025	0.030	(0.016)	0.036	0.121	74	0.024	0.032	0.031	0.029
2003	0.007	0.017	0.022	0.025	(0.014)	0.029	0.178	72	0.023	0.033	0.023	0.024
2004	0.007	0.015	0.020	0.022	(0.010)	0.026	0.077	101	0.026	0.026	0.021	0.019
2005	0.005	0.017	0.021	0.023	(0.009)	0.026	0.080	126	0.023	0.027	0.023	0.021
2006	0.003	0.017	0.023	0.024	(0.010)	0.029	0.070	154	0.032	0.026	0.025	0.023
2007	0.007	0.016	0.021	0.023	(0.009)	0.026	0.074	174	0.029	0.024	0.023	0.021
2008	0.006	0.023	0.031	0.037	(0.021)	0.046	0.189	107	0.030	0.034	0.039	0.037
2009	0.013	0.022	0.028	0.031	(0.014)	0.037	0.167	83	0.029	0.037	0.032	0.030
2010	0.003	0.018	0.023	0.025	(0.012)	0.030	0.158	107	0.031	0.028	0.023	0.021
2011	0.007	0.017	0.023	0.026	(0.015)	0.031	0.148	102	0.022	0.027	0.030	0.025
2012	0.002	0.016	0.019	0.022	(0.011)	0.026	0.095	94	0.020	0.030	0.022	0.020
2013	0.004	0.013	0.016	0.019	(0.010)	0.022	0.087	103	0.023	0.024	0.020	0.015
2014	0.002	0.013	0.017	0.021	(0.012)	0.026	0.085	116	0.028	0.026	0.021	0.018
2015	0.002	0.016	0.019	0.023	(0.012)	0.027	0.126	120	0.026	0.026	0.021	0.022
2016	0.002	0.016	0.020	0.024	(0.013)	0.028	0.132	122	0.024	0.027	0.024	0.022

Figure 75 Histogram Volatility - Subperiod

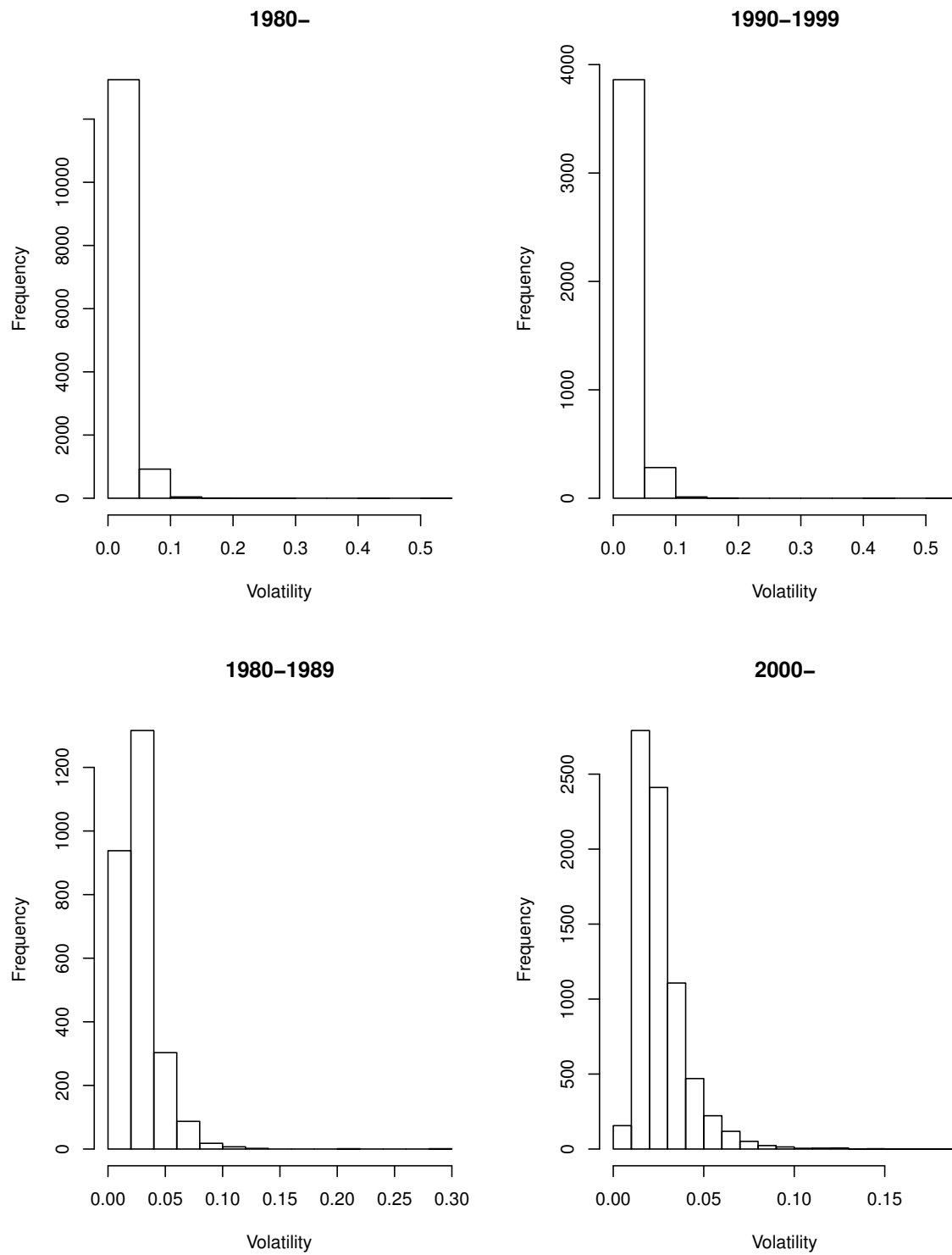


Figure 76 Histogram Volatility - By Size

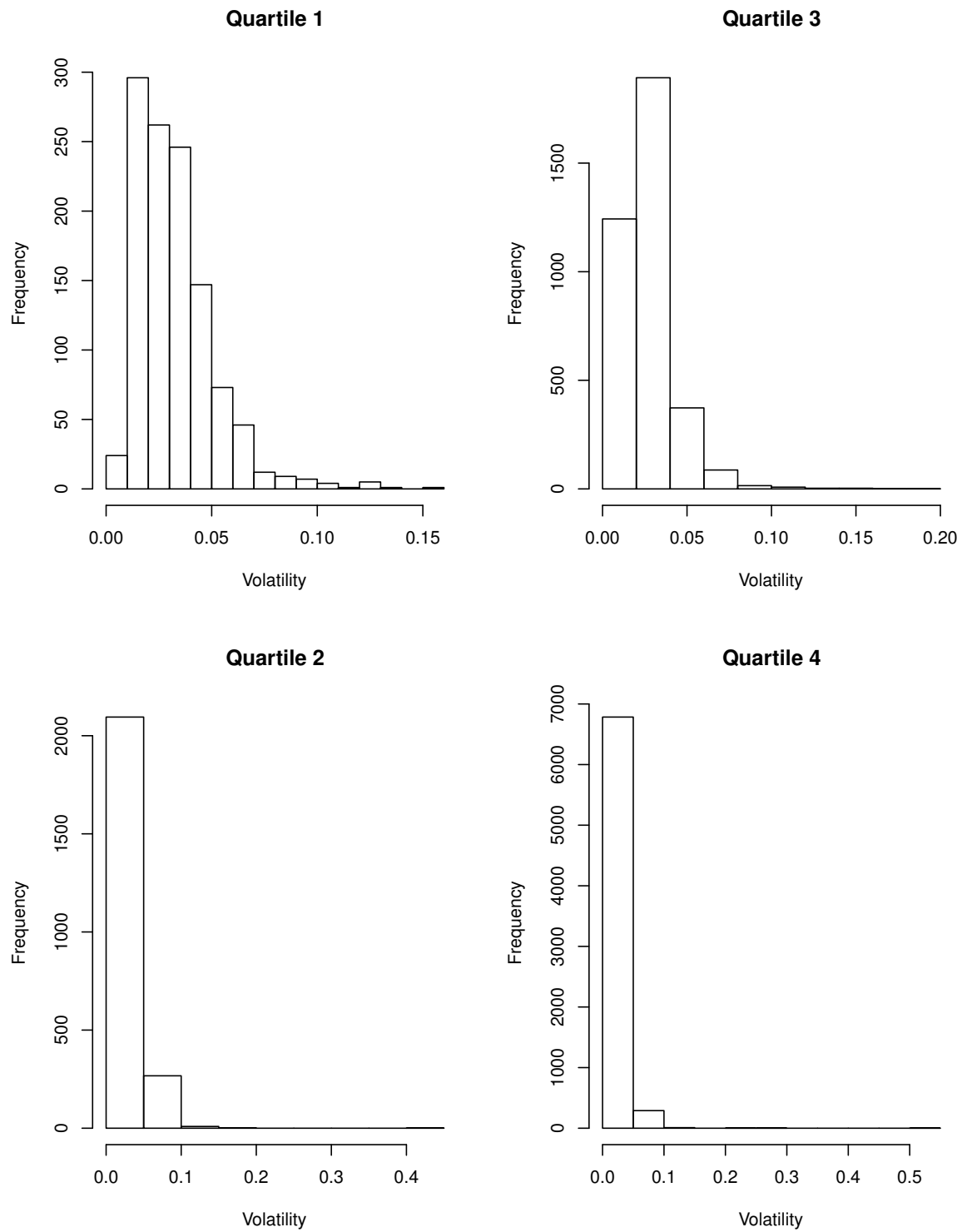
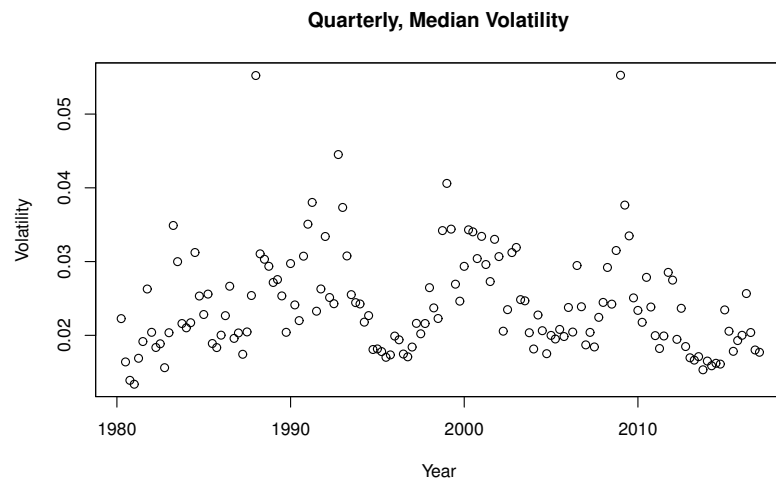
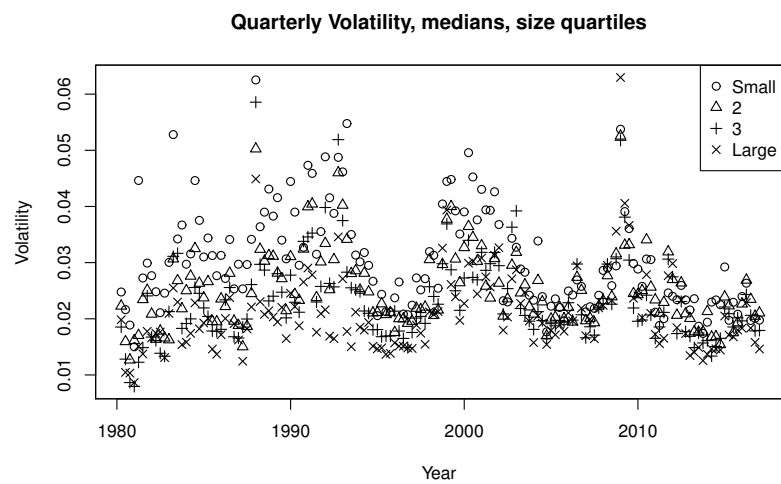


Figure 77 Time Series, Volatility, Quarterly Median

Panel A: Crossectional Median



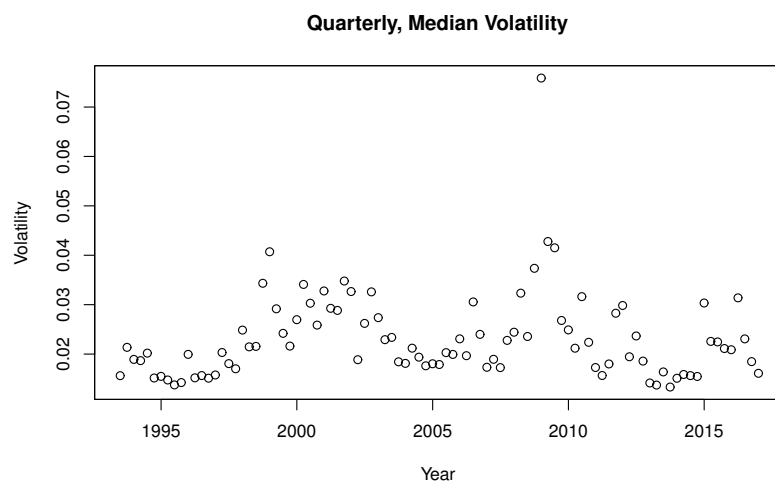
Panel B: Crossectional Median, Size sorted portfolios.



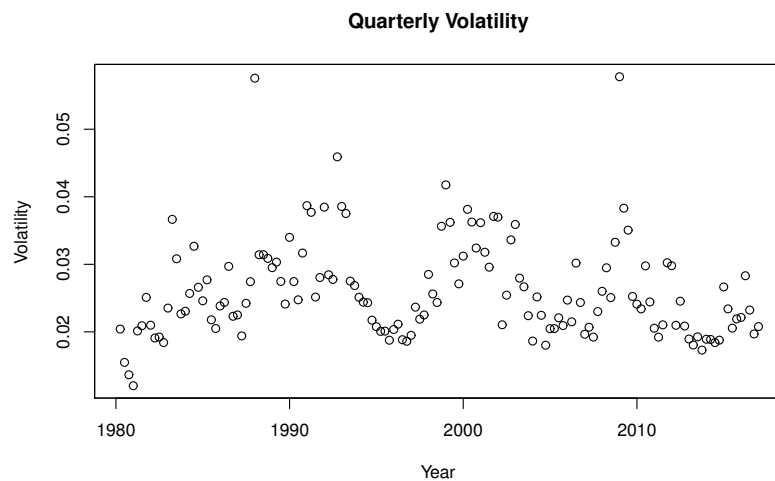
Time series plot of median volatility across stocks on the Oslo Stock Exchange.

Figure 78 Time Series, Volatility, Quarterly, Crossectional, OBX Constituents

Panel A: Crossectional Median



Panel B: Crossectional Average (trimmed)



Time series plot of median volatility across stocks on the Oslo Stock Exchange. The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

10.2 Realized Volatility

This section considers realized volatility.

Table 17 Descriptive, Realized Volatility

Period	min	Q1	med	mean	(std)	Q3	max	n	Size quartiles(means)			
									1(small)	2	3	4
1999-2012	0.0004	0.0042	0.0060	0.0074	(0.0054)	0.0088	0.0983	1570	0.0113	0.0084	0.0069	0.0054
1999	0.0017	0.0049	0.0070	0.0084	(0.0056)	0.0096	0.0531	171	0.0137	0.0086	0.0075	0.0060
2000	0.0011	0.0057	0.0077	0.0093	(0.0075)	0.0107	0.0983	169	0.0125	0.0100	0.0074	0.0069
2001	0.0011	0.0050	0.0075	0.0093	(0.0068)	0.0112	0.0789	156	0.0166	0.0117	0.0096	0.0069
2002	0.0012	0.0050	0.0074	0.0095	(0.0076)	0.0110	0.0486	139	0.0158	0.0131	0.0086	0.0063
2003	0.0013	0.0044	0.0063	0.0081	(0.0061)	0.0097	0.0430	134	0.0130	0.0093	0.0076	0.0055
2004	0.0009	0.0035	0.0050	0.0055	(0.0029)	0.0069	0.0228	150	0.0073	0.0064	0.0046	0.0038
2005	0.0013	0.0036	0.0049	0.0055	(0.0029)	0.0066	0.0260	182	0.0075	0.0069	0.0057	0.0044
2006	0.0009	0.0037	0.0048	0.0053	(0.0024)	0.0065	0.0200	202	0.0077	0.0057	0.0054	0.0045
2007	0.0006	0.0037	0.0047	0.0053	(0.0029)	0.0060	0.0397	212	0.0075	0.0055	0.0050	0.0046
2008	0.0004	0.0054	0.0074	0.0085	(0.0045)	0.0105	0.0320	200	0.0103	0.0090	0.0085	0.0074
2009	0.0013	0.0048	0.0066	0.0076	(0.0044)	0.0091	0.0410	179	0.0108	0.0086	0.0072	0.0058
2010	0.0008	0.0042	0.0059	0.0074	(0.0051)	0.0090	0.0389	204	0.0096	0.0073	0.0051	0.0043
2011	0.0007	0.0042	0.0062	0.0082	(0.0059)	0.0101	0.0426	208	0.0133	0.0106	0.0094	0.0050
2012	0.0004	0.0035	0.0054	0.0077	(0.0066)	0.0097	0.0667	196	0.0169	0.0101	0.0060	0.0040

Figure 79 Histogram Realized Volatility - Subperiod

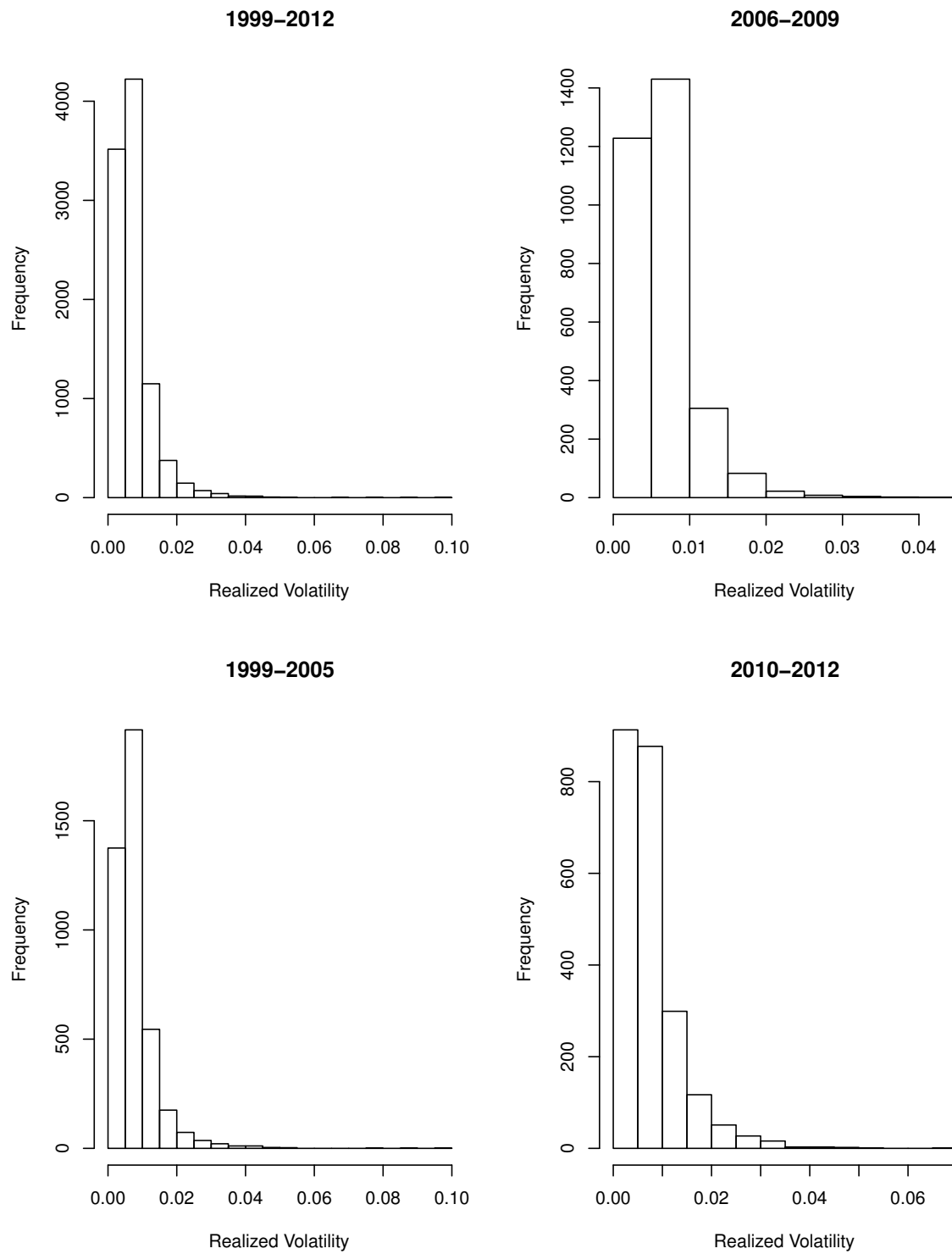


Figure 80 Histogram Realized Volatility - By Size

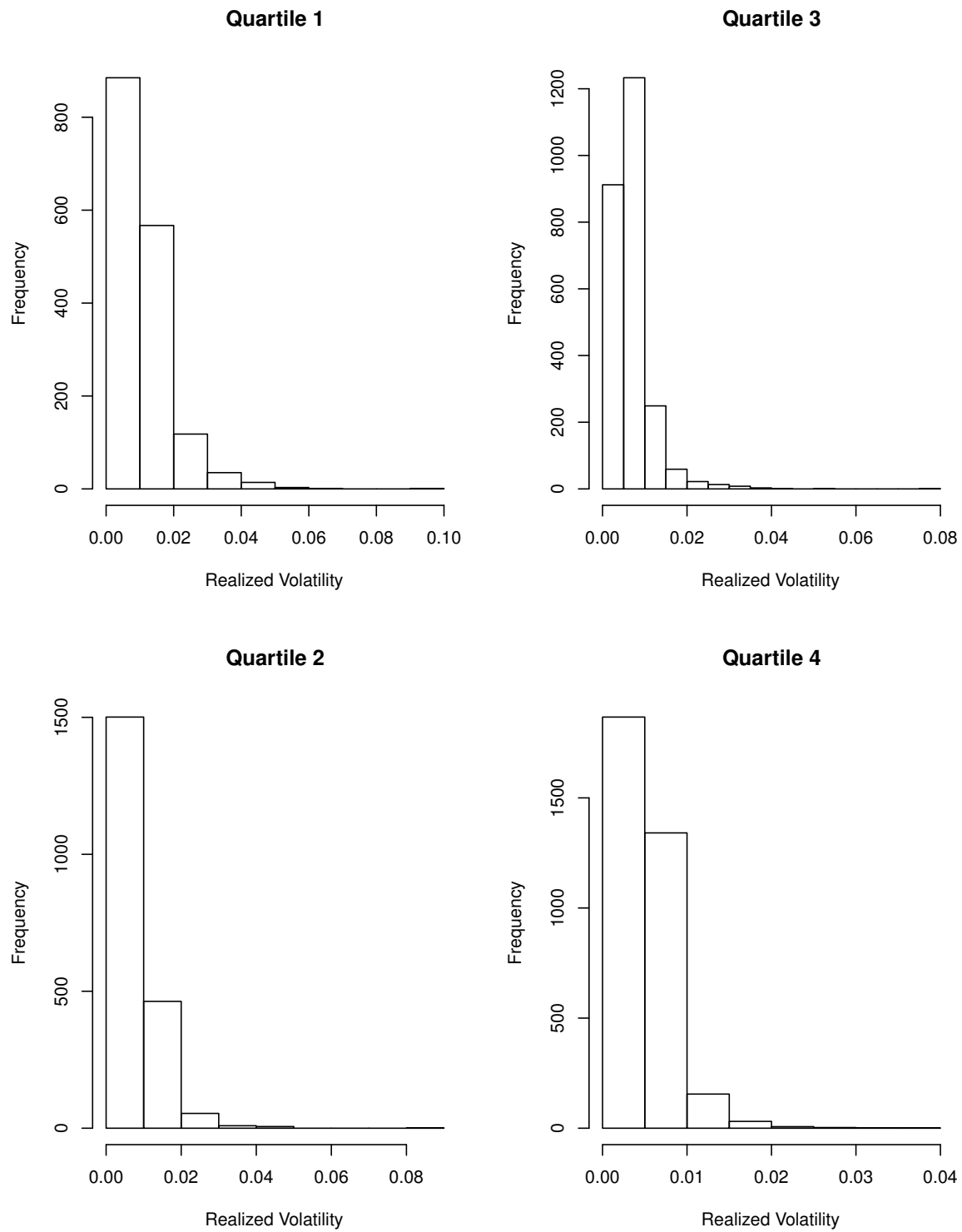
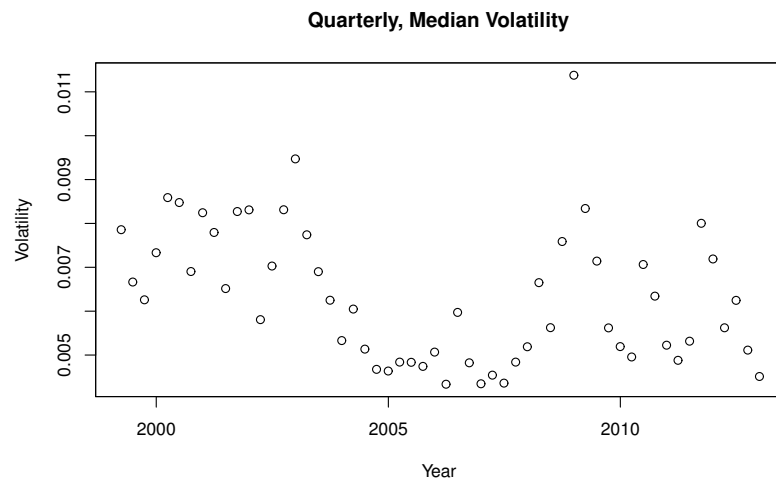
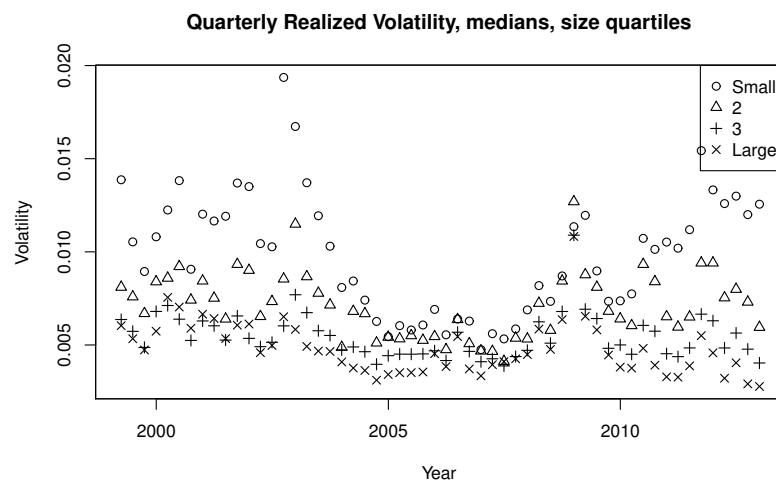


Figure 81 Time Series, Realized Volatility, Quarterly Median

Panel A: Crossectional Median



Panel B: Crossectional Median, Size sorted portfolios.

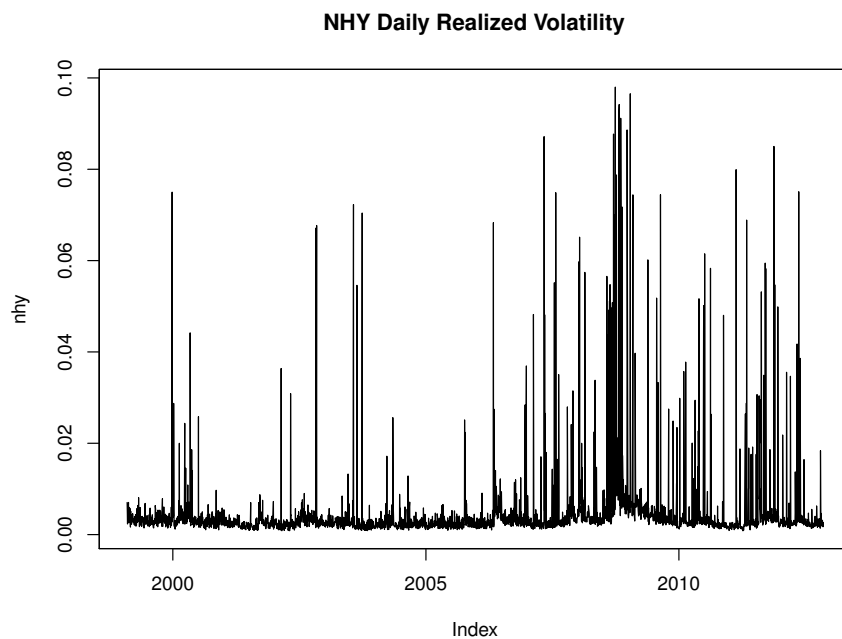


Time series plot of quarterly median of realized volatility across stocks on the Oslo Stock Exchange.

Figure 82 Example, Norsk Hydro and Statoil daily realized volatility

This figure illustrates the evolution of realized volatility for two example companies, Norsk Hydro and Statoil

Panel A: Norsk Hydro



Panel B: Statoil

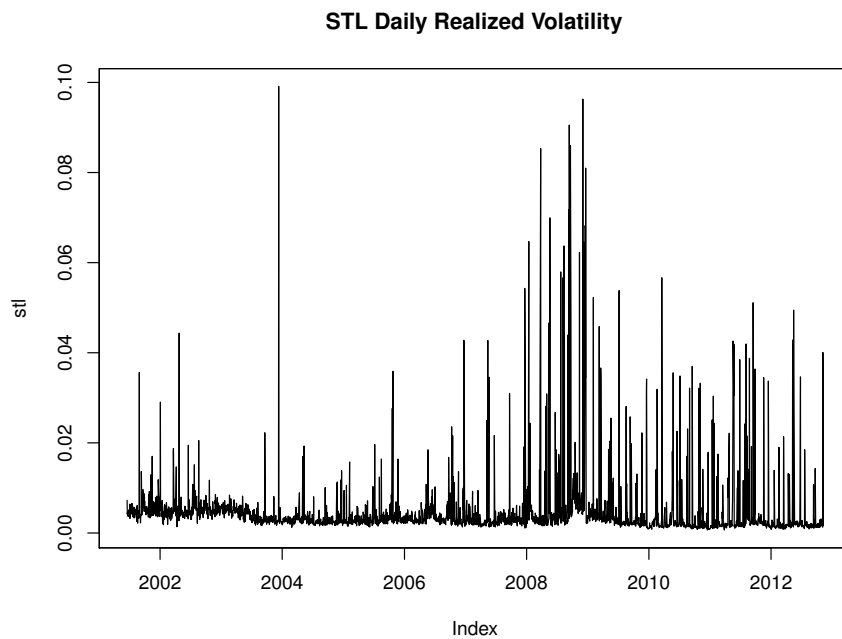
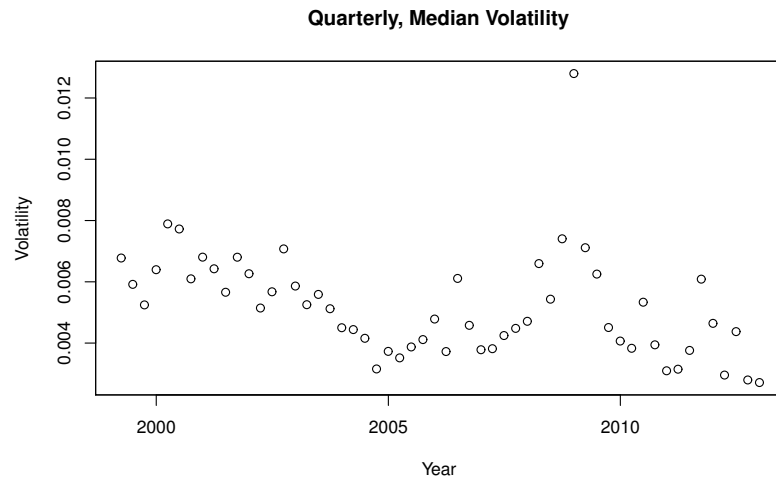
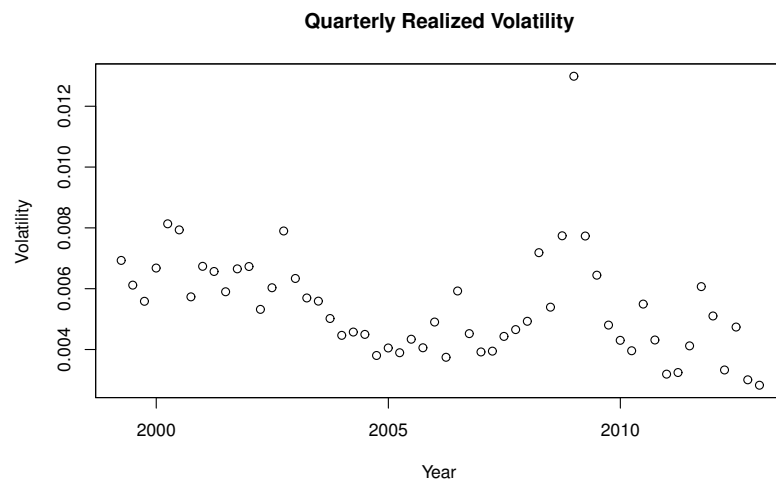


Figure 83 Time Series, Realized Volatility, Quarterly, OBX Constituents

Panel A: Crossectional Median



Panel B: Crossectional Average (trimmed)



Time series plot of quarterly median of realized volatility across stocks on the Oslo Stock Exchange.
The sample is restricted to stocks in the OBX index, an index of the 25 most active stocks on the OSE.

11 Fragmentation of trading

The discussion has so far been concentrated on trading at the Oslo Stock Exchange. In this section we start looking at trading of Norwegian equities (equities with their main listing at the Oslo Stock Exchange) outside of Oslo.

To do this we utilize data from Reuters, on trading on European exchanges. It is here necessary to give an idea of the structure of Reuters data. They provide data which are

- Summaries across exchanges
 - Lit exchanges (XBO)
 - Off exchange reported trades (Market BOAT)
- Data for individual exchanges

Let us first give an example, to illustrate the nature of the data. In table 19 we show summary numbers for trading in Statoil, the largest company listed on the OSE. The table lists *all* of Reuters identifiers related to Statoil (we leave out exchanges that have never exceeded a million traded for the five years.)

Table 18 Trading of Statoil (STL) on European exchanges. Annual number of shares (in millions) traded.

The table shows distribution of trading of Statoil across European exchanges by the various Reuters identifiers. Each row gives data for one Reuters identifier (RIC). The first column is the Reuters identifier (RIC). The second is the exchange code, the third the currency. For each of the years 2008–2012 we calculate the annual number of shares traded on the given exchange. The numbers are reported in millions. Observations indicated with – is a case where the numbers of shares traded is less than one million. Empties signifies no trading at all the given year. We have left out exchanges where the number of trades is never above one million shares.

RIC	Exchange	Curr	2008	2009	2010	2011	2012
STL.xNOK	<i>XDS</i> – > <i>TDS</i>	<i>NOK</i>	2999	173			
STL.xEUR	<i>XDS</i> – > <i>TDS</i>	<i>EUR</i>	239	2			
STLNOK.xbo	<i>XBO</i>	<i>NOK</i>		2149	2673	2826	1627
STL.mNOK	<i>BDS</i>	<i>NOK</i> – > <i>DKK</i> – > <i>NOK</i>	730	578	635	767	707
STL.mGBP	<i>BDS</i>	<i>GBP</i>	–		3	2	–
STL.mEUR	<i>BDS</i>	<i>EUR</i>	169	15	29	59	14
STLoI.SIG	<i>PAR</i> – > 1114 – > <i>SIG</i>	<i>NOK</i>				7	19
STL.TG	<i>FRA</i> – > <i>TDG</i>	<i>EUR</i>			–	3	3
STL.EU	<i>ITE</i>	<i>NOK</i>	1453				
STLoI.BD	<i>BUR</i>	<i>NOK</i>		–	12	15	6
STOHF.PK	<i>PNK</i> – > <i>QBB</i>	<i>USD</i>	–	2	7	4	3
STLo.CHI	<i>CHI</i>	<i>NOK</i>				51	206
STOL.DEU	<i>DEU</i>	<i>EUR</i>	7	5			
STLoI.TQ	<i>TRQ</i>	<i>NOK</i>	2	10	21	102	63
STLoI.NQX	763 – > <i>NQX</i>	<i>NOK</i>	–	1	–		
STLoI.BS	<i>BTE</i>	<i>NOK</i>	–	3	25	73	
STLo.BS	<i>BTE</i>	<i>NOK</i>				30	94
STLNOK.STp	<i>STO</i>	<i>NOK</i>	19	10	8	9	3
STLNOK.ST	<i>STO</i>	<i>NOK</i>		34	106	199	221
STLNOK.PAp	<i>PAR</i>	<i>NOK</i>	139	111	67	33	18
STLNOK.CHIp	<i>CHI</i>	<i>NOK</i>	–	–	–	2	–
0G57.L	<i>LSE</i>	<i>NOK</i>	130	227	42		
0M2Z.L	<i>LSE</i>	<i>NOK</i>			6	216	306
STOL.DE	<i>GER</i>	<i>EUR</i>	7	3			
STOL.F	<i>FRA</i>	<i>EUR</i>	4	1			
STLoI.CHI	<i>CHI</i>	<i>NOK</i>	15	43	87	159	
STLNOK.DEp	<i>GER</i>	<i>NOK</i>	12	5	3	7	2
STLEUR.DEp	<i>GER</i>	<i>EUR</i>	2	–	1	2	2
STL.OL	<i>OSL</i>	<i>NOK</i>	3417	2424	2523	2273	1107
STL.SG	<i>STU</i>	<i>EUR</i>		–	1	2	2
STL.F	<i>FRA</i>	<i>EUR</i>		–	3	8	6
STL.DE	<i>GER</i>	<i>EUR</i>		2	7	10	7
STL.DEU	<i>DEU</i>	<i>EUR</i>		3	13	21	15

Table 19 Trading of Norsk Hydro (NHY) on European exchanges. Annual number of shares (in millions) traded.

The table shows distribution of trading of Norsk Hydro across European exchanges by the various Reuters identifiers. Each row gives data for one Reuters identifier (RIC). The first column is the Reuters identifier (RIC). The second is the exchange code, the third the currency. For each of the years 2008–2012 we calculate the annual number of shares traded on the given exchange. The numbers are reported in millions. Observations indicated with – is a case where the numbers of shares traded is less than one million. Empties signifies no trading at all the given year. We have left out exchanges where the number of trades is never above one million shares.

RIC	Exchange	Curr	2008	2009	2010	2011	2012
NHYNOK.xbo	<i>XBO</i>	<i>NOK</i>		2321	2778	2513	1813
NHY.xNOK	<i>XDS– > TDS</i>	<i>NOK</i>	2010	129			
NHY.xEUR	<i>XDS– > TDS</i>	<i>EUR</i>	27	–			
NHY.mNOK	<i>BDS</i>	<i>NOK– > DKK– > NOK</i>	662	312	500	501	520
NHY.mGBP	<i>BDS</i>	<i>GBp</i>			4	4	–
NHY.mEUR	<i>BDS</i>	<i>EUR</i>	21	2	56	28	7
NHYN.L	<i>LSE</i>	<i>GBp– > NOK</i>			73	96	211
NHYol.TQ	<i>TRQ</i>	<i>NOK</i>	2	14	28	61	78
NHYol.SIG	<i>PAR– > 1114– > SIG</i>	<i>NOK</i>				–	4
NHYol.NQX	<i>763– > NQX</i>	<i>NOK</i>	–	1	–		
NHYol.CHI	<i>CHI</i>	<i>NOK</i>	17	59	105	139	
NHYol.BS	<i>BTE</i>	<i>NOK</i>	–	2	34	56	
NHYol.BD	<i>BUR</i>	<i>NOK</i>		–	3	5	2
NHYo.BS	<i>BTE</i>	<i>NOK</i>				25	112
NHYNOK.STp	<i>STO</i>	<i>NOK</i>	7	7	12	13	2
NHYNOK.ST	<i>STO</i>	<i>NOK</i>		25	83	106	118
NHYNOK.PAp	<i>PAR</i>	<i>NOK</i>	26	23	49	30	16
NHYNOK.DEp	<i>GER</i>	<i>NOK</i>	4	4	2	11	2
NHYEUR.PAp	<i>PAR</i>	<i>EUR</i>	52	14			
NHYEUR.DEp	<i>GER</i>	<i>EUR</i>	–	–	6	–	–
NHY.TG	<i>FRA– > TDG</i>	<i>EUR</i>			–	2	1
NHY.OL	<i>OSL</i>	<i>NOK</i>	2314	2518	2629	2134	1287
NHY.F	<i>FRA</i>	<i>EUR</i>	1	2	3	3	2
NHY.DEU	<i>DEU</i>	<i>EUR</i>	2	3	6	6	4
NHY.DE	<i>GER</i>	<i>EUR</i>	–	–	2	2	–
NHYKF.PK	<i>PNK– > PKC– > PNK– > PKC</i>	<i>USD</i>	–	3	–	–	1
NHY.PA	<i>PAR</i>	<i>EUR– > NOK</i>	2	1			
NHYq.L	<i>LSE</i>	<i>NOK– > GBp</i>	102	17	–		
NHYo.CHI	<i>CHI</i>	<i>NOK</i>				65	262

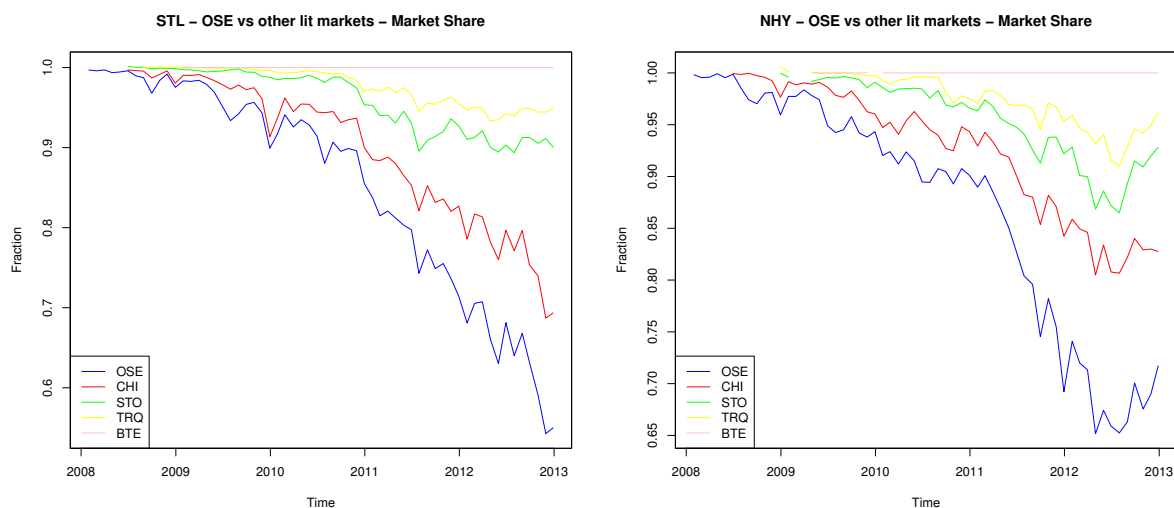
11.1 Examples, fragmentation

We look at some of the larger companies on the OSE, showing how trading is distributed among the lit exchanges (OSE, CHI, STO, TRQ and BTE). This is done using data from Reuters, where we calculate the daily market share for each of these five exchanges. To make the pictures easier to interpret, we calculate the monthly average of each market share (which explains why the market shares do not sum exactly to one).

Figure 84 Fragmentation of trading among lit exchanges - Example companies

The figures shows monthly averages of daily market shares. Each day, we sum the number of stocks traded across

Panel A: STL and NHY



Panel B: TEL and YAR

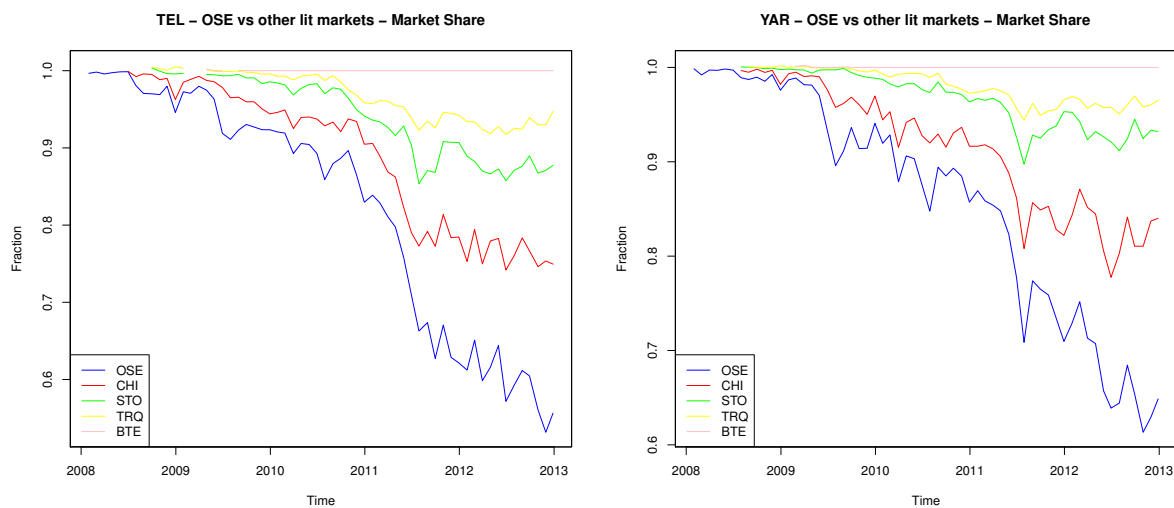


Figure 85 Fragmentation of trading among lit exchanges - Example companies - ctd

The figures shows monthly averages of daily market shares. Each day, we sum the number of stocks traded across

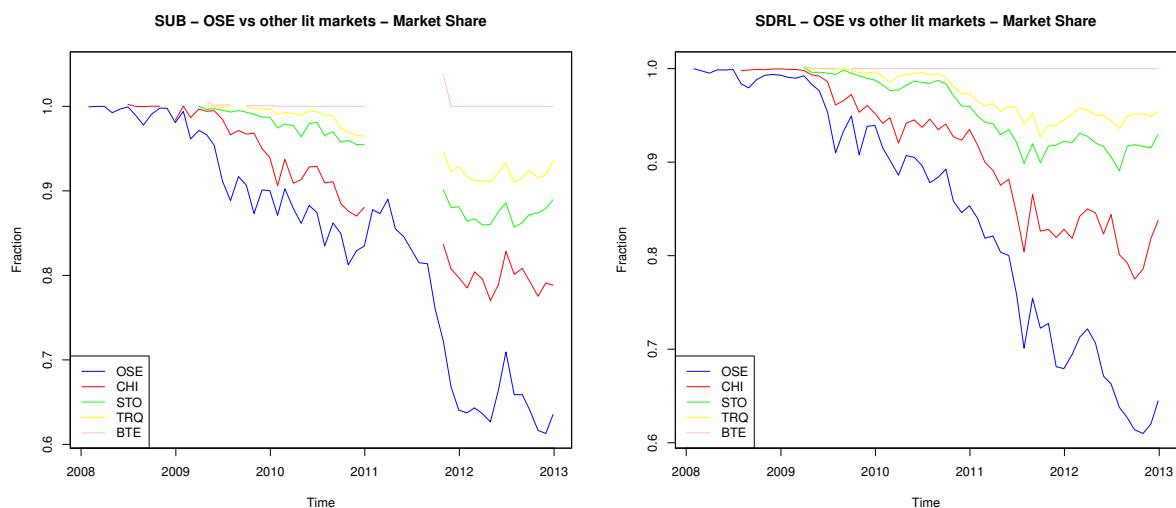
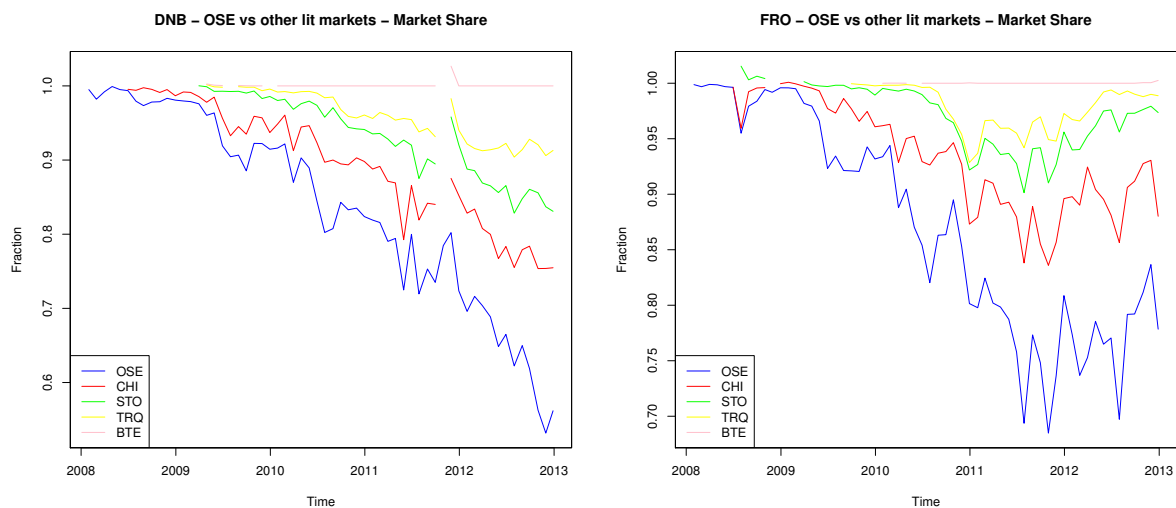
Panel A: SUB and SDRL**Panel B: DNB and FRO**

Figure 86 Fragmentation of trading among lit exchanges - Example companies - ctd

The figures shows monthly averages of daily market shares. Each day, we sum the number of stocks traded across

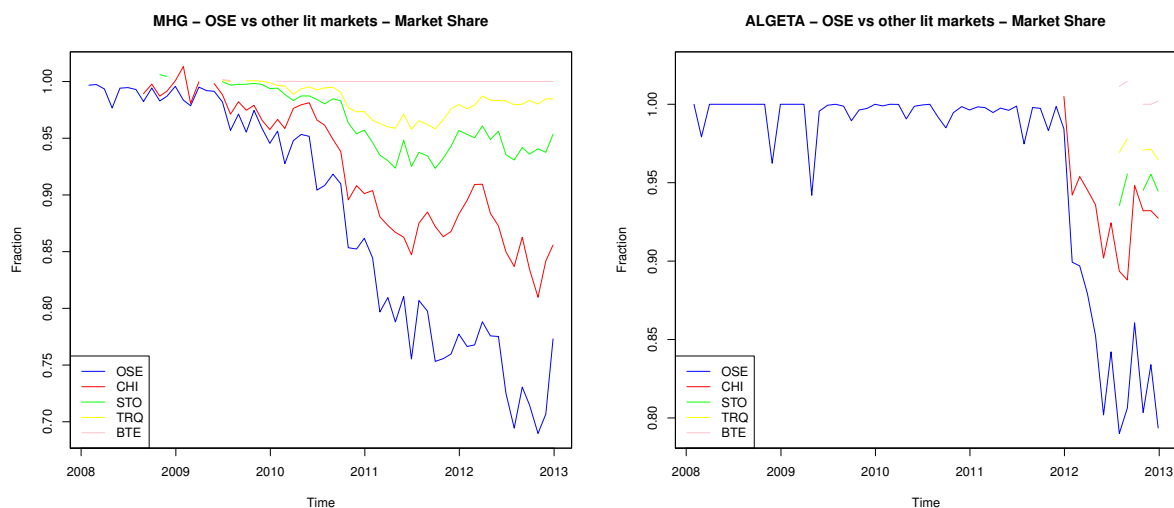
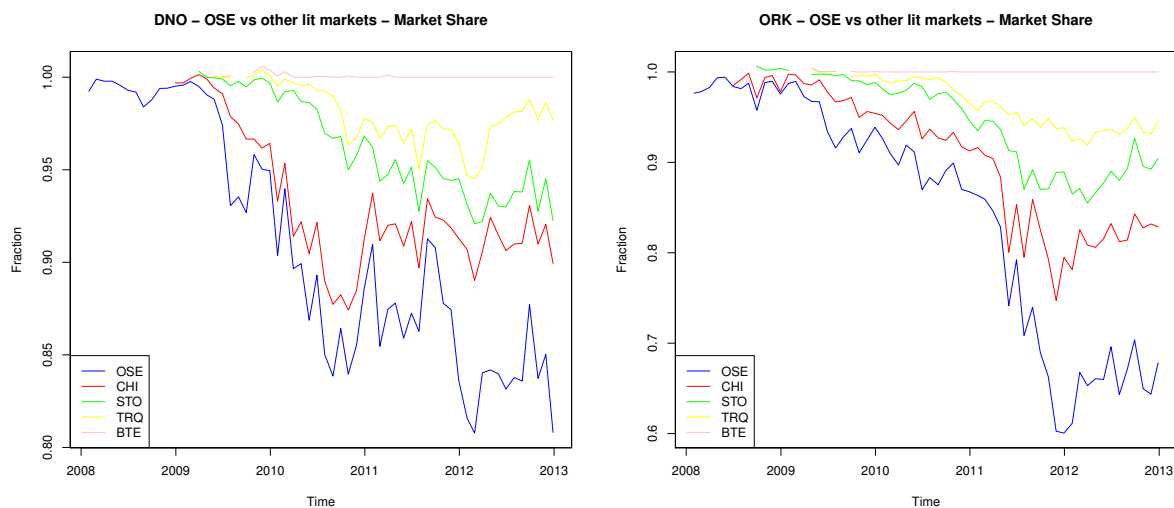
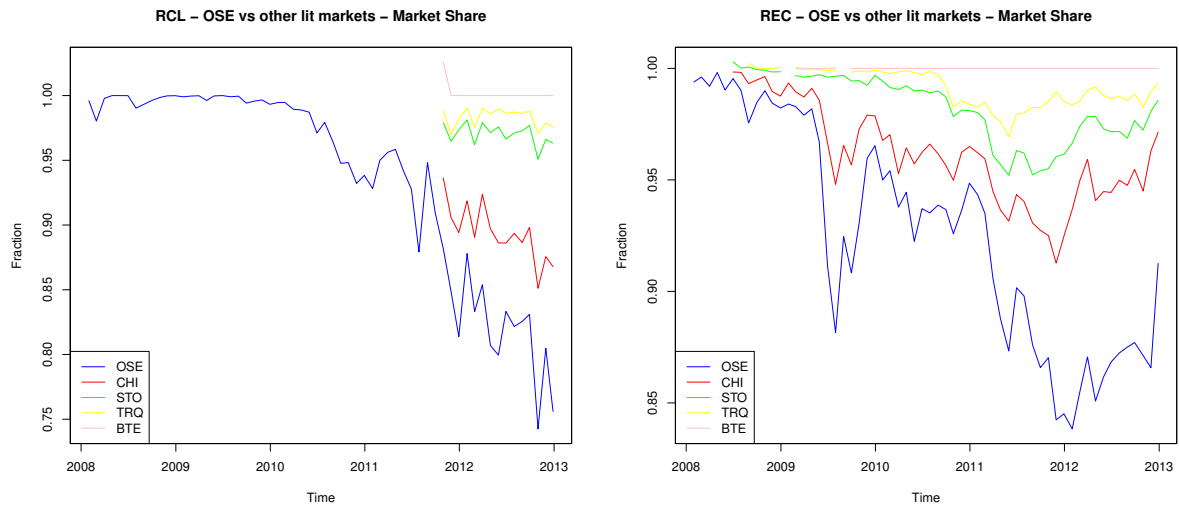
Panel A: MHG and ALGETA**Panel B: DNO and ORK**

Figure 87 Fragmentation of trading among lit exchanges - Example companies - ctd

The figures shows monthly averages of daily market shares. Each day, we sum the number of stocks traded across

Panel A: RCL and REC



Panel B: AKER and CEQ

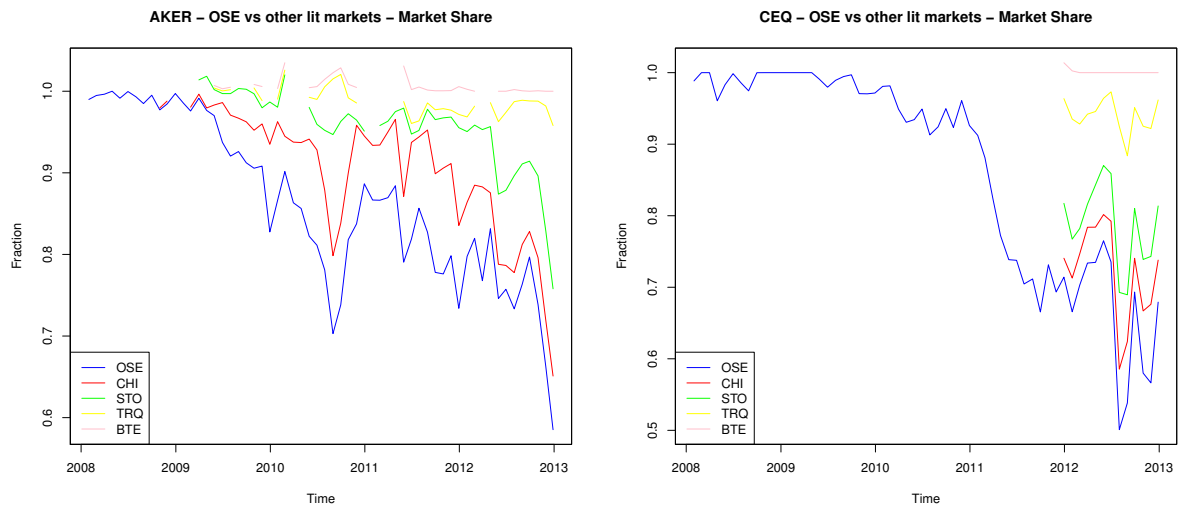
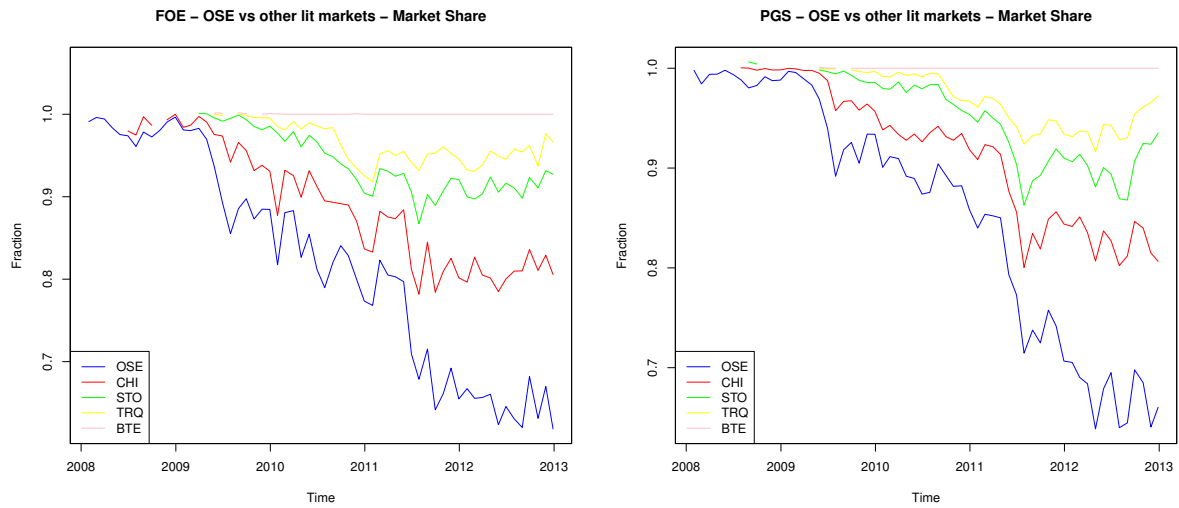


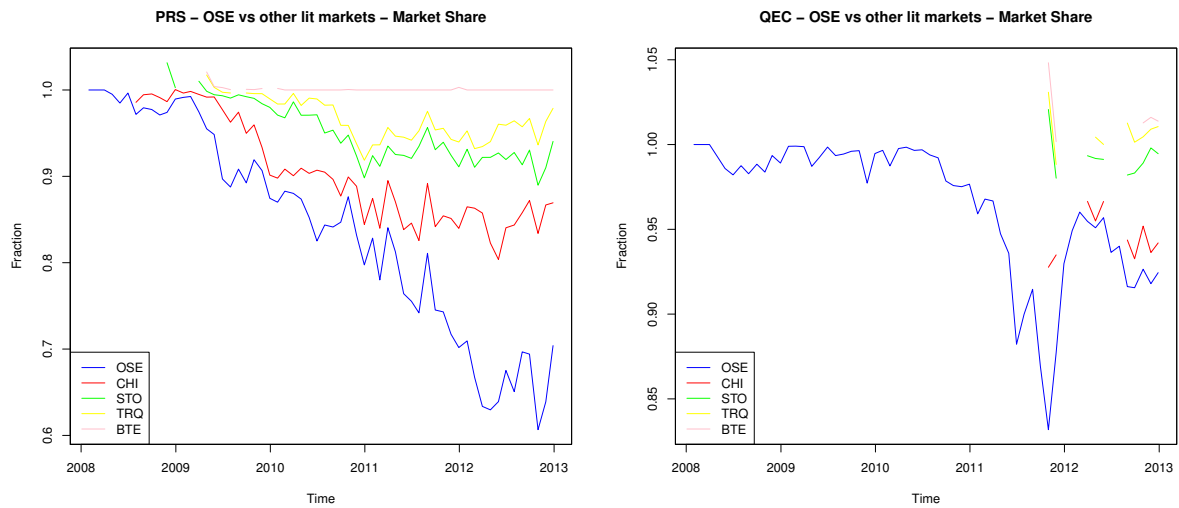
Figure 88 Fragmentation of trading among lit exchanges - Example companies - ctd

The figures shows monthly averages of daily market shares. Each day, we sum the number of stocks traded across

Panel A: FOE and PGS



Panel B: PRS and QEC



References

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