Empirics of the Oslo Stock Exchange:.
Asset pricing results. 1980–2016.

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Abstract

We show the results of numerous asset pricing specifications on the crossection of assets at the Oslo Stock Exchange using data from 1980–2016.

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Contents

1 Introduction

A prime prediction of any finance model is that there is relationship between risk and return, that more risky securities should require a higher return. Empirical asset pricing studies explore this relationship empirically. To take such a relationship to data one has to specify how risk is measured, and specify the relationship between the measured risk and asset prices (and returns). There is a large number of such empirical specifications.

In this paper we show a number of empirical asset pricing explorations using data from the Oslo Stock Exchange (OSE). This paper is not a self-contained study of asset pricing at the OSE, it is much more limited. Rather, it is a collection of results from applying standard asset pricing analysis to data from the OSE. A prime purpose of the paper is pedagogical, this paper contains a lot of results about the OSE which is useful when teaching asset pricing in the Norwegian context. As such the paper complements the analysis in Ødegaard (2016), which has a similar purpose, but is of a more descriptive nature. In this paper we concentrate on applications related to asset pricing.

A more complete analysis of asset pricing at the OSE was recently done in Næs, Skjeltorp, and Ødegaard (2008) (english version: Næs, Skjeltorp, and Ødegaard (2009)) Another purpose of the present paper is to update (some of) the analysis in Næs et al. (2008) with data through 2014, ie. it includes the recent crisis period.

1.1 Computer code

Reflecting the pedagogical purpose of this document, we also provide much of the computer code that has been used to do the estimation. The software package most commonly used to estimate these types of problems is R. For students and academics wanting to replicate the analysis done above we provide examples illustrating how it is estimated using R.

2 Describing Portfolios

We use a number of portfolios of OSE stocks. The portfolios are constructed by grouping the stocks on the exchange according some criterion.

2.1 Industry portfolios

For example, we construct ten industry portfolios by categorizing the stocks on the OSE according to the GICS standard, as shown in table 1.

Table 1 The GICS standard

GICS code	industry
10	Energy and consumption
15	Material/labor
20	Industrials
25	Consumer Discretionary
30	Consumer Staples
35	Health Care/liability
40	Financials
45	Information Technology (IT)
50	Telecommunication Services
55	Utilities

These 10 portfolios are characterized in table 2.

Table 2 Describing ten industry returns

Panel A: Returns

Statistic	Mean	St. Dev.	Min	Median	Max	N
Energy (10)	0.019	0.092	−0.283	0.016	0.654	444
Material (15)	0.019	0.117	−0.447	0.013	1.490	444
Industry (20)	0.017	0.060	−0.187	0.017	0.303	444
ConsDisc (25)	0.016	0.070	−0.203	0.014	0.433	444
ConsStapl (30)	0.021	0.065	−0.213	0.023	0.209	444
Health (35)	0.017	0.088	−0.330	0.011	0.686	444
Finan (40)	0.013	0.048	−0.147	0.011	0.267	444
IT (45)	0.024	0.106	−0.288	0.013	0.711	444
Telecom (50)	0.012	0.096	−0.454	0.007	0.328	260
Util (55)	0.010	0.061	−0.229	0.010	0.301	252

Panel B: Excess Returns

Statistic	Mean	St. Dev.	Min	Median	Max	N
Energy (10)	0.015	0.093	−0.288	0.013	0.645	410
Material (15)	0.010	0.117	−0.450	0.005	1.487	410
Industry (20)	0.011	0.061	−0.198	0.012	0.293	410
ConsDisc (25)	0.010	0.072	−0.207	0.008	0.430	410
ConsStapl (30)	0.013	0.066	−0.218	0.015	0.206	410
Health (35)	0.010	0.091	−0.342	0.005	0.681	410
Finan (40)	0.006	0.050	−0.156	0.006	0.259	410
IT (45)	0.017	0.107	−0.294	0.006	0.702	410
Telecom (50)	0.010	0.102	−0.460	0.002	0.321	227
Util (55)	0.004	0.064	−0.234	0.003	0.297	219

2.2 Size portfolios

An alternative sort is to rank the companies on the OSE by their size, and group them into ten size based portfolios, by increasing firm size. Table 3 describes these portfolios

Table 3 Describing ten size returns

Panel A: Returns

Statistic	Mean	St. Dev.	Min	Median	Max	N
1(small)	0.029	0.071	−0.181	0.019	0.467	432
2	0.021	0.067	−0.184	0.015	0.319	432
3	0.016	0.066	−0.241	0.015	0.323	432
4	0.015	0.067	−0.249	0.013	0.291	432
5	0.019	0.069	−0.192	0.018	0.533	432
6	0.017	0.064	−0.286	0.018	0.278	432
7	0.015	0.069	−0.242	0.015	0.490	432
8	0.014	0.069	−0.240	0.015	0.271	432
9	0.011	0.075	−0.285	0.013	0.228	432
10(large)	0.010	0.071	−0.339	0.012	0.249	432

Panel B: Excess Returns

Statistic	Mean	St. Dev.	Min	Median	Max	N
1(small)	0.023	0.072	−0.190	0.012	0.456	410
2	0.016	0.068	−0.188	0.010	0.311	410
3	0.010	0.067	−0.252	0.010	0.312	410
4	0.010	0.069	−0.257	0.010	0.282	410
5	0.013	0.070	−0.198	0.013	0.525	410
6	0.011	0.066	−0.295	0.011	0.269	410
7	0.009	0.071	−0.253	0.009	0.480	410
8	0.008	0.070	−0.249	0.011	0.265	410
9	0.006	0.077	−0.297	0.010	0.224	410
10(large)	0.004	0.073	−0.345	0.008	0.242	410

2.3 B/M portfolios

Another alternative sort is to rank the companies on the OSE by their B/M ratio, and group them into ten book/market based portfolios, by increasing B/M ratio. Table 4 describes these portfolios

Table 4 Describing ten bm returns

Panel A: Returns

Statistic	Mean	St. Dev.	Min	Median	Max	N
1(low b/m)	0.013	0.073	−0.305	0.013	0.325	432
2	0.020	0.083	−0.260	0.018	0.647	432
3	0.012	0.076	−0.225	0.011	0.427	432
4	0.016	0.066	−0.182	0.016	0.235	432
5	0.014	0.067	−0.241	0.015	0.318	432
6	0.016	0.068	−0.265	0.012	0.261	432
7	0.023	0.073	−0.209	0.022	0.382	432
8	0.022	0.074	−0.289	0.018	0.401	432
9	0.021	0.072	−0.252	0.020	0.410	432
10(high b/m)	0.024	0.075	−0.205	0.019	0.408	432

Panel B: Excess Returns

Statistic	Mean	St. Dev.	Min	Median	Max	N
1(low b/m)	0.008	0.075	−0.311	0.007	0.314	399
2	0.013	0.086	−0.264	0.013	0.638	399
3	0.006	0.078	−0.230	0.006	0.417	399
4	0.011	0.068	−0.193	0.012	0.229	399
5	0.009	0.069	−0.252	0.012	0.309	399
6	0.009	0.069	−0.272	0.005	0.250	399
7	0.016	0.074	−0.220	0.014	0.372	399
8	0.016	0.076	−0.298	0.014	0.391	399
9	0.016	0.074	−0.261	0.016	0.402	399
10(high b/m)	0.019	0.078	−0.214	0.017	0.400	399

2.4 Relative spread portfolios

We also sort portfolios on liquidity. We sort the companies on the OSE on a measure of the relative spread. We calculate average relative spread for the year before we form portfolios. Table 5 describes these portfolios

Table 5 Describing ten bm returns

Panel A: Returns

Statistic	Mean	St. Dev.	Min	Median	Max	N
1(low spread)	0.013	0.069	−0.255	0.018	0.229	432
2	0.013	0.068	−0.268	0.015	0.204	432
3	0.015	0.071	−0.255	0.016	0.324	432
4	0.014	0.061	−0.211	0.021	0.255	432
5	0.016	0.066	−0.241	0.013	0.483	432
6	0.014	0.061	−0.199	0.011	0.259	432
7	0.017	0.065	−0.169	0.011	0.317	432
8	0.018	0.066	−0.207	0.011	0.340	432
9	0.025	0.066	−0.188	0.017	0.336	432
10(high spread)	0.025	0.073	−0.206	0.015	0.543	432

Panel B: Excess Returns

Statistic	Mean	St. Dev.	Min	Median	Max	N
1(low spread)	0.007	0.072	−0.261	0.012	0.227	399
2	0.007	0.070	−0.280	0.010	0.194	399
3	0.010	0.073	−0.264	0.013	0.319	399
4	0.008	0.063	−0.217	0.014	0.244	399
5	0.009	0.067	−0.250	0.007	0.472	399
6	0.009	0.063	−0.204	0.005	0.249	399
7	0.010	0.066	−0.173	0.005	0.308	399
8	0.013	0.068	−0.219	0.008	0.332	399
9	0.018	0.067	−0.197	0.012	0.329	399
10(high spread)	0.020	0.075	−0.217	0.011	0.533	399

3 Pricing Factors for Asset Pricing

In this chapter we discuss construction of pricing factors a la Fama and French (1996) and Carhart (1997). Using the definitions in these papers similar algorithms are applied to asset pricing data for the Oslo Stock Exchange. We then see whether these factor portfolios are helpful in describing the crossection of Norwegian asset returns.

3.1 Fama French factors

The two factors `SMB` and `HML` were introduced in Fama and French (1996). For the construction they split data for the US stock market as shown in figure 1.

Figure 1 The construction of the two Fama and French (1996) factors

		Book/Market		
		L	H	M
Size	Small	S/L	S/M	S/H
	Big	B/L	B/M	B/H

The pricing factors are then constructed as:

$$\texttt{SMB} = \text{average}(S/L, S/M, S/H) - \text{average}(B/L, B/M, B/H)$$

$$\texttt{HML} = \text{average}(S/H, B/H) - \text{average}(S/L, B/L)$$

Similar factors are constructed for the Norwegian stock market by doing a split just like that done by FF, a double sort into six different portfolios. End of June values of the stock and B/M are used to perform the sorting. Within each portfolio returns are calculated as the value weighted average of the constituent stocks. Table 6 describes these six portfolios.

Table 6 Average returns for the six portfolios used in the FF construction

1980–2016

SL		SM		SH	
2.36	(7.32)	2.98	(7.39)	2.81	(6.62)
BL		BM		BH	
1.68	(7.45)	1.85	(6.42)	2.10	(8.00)

1980–1989

SL		SM		SH	
2.50	(8.28)	4.13	(9.18)	4.37	(7.73)
BL		BM		BH	
2.24	(8.12)	2.64	(7.45)	3.46	(8.89)

1990–1999

SL		SM		SH	
2.57	(7.90)	2.97	(7.67)	3.08	(7.35)
BL		BM		BH	
1.97	(6.59)	1.50	(6.77)	1.60	(8.87)

2000–2016

SL		SM		SH	
2.16	(6.38)	2.40	(6.02)	1.86	(5.26)
BL		BM		BH	
1.22	(7.54)	1.66	(5.57)	1.72	(6.84)

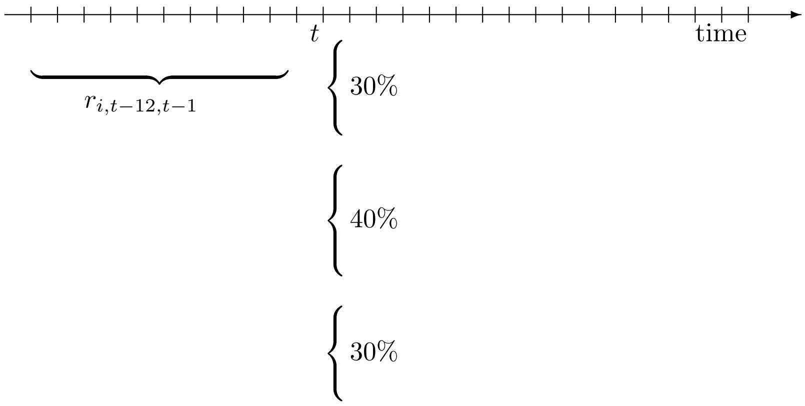
The table shows average returns for the six portfolios S/L, S/M, S/H, B/L, B/M and B/H.

3.2 Momentum

3.2.1 The Carhart factor PR1YR

Carhart (1997) introduced an additional factor that accounts for momentum. Figure 2 illustrates this factor construction. Each month the stock return is calculated over the previous eleven months. The returns are ranked, and split into three portfolios: The top 30%, the median 40% and the bottom 30%. The Carhart (1997) factor PR1YR is the difference between the average return of the top and the bottom portfolios. The ranking is recalculated every month.

Figure 2 The construction of the Carhart (1997) factor PR1YR



3.2.2 An alternative momentum factor: UMD

Ken French introduces an alternative momentum factor UMD, which he describes as follows:

> *....a momentum factor, constructed from six value-weight portfolios formed using independent sorts on size and prior return of NYSE, AMEX, and NASDAQ stocks. Mom is the average of the returns on two (big and small) high prior return portfolios minus the average of the returns on two low prior return portfolios. The portfolios are constructed monthly. Big means a firm is above the median market cap on the NYSE at the end of the previous month; small firms are below the median NYSE market cap. Prior return is measured from month -12 to - 2. Firms in the low prior return portfolio are below the 30th NYSE percentile. Those in the high portfolio are above the 70th NYSE percentile.* (from Ken French's web site)

3.3 Liquidty

In Næs et al. (2009) a *liquidity* factor is constructed.

3.4 Describing the calculated factors

Table 7 gives some descriptive statistics for the calculated factors. The averages seem to be significantly different from zero, at least for some of them, and they are relatively little correlated.

Table 7 Descriptive statistics for asset pricing factors.

Average

	SMB		HML		PR1YR		UMD	
1980–2016	0.84	(0.00)	0.44	(0.07)	1.03	(0.00)	0.89	(0.00)
1980–1989	0.89	(0.07)	1.55	(0.00)	2.28	(0.00)	1.85	(0.00)
1990–1999	1.19	(0.01)	0.07	(0.89)	-0.17	(0.71)	-0.21	(0.70)
2000–2016	0.61	(0.03)	0.10	(0.75)	1.07	(0.00)	1.03	(0.01)

Correlations

	SMB	HML	PR1YR
HML	-0.12		
PR1YR	0.13	-0.03	
UMD	0.13	-0.04	0.78

The table describes the calculated asset pricing factors. `SMB` and `HML` are the Fama and French (1996) pricing factors. `PR1YR` is the Carhart (1997) factor. The table list the average percentage monthly return, and in parenthesis the p-value for a test of difference from zero.

4 Ex Post Mean Variance Optimal Portfolios

A useful way of getting some understanding of the properties of portfolios sorted by some criteria is to investigate how they are mixed in mean-variance optimal portfolios.

Suppose we have $n \geq 2$ risky securities, with expected returns $\mathbf{e}$:

$$\mathbf{e} = \begin{bmatrix} E[r_1] \\ E[r_2] \\ \vdots \\ E[r_n] \end{bmatrix}$$

and covariance matrix $\mathbf{V}$:

$$\mathbf{V} = \begin{bmatrix} \sigma(r_1, r_1) & \sigma(r_1, r_2) & \dots & \\ \sigma(r_2, r_1) & \sigma(r_2, r_2) & \dots & \\ \vdots & & & \\ \sigma(r_n, r_1) & & \dots & \sigma(r_n, r_n) \end{bmatrix}$$

The covariance matrix $\mathbf{V}$ is assumed invertible.

A portfolio p is defined by the set of weights $\mathbf{w}$ invested in the n risky assets.

$$\mathbf{w} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix}$$

The expected return on a portfolio is calculated as

$$E[r_p] = \mathbf{w}'\mathbf{e}$$

and the variance of the portfolio is

$$\sigma^2(r_p) = \mathbf{w}'\mathbf{V}\mathbf{w}$$

A portfolio is a *frontier* portfolio if it minimizes the variance for a given expected return. That is, a frontier portfolio p solves

$$\mathbf{w}_p = \arg\min_{\mathbf{w}} \frac{1}{2}\mathbf{w}'\mathbf{V}\mathbf{w}$$

s.t.

$$\mathbf{w}'\mathbf{e} = E[\tilde{r}_p]$$

$$\mathbf{w}'\mathbf{1} = 1$$

The set of all frontier portfolios is called the *minimum variance frontier.*

If in addition a constraint of *no short sales* is imposed, the minimization problem has the additional constraints

$$w_i \geq 0 \; \forall \; i$$

In this section we use actual portfolios at the Oslo Stock Exchange and construct the optimal frontier combinations. To do this calculation we need estimates of expected returns $\mathbf{e}$ and the covariance matrix $\mathbf{V}$. In the following calculations empirical data on monthly returns from a given subperiod is used to to find means and covariances. Given these estimates of $\mathbf{e}$ and $\mathbf{V}$ we calculate the resulting (ex post) mean-variance optimized portfolios. Three subperiods, 1980-89, 1990-99 and 2000-2016 are considered.

4.1 Size portfolios

In this section we consider the portfolios sorted by equity size.

Table 8 Mean variance optimal portfolios. 10 portfolios. Using data from 1980 to 1989

Panel A: Expected Returns

Asset	mean	std
1 (small)	0.048	0.098
2	0.031	0.077
3	0.020	0.083
4	0.020	0.078
5	0.023	0.083
6	0.025	0.066
7	0.020	0.083
8	0.018	0.067
9	0.018	0.081
10	0.012	0.074

Panel B: Correlations matrix

$\rho(i,j)$	1 (small)	2	3	4	5	6	7	8	9	10
1 (small)	1	0.494	0.613	0.543	0.586	0.539	0.453	0.452	0.483	0.387
2	0.494	1	0.55	0.528	0.453	0.443	0.459	0.522	0.476	0.448
3	0.613	0.55	1	0.657	0.66	0.657	0.615	0.657	0.663	0.566
4	0.543	0.528	0.657	1	0.591	0.646	0.636	0.676	0.703	0.571
5	0.586	0.453	0.66	0.591	1	0.599	0.512	0.601	0.543	0.439
6	0.539	0.443	0.657	0.646	0.599	1	0.611	0.626	0.638	0.571
7	0.453	0.459	0.615	0.636	0.512	0.611	1	0.729	0.733	0.593
8	0.452	0.522	0.657	0.676	0.601	0.626	0.729	1	0.772	0.685
9	0.483	0.476	0.663	0.703	0.543	0.638	0.733	0.772	1	0.703
10	0.387	0.448	0.566	0.571	0.439	0.571	0.593	0.685	0.703	1

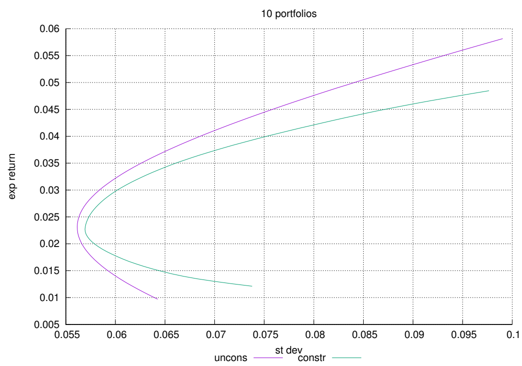
Panel C: Optimal unconstrained portfolios

Asset	Expected Return								
	0.00969	0.0157	0.0218	0.0279	0.0339	0.04	0.046	0.0521	0.0582
1 (small)	-0.336	-0.18	-0.0245	0.131	0.287	0.443	0.599	0.754	0.91
2	0.108	0.166	0.224	0.283	0.341	0.399	0.457	0.516	0.574
3	0.0751	-0.00828	-0.0917	-0.175	-0.258	-0.342	-0.425	-0.509	-0.592
4	0.138	0.0836	0.0294	-0.0247	-0.0789	-0.133	-0.187	-0.242	-0.296
5	0.141	0.104	0.068	0.0317	-0.00466	-0.041	-0.0773	-0.114	-0.15
6	0.297	0.354	0.41	0.466	0.522	0.579	0.635	0.691	0.748
7	-0.034	-0.0299	-0.0258	-0.0216	-0.0175	-0.0134	-0.00926	-0.00513	-0.001
8	0.344	0.338	0.332	0.326	0.32	0.314	0.307	0.301	0.295
9	-0.164	-0.152	-0.139	-0.127	-0.115	-0.103	-0.0905	-0.0782	-0.066
10	0.431	0.324	0.218	0.111	0.00438	-0.102	-0.209	-0.315	-0.422

Panel D: Optimal short sale restricted portfolios

Asset	Expected Return								
	0.0121	0.0167	0.0212	0.0257	0.0303	0.0348	0.0394	0.0439	0.0485
1 (small)	-	-	-	0.071	0.195	0.347	0.546	0.747	1
2	-	-	0.165	0.256	0.289	0.311	0.285	0.253	-
3	-	-	-	-	-	-	-	-	-
4	-	0.0405	-	-	-	-	-	-	-
5	-	0.0283	0.0312	-	-	-	-	-	-
6	-	0.15	0.322	0.398	0.393	0.342	0.169	-	-
7	-	-	-	-	-	-	-	-	-
8	-	0.332	0.262	0.21	0.123	-	-	-	-
9	-	-	-	-	-	-	-	-	-
10	1	0.449	0.22	0.0663	-	-	-	-	-

Panel E: Illustrating portfolio frontiers



The table describes the construction of mean-variance optimal portfolios using historical data from the Oslo Stock Exchange.

Table 9 Mean variance optimal portfolios. 10 portfolios. Using data from 1990 to 1999

Panel A: Expected Returns

Asset	mean	std
1 (small)	0.031	0.076
2	0.023	0.076
3	0.015	0.069
4	0.018	0.079
5	0.020	0.063
6	0.012	0.074
7	0.006	0.070
8	0.011	0.081
9	0.010	0.077
10	0.010	0.070

Panel B: Correlations matrix

$\rho(i,j)$	1 (small)	2	3	4	5	6	7	8	9	10
1 (small)	1	0.487	0.39	0.443	0.471	0.421	0.451	0.328	0.445	0.329
2	0.487	1	0.694	0.7	0.7	0.698	0.656	0.593	0.557	0.385
3	0.39	0.694	1	0.68	0.662	0.614	0.683	0.659	0.607	0.466
4	0.443	0.7	0.68	1	0.716	0.704	0.684	0.62	0.608	0.515
5	0.471	0.7	0.662	0.716	1	0.743	0.716	0.718	0.66	0.528
6	0.421	0.698	0.614	0.704	0.743	1	0.75	0.677	0.734	0.609
7	0.451	0.656	0.683	0.684	0.716	0.75	1	0.728	0.815	0.68
8	0.328	0.593	0.659	0.62	0.718	0.677	0.728	1	0.728	0.669
9	0.445	0.557	0.607	0.608	0.66	0.734	0.815	0.728	1	0.748
10	0.329	0.385	0.466	0.515	0.528	0.609	0.68	0.669	0.748	1

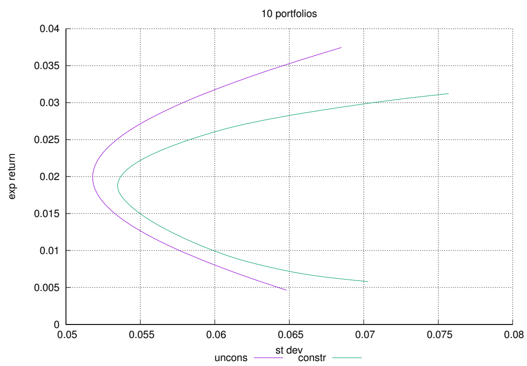
Panel C: Optimal unconstrained portfolios

Asset	Expected Return								
	0.00463	0.00874	0.0128	0.0169	0.0211	0.0252	0.0293	0.0334	0.0375
1 (small)	-0.0654	0.0124	0.0903	0.168	0.246	0.324	0.402	0.48	0.557
2	-0.229	-0.155	-0.0809	-0.00668	0.0676	0.142	0.216	0.29	0.365
3	0.386	0.361	0.336	0.311	0.286	0.262	0.237	0.212	0.187
4	-0.198	-0.176	-0.154	-0.132	-0.11	-0.0876	-0.0656	-0.0436	-0.0216
5	0.153	0.228	0.303	0.379	0.454	0.53	0.605	0.68	0.756
6	0.177	0.128	0.0784	0.0292	-0.02	-0.0692	-0.118	-0.168	-0.217
7	0.719	0.529	0.338	0.148	-0.0425	-0.233	-0.423	-0.614	-0.804
8	-0.117	-0.129	-0.14	-0.152	-0.164	-0.175	-0.187	-0.199	-0.21
9	-0.278	-0.25	-0.221	-0.193	-0.165	-0.137	-0.108	-0.0802	-0.052
10	0.453	0.451	0.45	0.448	0.446	0.445	0.443	0.442	0.44

Panel D: Optimal short sale restricted portfolios

Asset	Expected Return								
	0.00579	0.00897	0.0121	0.0153	0.0185	0.0217	0.0249	0.028	0.0312
1 (small)	-	-	0.0356	0.118	0.221	0.351	0.476	0.688	1
2	-	-	-	-	-	0.0346	0.0889	0.124	-
3	-	0.182	0.225	0.229	0.206	0.119	0.0247	-	-
4	-	-	-	-	-	-	-	-	-
5	-	-	0.134	0.213	0.285	0.326	0.357	0.188	-
6	-	-	-	-	-	-	-	-	-
7	1	0.469	0.252	0.0989	-	-	-	-	-
8	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-
10	-	0.349	0.353	0.341	0.289	0.17	0.0536	-	-

Panel E: Illustrating portfolio frontiers



The table describes the construction of mean-variance optimal portfolios using historical data from the Oslo Stock Exchange.

Table 10 Mean variance optimal portfolios. 10 portfolios. Using data from 2000 to 2016

Panel A: Expected Returns

Asset	mean	std
1 (small)	0.016	0.037
2	0.015	0.052
3	0.015	0.050
4	0.011	0.049
5	0.016	0.061
6	0.015	0.055
7	0.017	0.057
8	0.014	0.059
9	0.006	0.069
10	0.009	0.068

Panel B: Correlations matrix

$\rho(i,j)$	1 (small)	2	3	4	5	6	7	8	9	10
1 (small)	1	0.42	0.361	0.388	0.395	0.405	0.441	0.424	0.375	0.278
2	0.42	1	0.555	0.521	0.567	0.532	0.596	0.606	0.542	0.484
3	0.361	0.555	1	0.586	0.581	0.606	0.67	0.658	0.656	0.614
4	0.388	0.521	0.586	1	0.564	0.64	0.624	0.654	0.683	0.614
5	0.395	0.567	0.581	0.564	1	0.653	0.706	0.68	0.705	0.673
6	0.405	0.532	0.606	0.64	0.653	1	0.78	0.738	0.707	0.659
7	0.441	0.596	0.67	0.624	0.706	0.78	1	0.831	0.804	0.743
8	0.424	0.606	0.658	0.654	0.68	0.738	0.831	1	0.792	0.763
9	0.375	0.542	0.656	0.683	0.705	0.707	0.804	0.792	1	0.781
10	0.278	0.484	0.614	0.614	0.673	0.659	0.743	0.763	0.781	1

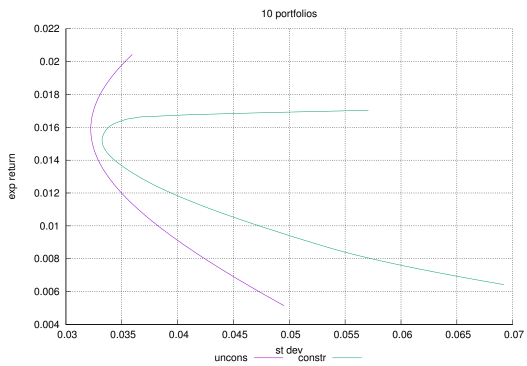
Panel C: Optimal unconstrained portfolios

Asset	Expected Return								
	0.00514	0.00705	0.00896	0.0109	0.0128	0.0147	0.0166	0.0185	0.0204
1 (small)	0.593	0.597	0.602	0.606	0.61	0.614	0.618	0.622	0.627
2	0.149	0.141	0.132	0.124	0.115	0.107	0.0981	0.0896	0.0811
3	0.0979	0.115	0.133	0.151	0.168	0.186	0.203	0.221	0.238
4	0.451	0.408	0.365	0.323	0.28	0.237	0.194	0.151	0.109
5	-0.259	-0.212	-0.165	-0.118	-0.0708	-0.0238	0.0233	0.0703	0.117
6	0.112	0.106	0.0994	0.0932	0.087	0.0808	0.0745	0.0683	0.0621
7	-0.828	-0.685	-0.543	-0.401	-0.258	-0.116	0.0265	0.169	0.311
8	-0.238	-0.211	-0.184	-0.157	-0.131	-0.104	-0.0772	-0.0505	-0.0238
9	0.576	0.443	0.31	0.178	0.0448	-0.088	-0.221	-0.354	-0.486
10	0.345	0.298	0.25	0.203	0.155	0.108	0.06	0.0124	-0.0352

Panel D: Optimal short sale restricted portfolios

Asset	Expected Return								
	0.00642	0.00775	0.00908	0.0104	0.0117	0.0131	0.0144	0.0157	0.017
1 (small)	-	-	0.0437	0.19	0.337	0.472	0.554	0.681	-
2	-	-	-	-	-	-	0.0394	0.0907	-
3	-	-	-	-	-	0.0167	0.1	0.164	-
4	-	0.315	0.522	0.475	0.428	0.378	0.275	0.0518	-
5	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	0.012	-
7	-	-	-	-	-	-	-	-	1
8	-	-	-	-	-	-	-	-	-
9	1	0.685	0.428	0.306	0.184	0.0638	-	-	-
10	-	-	0.00616	0.0287	0.0512	0.0696	0.0306	-	-

Panel E: Illustrating portfolio frontiers



The table describes the construction of mean-variance optimal portfolios using historical data from the Oslo Stock Exchange.

4.2 B/M portfolios

In this section we use the portfolios sorted by B/M.

Table 11 Mean variance optimal portfolios. 10 portfolios. Using data from 1981 to 1989

Panel A: Expected Returns

Asset	mean	std
1 (small)	0.018	0.077
2	0.027	0.111
3	0.015	0.102
4	0.023	0.079
5	0.025	0.081
6	0.024	0.076
7	0.037	0.088
8	0.030	0.096
9	0.034	0.079
10	0.040	0.074

Panel B: Correlations matrix

$\rho(i,j)$	1 (small)	2	3	4	5	6	7	8	9	10
1 (small)	1	0.541	0.531	0.582	0.631	0.688	0.53	0.594	0.549	0.57
2	0.541	1	0.643	0.5	0.671	0.45	0.43	0.435	0.386	0.418
3	0.531	0.643	1	0.55	0.622	0.578	0.525	0.597	0.609	0.584
4	0.582	0.5	0.55	1	0.653	0.69	0.656	0.706	0.637	0.628
5	0.631	0.671	0.622	0.653	1	0.665	0.6	0.69	0.583	0.691
6	0.688	0.45	0.578	0.69	0.665	1	0.771	0.728	0.692	0.705
7	0.53	0.43	0.525	0.656	0.6	0.771	1	0.648	0.628	0.691
8	0.594	0.435	0.597	0.706	0.69	0.728	0.648	1	0.695	0.716
9	0.549	0.386	0.609	0.637	0.583	0.692	0.628	0.695	1	0.701
10	0.57	0.418	0.584	0.628	0.691	0.705	0.691	0.716	0.701	1

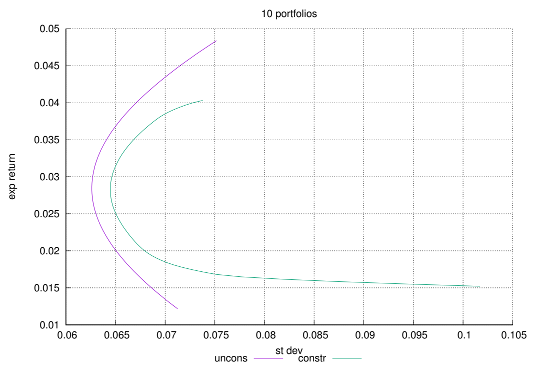
Panel C: Optimal unconstrained portfolios

Asset	Expected Return								
	0.0122	0.0167	0.0212	0.0258	0.0303	0.0348	0.0393	0.0439	0.0484
1 (small)	0.494	0.432	0.37	0.308	0.246	0.184	0.122	0.0596	-0.00245
2	-0.176	-0.122	-0.0675	-0.0132	0.0411	0.0953	0.15	0.204	0.258
3	0.233	0.156	0.0791	0.00238	-0.0744	-0.151	-0.228	-0.305	-0.381
4	0.439	0.384	0.329	0.275	0.22	0.165	0.111	0.0562	0.00162
5	0.258	0.213	0.169	0.124	0.0796	0.0351	-0.00949	-0.054	-0.0986
6	0.368	0.309	0.251	0.192	0.133	0.0739	0.015	-0.0438	-0.103
7	-0.22	-0.17	-0.12	-0.0695	-0.0193	0.0308	0.081	0.131	0.181
8	-0.326	-0.308	-0.291	-0.273	-0.256	-0.238	-0.22	-0.203	-0.185
9	0.0297	0.0774	0.125	0.173	0.221	0.268	0.316	0.364	0.412
10	-0.0995	0.0276	0.155	0.282	0.409	0.536	0.663	0.79	0.918

Panel D: Optimal short sale restricted portfolios

Asset	Expected Return								
	0.0152	0.0184	0.0215	0.0246	0.0278	0.0309	0.0341	0.0372	0.0403
1 (small)	-	0.581	0.405	0.356	0.291	0.218	0.136	0.0328	-
2	-	-	-	-	0.0121	0.0382	0.0518	0.0664	-
3	1	0.19	0.0567	0.01	-	-	-	-	-
4	-	0.229	0.251	0.221	0.18	0.134	0.0671	-	-
5	-	-	0.0743	0.0699	0.0395	-	-	-	-
6	-	-	0.146	0.113	0.0756	0.0364	-	-	-
7	-	-	-	-	-	0.00366	0.0476	0.08	-
8	-	-	-	-	-	-	-	-	-
9	-	-	0.0573	0.0978	0.13	0.159	0.18	0.194	-
10	-	-	0.00948	0.132	0.272	0.411	0.517	0.627	1

Panel E: Illustrating portfolio frontiers



The table describes the construction of mean-variance optimal portfolios using historical data from the Oslo Stock Exchange.

Table 12 Mean variance optimal portfolios. 10 portfolios. Using data from 1990 to 1999

Panel A: Expected Returns

Asset	mean	std
1 (small)	0.016	0.070
2	0.020	0.077
3	0.010	0.068
4	0.012	0.071
5	0.011	0.072
6	0.015	0.071
7	0.016	0.075
8	0.018	0.075
9	0.017	0.088
10	0.025	0.094

Panel B: Correlations matrix

$\rho(i,j)$	1 (small)	2	3	4	5	6	7	8	9	10
1 (small)	1	0.611	0.571	0.546	0.614	0.594	0.538	0.395	0.52	0.342
2	0.611	1	0.654	0.651	0.613	0.635	0.603	0.549	0.571	0.409
3	0.571	0.654	1	0.725	0.69	0.642	0.659	0.622	0.561	0.533
4	0.546	0.651	0.725	1	0.704	0.674	0.658	0.63	0.625	0.63
5	0.614	0.613	0.69	0.704	1	0.693	0.597	0.681	0.62	0.592
6	0.594	0.635	0.642	0.674	0.693	1	0.621	0.609	0.626	0.54
7	0.538	0.603	0.659	0.658	0.597	0.621	1	0.592	0.656	0.578
8	0.395	0.549	0.622	0.63	0.681	0.609	0.592	1	0.675	0.661
9	0.52	0.571	0.561	0.625	0.62	0.626	0.656	0.675	1	0.744
10	0.342	0.409	0.533	0.63	0.592	0.54	0.578	0.661	0.744	1

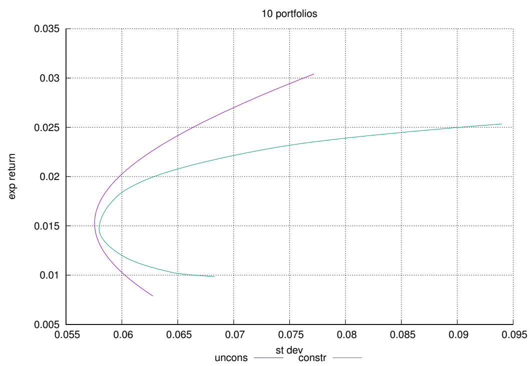
Panel C: Optimal unconstrained portfolios

Asset	Expected Return								
	0.00788	0.0107	0.0135	0.0163	0.0191	0.022	0.0248	0.0276	0.0304
1 (small)	0.236	0.281	0.327	0.372	0.418	0.463	0.509	0.554	0.599
2	-0.253	-0.152	-0.0508	0.0502	0.151	0.252	0.353	0.454	0.555
3	0.456	0.35	0.243	0.137	0.0305	-0.0759	-0.182	-0.289	-0.395
4	0.272	0.206	0.14	0.074	0.00791	-0.0582	-0.124	-0.19	-0.256
5	0.204	0.115	0.0264	-0.0625	-0.151	-0.24	-0.329	-0.418	-0.507
6	0.123	0.133	0.142	0.152	0.161	0.171	0.18	0.19	0.199
7	0.0742	0.0817	0.0893	0.0968	0.104	0.112	0.119	0.127	0.134
8	0.123	0.166	0.209	0.253	0.296	0.34	0.383	0.427	0.47
9	0.0363	-0.0294	-0.0951	-0.161	-0.227	-0.292	-0.358	-0.424	-0.49
10	-0.272	-0.152	-0.0315	0.0886	0.209	0.329	0.449	0.569	0.689

Panel D: Optimal short sale restricted portfolios

Asset	Expected Return								
	0.00985	0.0118	0.0137	0.0157	0.0176	0.0195	0.0215	0.0234	0.0253
1 (small)	-	0.2	0.289	0.335	0.348	0.316	0.171	-	-
2	-	-	-	0.0408	0.123	0.243	0.36	0.345	-
3	1	0.45	0.284	0.132	0.0234	-	-	-	-
4	-	0.129	0.113	0.0581	-	-	-	-	-
5	-	0.177	0.0437	-	-	-	-	-	-
6	-	0.0444	0.109	0.124	0.11	0.0126	-	-	-
7	-	-	0.0371	0.0702	0.0686	-	-	-	-
8	-	-	0.124	0.21	0.219	0.173	0.0261	-	-
9	-	-	-	-	-	-	-	-	-
10	-	-	-	0.0297	0.108	0.255	0.443	0.655	1

Panel E: Illustrating portfolio frontiers



The table describes the construction of mean-variance optimal portfolios using historical data from the Oslo Stock Exchange.

Table 13 Mean variance optimal portfolios. 10 portfolios. Using data from 2000 to 2016

Panel A: Expected Returns

Asset	mean	std
1 (small)	0.008	0.072
2	0.016	0.068
3	0.011	0.062
4	0.015	0.054
5	0.009	0.052
6	0.012	0.061
7	0.019	0.061
8	0.020	0.056
9	0.017	0.053
10	0.015	0.060

Panel B: Correlations matrix

$\rho(i,j)$	1 (small)	2	3	4	5	6	7	8	9	10
1 (small)	1	0.731	0.702	0.546	0.605	0.565	0.551	0.507	0.564	0.553
2	0.731	1	0.751	0.557	0.604	0.578	0.582	0.525	0.542	0.594
3	0.702	0.751	1	0.689	0.68	0.678	0.638	0.639	0.657	0.65
4	0.546	0.557	0.689	1	0.679	0.685	0.644	0.592	0.654	0.619
5	0.605	0.604	0.68	0.679	1	0.676	0.645	0.647	0.662	0.691
6	0.565	0.578	0.678	0.685	0.676	1	0.688	0.6	0.621	0.699
7	0.551	0.582	0.638	0.644	0.645	0.688	1	0.634	0.657	0.688
8	0.507	0.525	0.639	0.592	0.647	0.6	0.634	1	0.695	0.655
9	0.564	0.542	0.657	0.654	0.662	0.621	0.657	0.695	1	0.682
10	0.553	0.594	0.65	0.619	0.691	0.699	0.688	0.655	0.682	1

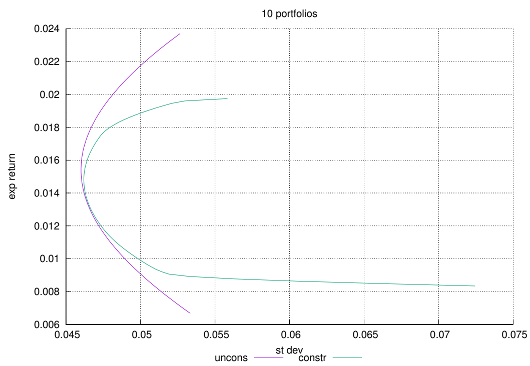
Panel C: Optimal unconstrained portfolios

Asset	Expected Return								
	0.00667	0.0088	0.0109	0.0131	0.0152	0.0173	0.0194	0.0216	0.0237
1 (small)	0.17	0.122	0.0752	0.0278	-0.0195	-0.0668	-0.114	-0.161	-0.209
2	-0.167	-0.105	-0.0432	0.0185	0.0802	0.142	0.204	0.265	0.327
3	0.14	0.0784	0.0164	-0.0455	-0.107	-0.169	-0.231	-0.293	-0.355
4	0.112	0.154	0.196	0.238	0.28	0.322	0.364	0.406	0.448
5	0.773	0.655	0.538	0.421	0.303	0.186	0.0687	-0.0486	-0.166
6	0.199	0.157	0.115	0.0722	0.0299	-0.0124	-0.0546	-0.0969	-0.139
7	-0.243	-0.179	-0.115	-0.0505	0.0135	0.0776	0.142	0.206	0.27
8	-0.143	-0.06	0.0231	0.106	0.189	0.272	0.356	0.439	0.522
9	0.152	0.175	0.197	0.22	0.243	0.266	0.289	0.311	0.334
10	0.00664	0.00182	-0.003	-0.00783	-0.0126	-0.0175	-0.0223	-0.0271	-0.0319

Panel D: Optimal short sale restricted portfolios

Asset	Expected Return								
	0.00834	0.00977	0.0112	0.0126	0.014	0.0155	0.0169	0.0183	0.0197
1 (small)	1	0.109	0.0687	0.0316	-	-	-	-	-
2	-	-	-	-	0.0216	0.0332	0.0404	-	-
3	-	0.0265	0.00109	-	-	-	-	-	-
4	-	0.0476	0.143	0.205	0.245	0.269	0.282	0.182	-
5	-	0.701	0.57	0.451	0.341	0.209	0.0608	-	-
6	-	0.111	0.0888	0.0613	0.0298	-	-	-	-
7	-	-	-	-	-	0.0411	0.0901	0.205	-
8	-	-	-	0.0571	0.142	0.218	0.292	0.457	1
9	-	0.00529	0.128	0.193	0.22	0.229	0.234	0.156	-
10	-	-	-	-	-	-	-	-	-

Panel E: Illustrating portfolio frontiers



The table describes the construction of mean-variance optimal portfolios using historical data from the Oslo Stock Exchange.

4.3 Momentum portfolios

In this section we use portfolios sorted by momentum.

Table 14 Mean variance optimal portfolios. 10 portfolios. Using data from 1980 to 1989

Panel A: Expected Returns

Asset	mean	std
1 (small)	0.016	0.079
2	0.023	0.083
3	0.020	0.070
4	0.021	0.077
5	0.021	0.063
6	0.022	0.067
7	0.025	0.068
8	0.019	0.074
9	0.033	0.097
10	0.034	0.092

Panel B: Correlations matrix

$\rho(i,j)$	1 (small)	2	3	4	5	6	7	8	9	10
1 (small)	1	0.613	0.568	0.637	0.537	0.542	0.548	0.572	0.57	0.581
2	0.613	1	0.567	0.629	0.606	0.532	0.498	0.507	0.488	0.429
3	0.568	0.567	1	0.697	0.607	0.589	0.578	0.593	0.462	0.444
4	0.637	0.629	0.697	1	0.705	0.671	0.617	0.718	0.65	0.604
5	0.537	0.606	0.607	0.705	1	0.663	0.564	0.63	0.538	0.528
6	0.542	0.532	0.589	0.671	0.663	1	0.613	0.74	0.595	0.552
7	0.548	0.498	0.578	0.617	0.564	0.613	1	0.693	0.602	0.621
8	0.572	0.507	0.593	0.718	0.63	0.74	0.693	1	0.665	0.554
9	0.57	0.488	0.462	0.65	0.538	0.595	0.602	0.665	1	0.768
10	0.581	0.429	0.444	0.604	0.528	0.552	0.621	0.554	0.768	1

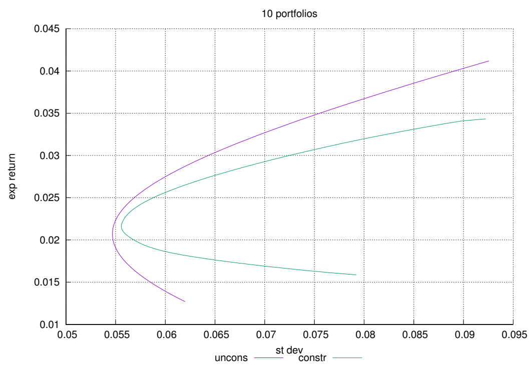
Panel C: Optimal unconstrained portfolios

Asset	Expected Return								
	0.0127	0.0163	0.0198	0.0234	0.0269	0.0305	0.0341	0.0376	0.0412
1 (small)	0.434	0.291	0.148	0.00544	-0.138	-0.28	-0.423	-0.566	-0.709
2	-0.102	-0.0496	0.00262	0.0549	0.107	0.159	0.212	0.264	0.316
3	0.156	0.17	0.184	0.198	0.211	0.225	0.239	0.253	0.267
4	-0.0788	-0.113	-0.146	-0.18	-0.214	-0.248	-0.282	-0.315	-0.349
5	0.506	0.465	0.424	0.384	0.343	0.303	0.262	0.222	0.181
6	0.187	0.211	0.235	0.259	0.283	0.307	0.331	0.355	0.379
7	0.16	0.206	0.252	0.298	0.344	0.39	0.436	0.482	0.528
8	0.227	0.124	0.0218	-0.0809	-0.184	-0.286	-0.389	-0.492	-0.594
9	-0.258	-0.179	-0.1	-0.0212	0.0578	0.137	0.216	0.295	0.374
10	-0.231	-0.126	-0.0215	0.0833	0.188	0.293	0.398	0.502	0.607

Panel D: Optimal short sale restricted portfolios

Asset	Expected Return								
	0.0159	0.0182	0.0205	0.0228	0.0251	0.0274	0.0297	0.032	0.0343
1 (small)	1	0.476	0.172	-	-	-	-	-	-
2	-	-	-	0.0281	0.048	0.0597	0.0648	0.0157	-
3	-	0.102	0.173	0.138	0.0779	0.0234	-	-	-
4	-	-	-	-	-	-	-	-	-
5	-	0.276	0.383	0.331	0.23	0.137	0.0319	-	-
6	-	-	0.151	0.193	0.158	0.114	0.0611	-	-
7	-	-	0.1	0.266	0.276	0.279	0.273	0.201	-
8	-	0.146	0.0199	-	-	-	-	-	-
9	-	-	-	-	-	0.0543	0.115	0.186	-
10	-	-	-	0.0439	0.21	0.332	0.454	0.597	1

Panel E: Illustrating portfolio frontiers



The table describes the construction of mean-variance optimal portfolios using historical data from the Oslo Stock Exchange.

Table 15 Mean variance optimal portfolios. 10 portfolios. Using data from 1990 to 1999

Panel A: Expected Returns

Asset	mean	std
1 (small)	0.019	0.073
2	0.030	0.104
3	0.013	0.080
4	0.013	0.065
5	0.015	0.071
6	0.011	0.055
7	0.010	0.066
8	0.012	0.059
9	0.016	0.070
10	0.018	0.072

Panel B: Correlations matrix

$\rho(i,j)$	1 (small)	2	3	4	5	6	7	8	9	10
1 (small)	1	0.635	0.689	0.598	0.724	0.644	0.67	0.62	0.669	0.707
2	0.635	1	0.717	0.619	0.646	0.623	0.646	0.541	0.558	0.487
3	0.689	0.717	1	0.725	0.745	0.774	0.723	0.696	0.633	0.55
4	0.598	0.619	0.725	1	0.664	0.715	0.568	0.634	0.585	0.49
5	0.724	0.646	0.745	0.664	1	0.794	0.756	0.734	0.641	0.606
6	0.644	0.623	0.774	0.715	0.794	1	0.762	0.746	0.674	0.602
7	0.67	0.646	0.723	0.568	0.756	0.762	1	0.752	0.715	0.658
8	0.62	0.541	0.696	0.634	0.734	0.746	0.752	1	0.674	0.617
9	0.669	0.558	0.633	0.585	0.641	0.674	0.715	0.674	1	0.646
10	0.707	0.487	0.55	0.49	0.606	0.602	0.658	0.617	0.646	1

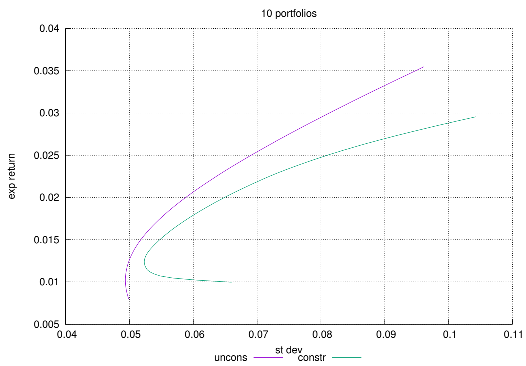
Panel C: Optimal unconstrained portfolios

Asset	Expected Return								
	0.00797	0.0114	0.0148	0.0183	0.0217	0.0252	0.0286	0.032	0.0355
1 (small)	0.127	0.145	0.162	0.179	0.197	0.214	0.232	0.249	0.267
2	-0.174	-0.0501	0.0734	0.197	0.32	0.444	0.568	0.691	0.815
3	-0.224	-0.276	-0.329	-0.381	-0.433	-0.486	-0.538	-0.59	-0.643
4	0.288	0.245	0.202	0.16	0.117	0.0742	0.0316	-0.0111	-0.0537
5	-0.213	-0.157	-0.1	-0.0436	0.0131	0.0697	0.126	0.183	0.24
6	0.697	0.626	0.554	0.483	0.412	0.341	0.27	0.198	0.127
7	0.224	0.0451	-0.133	-0.312	-0.49	-0.669	-0.848	-1.03	-1.2
8	0.283	0.309	0.335	0.361	0.387	0.413	0.439	0.465	0.491
9	-0.0264	0.0315	0.0893	0.147	0.205	0.263	0.321	0.379	0.437
10	0.0195	0.0827	0.146	0.209	0.272	0.336	0.399	0.462	0.525

Panel D: Optimal short sale restricted portfolios

Asset	Expected Return								
	0.00996	0.0124	0.0149	0.0173	0.0198	0.0222	0.0247	0.0271	0.0296
1 (small)	-	-	0.0815	0.0967	0.111	0.118	0.053	-	-
2	-	-	0.0471	0.15	0.258	0.382	0.577	0.79	1
3	-	-	-	-	-	-	-	-	-
4	-	0.13	0.136	0.109	0.0674	-	-	-	-
5	-	-	-	-	-	-	-	-	-
6	-	0.485	0.253	0.11	-	-	-	-	-
7	1	-	-	-	-	-	-	-	-
8	-	0.264	0.2	0.173	0.128	0.0045	-	-	-
9	-	0.0135	0.0871	0.115	0.139	0.148	0.00926	-	-
10	-	0.107	0.196	0.246	0.297	0.347	0.361	0.21	-

Panel E: Illustrating portfolio frontiers



The table describes the construction of mean-variance optimal portfolios using historical data from the Oslo Stock Exchange.

Table 16 Mean variance optimal portfolios. 10 portfolios. Using data from 2000 to 2016

Panel A: Expected Returns

Asset	mean	std
1 (small)	0.020	0.072
2	0.014	0.069
3	0.012	0.054
4	0.009	0.048
5	0.011	0.045
6	0.012	0.045
7	0.012	0.045
8	0.012	0.045
9	0.012	0.055
10	0.020	0.071

Panel B: Correlations matrix

$\rho(i,j)$	1 (small)	2	3	4	5	6	7	8	9	10
1 (small)	1	0.688	0.609	0.594	0.587	0.638	0.548	0.539	0.584	0.589
2	0.688	1	0.693	0.647	0.605	0.614	0.552	0.577	0.555	0.582
3	0.609	0.693	1	0.683	0.638	0.643	0.574	0.612	0.555	0.565
4	0.594	0.647	0.683	1	0.698	0.716	0.651	0.649	0.648	0.628
5	0.587	0.605	0.638	0.698	1	0.685	0.675	0.668	0.665	0.635
6	0.638	0.614	0.643	0.716	0.685	1	0.741	0.704	0.71	0.706
7	0.548	0.552	0.574	0.651	0.675	0.741	1	0.765	0.744	0.712
8	0.539	0.577	0.612	0.649	0.668	0.704	0.765	1	0.711	0.696
9	0.584	0.555	0.555	0.648	0.665	0.71	0.744	0.711	1	0.778
10	0.589	0.582	0.565	0.628	0.635	0.706	0.712	0.696	0.778	1

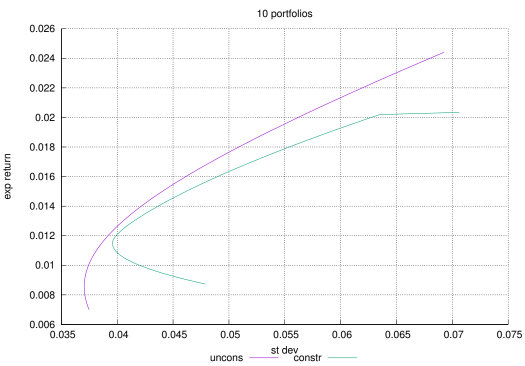
Panel C: Optimal unconstrained portfolios

Asset	Expected Return								
	0.00699	0.00917	0.0113	0.0135	0.0157	0.0179	0.0201	0.0222	0.0244
1 (small)	-0.117	-0.0329	0.0512	0.135	0.219	0.303	0.388	0.472	0.556
2	-0.0672	-0.0755	-0.0838	-0.0921	-0.1	-0.109	-0.117	-0.125	-0.134
3	0.0872	0.0883	0.0893	0.0904	0.0915	0.0926	0.0937	0.0947	0.0958
4	0.238	0.122	0.00566	-0.111	-0.227	-0.344	-0.46	-0.576	-0.693
5	0.289	0.292	0.295	0.299	0.302	0.306	0.309	0.312	0.316
6	0.294	0.273	0.252	0.231	0.21	0.189	0.168	0.147	0.126
7	0.291	0.305	0.318	0.332	0.346	0.359	0.373	0.387	0.4
8	0.254	0.267	0.28	0.294	0.307	0.321	0.334	0.348	0.361
9	0.0794	-0.0204	-0.12	-0.22	-0.32	-0.419	-0.519	-0.619	-0.719
10	-0.348	-0.219	-0.0887	0.0411	0.171	0.301	0.431	0.561	0.69

Panel D: Optimal short sale restricted portfolios

Asset	Expected Return								
	0.00874	0.0102	0.0116	0.0131	0.0145	0.016	0.0174	0.0189	0.0203
1 (small)	-	-	-	0.128	0.2	0.27	0.336	0.399	-
2	-	-	-	-	-	-	-	-	-
3	-	-	0.0409	0.0207	-	-	-	-	-
4	1	0.52	0.0634	-	-	-	-	-	-
5	-	0.207	0.267	0.206	0.161	0.107	0.0307	-	-
6	-	0.0172	0.183	0.137	0.0693	-	-	-	-
7	-	0.163	0.239	0.254	0.23	0.203	0.147	0.065	-
8	-	0.0925	0.207	0.236	0.222	0.201	0.163	0.101	-
9	-	-	-	-	-	-	-	-	-
10	-	-	-	0.018	0.117	0.219	0.323	0.434	1

Panel E: Illustrating portfolio frontiers



The table describes the construction of mean-variance optimal portfolios using historical data from the Oslo Stock Exchange.

5 Black Jensen Scholes(1972) analysis of the OSE

5.1 Introduction

The analysis of Black, Jensen, and Scholes (1972) was the first to formulate the testing of the CAPM in a time series framework. Let us start by giving discussing it in that setting. Consider the regression

$$er_{it} = \alpha_i + \beta_i er_{mt} + \varepsilon_{it} \tag{1}$$

where $er_{it} = r_{it} - r_{ft}$ is the equity exess return (return in excess of the risk free rate), and $er_{mt} = r_{mt} - r_{ft}$ is the corresponding excess return of a stock market portfolio.

Comparing this specification to the CAPM in expectation form

$$E[r_i] = r_f + \beta_i(E[r_m] - r_f),$$

which can be rewritten as

$$E[r_i] - r_f = \beta_i(E[r_m] - r_f),$$

we see that the CAPM imposes the restriction $\alpha_i = 0$ in equation (1).

This regression is called often termed *the* Black Jensen Scholes analysis, and is typically estimated either for single stocks, or (more typically) for stock portfolios, where the data is time series of equity and market returns, from which one subtract a risk free rate to get the excess returns.

The regression is not restricted to having just the market return as an explanatory variable. In more recent asset pricing analyses, particularly in the US, one tend to add two additional factors (The Fama French factors) *SMB* and `HML` (Fama and French, 1993) to get the "tree factor model:"

$$er_{it} = \alpha_i + \beta_i er_{mt} + b_1 SMB_t + b_2 HML_t + \varepsilon_{it} \tag{2}$$

The "four factor model" adds a fourth factor `MOM` related to momentum, (Carhart, 1997)

$$er_{it} = \alpha_i + \beta_i er_{mt} + b_1 SMB_t + b_2 HML_t + b_3 MOM_t + \varepsilon_{it} \tag{3}$$

One can also add non-financial assets as explantory variables, such as for example the oil price. But one should be careful about interpretation of such non-asset variables.

5.2 Industry Portfolios

We use 10 industry portfolios from the Oslo Stock Exchange, in the period after 1980.

Table 17 BJS analysis of OSE portfolios

Results of running the BJS estimations on 10 different industry based portfolios at the OSE. Panel A: Estimation of $er_{it} = a_i + b_i er_{mt} + \varepsilon_t$ Panel B: Estimation of $er_{it} = a_i + b_{m,i} er_{mt} + b_{smb,i} SMB_t + b_{hml,i} HML_t + \varepsilon_t$ Panel C: Estimation of $er_{it} = a_i + b_{m,i} er_{mt} + b_{smb,i} SMB_t + b_{hml,i} HML_t + b_{umd,i} UMD_t + \varepsilon_t$ Data 1980–2013.

Panel A: CAPM

	Dependent variable:									
	Enrg(10)	Matr(15)	Indu(20)	CnsD(25)	CnsS(30)	Hlth(35)	Fin(40)	IT(45)	Tele(50)	Util(55)
eRm	1.397***	1.170***	0.995***	0.915***	0.840***	0.924***	0.737***	1.255***	0.983***	0.666***
	(0.045)	(0.086)	(0.022)	(0.043)	(0.040)	(0.063)	(0.024)	(0.070)	(0.104)	(0.070)
Constant	−0.002	0.0003	0.0003	0.001	0.006**	0.001	−0.001	0.005	−0.001	−0.001
	(0.002)	(0.005)	(0.001)	(0.002)	(0.002)	(0.004)	(0.001)	(0.004)	(0.005)	(0.003)
Observations	444	444	444	444	444	444	444	444	260	252
Adjusted R^2	0.685	0.294	0.821	0.506	0.495	0.324	0.681	0.417	0.256	0.264
Note:										*p<0.1; **p<0.05; ***p<0.01

Panel B: FF

	Enrg(10)	Matr(15)	Indu(20)	CnsD(25)	CnsS(30)	Hlth(35)	Fin(40)	IT(45)	Tele(50)	Util(55)
eRm	1.326***	1.146***	0.996***	0.919***	0.837***	0.940***	0.750***	1.182***	0.809***	0.649***
	(0.040)	(0.087)	(0.022)	(0.044)	(0.042)	(0.063)	(0.024)	(0.051)	(0.103)	(0.072)
SMB	−0.098*	−0.242**	0.007	0.039	−0.138***	−0.042	0.104***	0.052	−0.391***	−0.238***
	(0.050)	(0.108)	(0.027)	(0.054)	(0.051)	(0.078)	(0.030)	(0.064)	(0.127)	(0.085)
HML	−0.074*	0.469***	0.054**	−0.024	−0.020	−0.438***	0.132***	−0.386***	−0.557***	0.080
	(0.044)	(0.096)	(0.024)	(0.048)	(0.046)	(0.069)	(0.026)	(0.056)	(0.114)	(0.076)
Constant	−0.001	0.002	0.0001	0.0003	0.007***	0.004	−0.003**	−0.001	0.003	0.001
	(0.002)	(0.005)	(0.001)	(0.002)	(0.002)	(0.004)	(0.001)	(0.003)	(0.005)	(0.003)
N	426	426	426	426	426	426	426	426	260	252
Adjusted R^2	0.725	0.339	0.829	0.506	0.502	0.373	0.707	0.569	0.332	0.285

Panel C: FF+UMD

	Enrg(10)	Matr(15)	Indu(20)	CnsD(25)	CnsS(30)	Hlth(35)	Fin(40)	IT(45)	Tele(50)	Util(55)
eRm	1.322***	1.137***	0.995***	0.914***	0.849***	0.940***	0.753***	1.175***	0.809***	0.658***
	(0.040)	(0.088)	(0.022)	(0.044)	(0.041)	(0.064)	(0.024)	(0.052)	(0.103)	(0.073)
SMB	−0.091*	−0.228**	0.008	0.046	−0.156***	−0.042	0.099***	0.062	−0.391***	−0.246***
	(0.050)	(0.109)	(0.028)	(0.055)	(0.051)	(0.079)	(0.030)	(0.064)	(0.127)	(0.086)
HML	−0.075*	0.466***	0.054**	−0.025	−0.017	−0.438***	0.133***	−0.388***	−0.557***	0.088
	(0.044)	(0.096)	(0.024)	(0.048)	(0.045)	(0.070)	(0.026)	(0.056)	(0.114)	(0.077)
UMD	−0.040	−0.087	−0.005	−0.049	0.119***	−0.001	0.036	−0.069		0.057
	(0.038)	(0.082)	(0.021)	(0.041)	(0.039)	(0.060)	(0.022)	(0.048)		(0.059)
Constant	−0.001	0.002	0.0001	0.001	0.006***	0.004	−0.003**	−0.001	0.003	0.0003
	(0.002)	(0.005)	(0.001)	(0.002)	(0.002)	(0.004)	(0.001)	(0.003)	(0.005)	(0.004)
N	426	426	426	426	426	426	426	426	260	252
Adjusted R^2	0.725	0.339	0.829	0.507	0.512	0.371	0.708	0.570	0.332	0.285

5.3 Size Portfolios

We use 10 size portfolios from the Oslo Stock Exchange, in the period after 1980.

Table 18 BJS analysis of OSE portfolios

Results of running the estimation $er_{it} = \alpha_i + \beta_i er_{mt} + \varepsilon_t$ on 10 different size based portfolios at the OSE. Data 1980–2016.

Panel A: CAPM

	Dependent variable:									
	1(small)	2	3	4	5	6	7	8	9	10(large)
eRm(ew)	0.794***	0.895***	0.981***	0.997***	1.008***	0.978***	1.084***	1.065***	1.183***	0.993***
	(0.048)	(0.040)	(0.033)	(0.034)	(0.036)	(0.031)	(0.031)	(0.031)	(0.033)	(0.039)
α	0.015***	0.006***	−0.0005	−0.002	0.002	0.0002	−0.003	−0.003*	−0.008***	−0.007***
	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Observations	444	444	444	444	444	444	444	444	444	444
Adjusted R^2	0.380	0.536	0.660	0.661	0.644	0.696	0.731	0.724	0.742	0.588

Panel B: FF

	1(small)	2	3	4	5	6	7	8	9	10(large)
eRm(ew)	0.741***	0.918***	0.972***	1.044***	0.983***	0.964***	1.098***	1.066***	1.207***	0.967***
	(0.046)	(0.038)	(0.033)	(0.032)	(0.031)	(0.031)	(0.030)	(0.031)	(0.031)	(0.030)
SMB	0.209***	0.335***	0.191***	0.300***	0.235***	0.021	−0.170***	−0.209***	−0.321***	−0.669***
	(0.057)	(0.047)	(0.041)	(0.039)	(0.039)	(0.038)	(0.037)	(0.038)	(0.038)	(0.037)
HML	0.114**	0.056	0.022	−0.044	−0.057*	0.001	0.083**	0.032	−0.035	−0.120***
	(0.051)	(0.042)	(0.037)	(0.035)	(0.034)	(0.034)	(0.033)	(0.034)	(0.034)	(0.033)
α	0.011***	0.002	−0.002	−0.004**	0.0003	−0.001	−0.001	−0.001	−0.005***	0.001
	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
N	426	426	426	426	426	426	426	426	426	426
Adjusted R^2	0.383	0.587	0.666	0.719	0.699	0.701	0.768	0.753	0.801	0.781

Note: ***p < .01; **p < .05; *p < .1

Panel C: FF+UMD

	1(small)	2	3	4	5	6	7	8	9	10(large)
eRm(ew)	0.739***	0.917***	0.978***	1.040***	0.984***	0.968***	1.092***	1.066***	1.202***	0.969***
	(0.047)	(0.038)	(0.033)	(0.032)	(0.032)	(0.031)	(0.030)	(0.031)	(0.031)	(0.030)
SMB	0.213***	0.336***	0.181***	0.306***	0.234***	0.016	−0.162***	−0.209***	−0.313***	−0.671***
	(0.058)	(0.047)	(0.041)	(0.040)	(0.039)	(0.038)	(0.038)	(0.038)	(0.038)	(0.037)
HML	0.114**	0.056	0.024	−0.045	−0.057*	0.002	0.082**	0.032	−0.036	−0.120***
	(0.051)	(0.042)	(0.037)	(0.035)	(0.035)	(0.034)	(0.033)	(0.034)	(0.034)	(0.033)
α	−0.024	−0.011	0.061*	−0.039	0.010	0.033	−0.055*	−0.002	−0.050*	0.012
	(0.044)	(0.036)	(0.031)	(0.030)	(0.030)	(0.029)	(0.028)	(0.029)	(0.029)	(0.028)
Constant	0.011***	0.002	−0.002	−0.004**	0.0002	−0.001	−0.001	−0.001	−0.004**	0.001
	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
N	426	426	426	426	426	426	426	426	426	426
Adjusted R^2	0.382	0.586	0.669	0.720	0.698	0.701	0.770	0.752	0.802	0.781

Note: ***p < .01; **p < .05; *p < .1

Table 19 BJS analysis of OSE portfolios

Results of running the estimation $er_{it} = \alpha_i + \beta_i er_{mt} + \varepsilon_t$ on 10 different size based portfolios at the OSE. Data 1980–2016.

Panel A: eR_m+LIQ

	1(small)	2	3	4	5	6	7	8	9	10(large)
$eRm(ew)$	0.912^{***}	0.982^{***}	1.019^{***}	1.041^{***}	1.048^{***}	0.981^{***}	1.038^{***}	1.003^{***}	1.088^{***}	0.858^{***}
	(0.045)	(0.037)	(0.034)	(0.034)	(0.037)	(0.032)	(0.031)	(0.030)	(0.029)	(0.034)
LIQ	0.493^{***}	0.376^{***}	0.188^{***}	0.155^{***}	0.157^{***}	0.013	-0.191^{***}	-0.255^{***}	-0.421^{***}	-0.534^{***}
	(0.050)	(0.042)	(0.037)	(0.037)	(0.041)	(0.036)	(0.035)	(0.034)	(0.033)	(0.038)
α	0.011^{***}	0.003	−0.001	−0.002	0.002	0.0003	−0.001	−0.002	-0.006^{***}	-0.003^{*}
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
N	432	432	432	432	432	432	432	432	432	432
Adjusted R^2	0.498	0.617	0.682	0.693	0.658	0.696	0.752	0.761	0.817	0.720

Note: $^{***}p < .01$; $^{**}p < .05$; $^{*}p < .1$

Panel B: FF+LIQ

	1(small)	2	3	4	5	6	7	8	9	10(large)
$eRm(ew)$	0.870^{***}	1.036^{***}	1.004^{***}	1.066^{***}	0.938^{***}	0.939^{***}	1.077^{***}	1.013^{***}	1.120^{***}	0.904^{***}
	(0.048)	(0.039)	(0.036)	(0.034)	(0.034)	(0.033)	(0.033)	(0.033)	(0.031)	(0.031)
SMB	−0.065	0.086	0.123^{**}	0.253^{***}	0.330^{***}	0.075	-0.126^{***}	-0.096^{**}	-0.135^{***}	-0.536^{***}
	(0.067)	(0.054)	(0.051)	(0.048)	(0.047)	(0.047)	(0.046)	(0.046)	(0.044)	(0.044)
HML	0.059	0.006	0.009	−0.054	−0.038	0.012	0.092^{***}	0.055	0.003	-0.093^{***}
	(0.049)	(0.040)	(0.037)	(0.035)	(0.034)	(0.034)	(0.034)	(0.034)	(0.032)	(0.032)
LIQ	0.471^{***}	0.429^{***}	0.116^{**}	0.081^{*}	-0.164^{***}	-0.092^{**}	−0.075	-0.195^{***}	-0.321^{***}	-0.230^{***}
	(0.067)	(0.055)	(0.051)	(0.049)	(0.047)	(0.047)	(0.046)	(0.046)	(0.044)	(0.045)
α	0.012^{***}	0.003	−0.002	-0.004^{**}	0.0002	−0.001	−0.001	−0.001	-0.005^{***}	0.001
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
N	426	426	426	426	426	426	426	426	426	426
Adjusted R^2	0.446	0.639	0.670	0.721	0.707	0.703	0.769	0.762	0.822	0.794

Note: $^{***}p < .01$; $^{**}p < .05$; $^{*}p < .1$

program 1 R program producing the first table.

```
library(zoo)
library(stargazer)

source ("../../../data/read_ose_data.R")
outdir <- "../../results/2017_02_bjs_size_portfolios/"

head(eRsize)

reg1 <- lm(eRsize[,1]~eRmew)
reg2 <- lm(eRsize[,2]~eRmew)
reg3 <- lm(eRsize[,3]~eRmew)
reg4 <- lm(eRsize[,4]~eRmew)
reg5 <- lm(eRsize[,5]~eRmew)
reg6 <- lm(eRsize[,6]~eRmew)
reg7 <- lm(eRsize[,7]~eRmew)
reg8 <- lm(eRsize[,8]~eRmew)
reg9 <- lm(eRsize[,9]~eRmew)
reg10 <- lm(eRsize[,10]~eRmew)
ColLabels <- c("1(small)","2","3","4","5","6","7","8","9","10(large)")
filename <- paste0(outdir,"bjs_capm_ew_10_size.tex")
CovLabels <- c("$eR_m(ew)$","$\\alpha$")
CovLabels <- c("eRm(ew)","$\\alpha$")
stargazer(reg1,reg2,reg3,reg4,reg5,reg6,reg7,reg8,reg9,reg10,
          column.labels = ColLabels,
          dep.var.labels.include = FALSE,
          float=FALSE,
          font.size="small",
          column.sep.width="1pt",
          omit.stat=c("rsq","f","chi2","ser"),
          model.numbers=FALSE,
          covariate.labels=CovLabels,
#          style="jpam",
          omit.table.layout="n",
          out=filename)
```

5.4 Black Jensen Scholes analysis - oil prices

A claim one often hears is that the Oslo Stock Exchange is very influenced by oil prices. Let us investigate that in the context of a Black Jensen Scholes analysis, by introducing (contemporaneous) changes in oil prices as an explanatory factor.

Let us look at adding (log) changes in the oil prices as an explanatory factor in addition to the market portfolio.

Table 20 Add oil as an explanatory variable

Panel A: Industry Portfolios

	Dependent variable:									
	Enrg(10)	Matr(15)	Indu(20)	CnsD(25)	CnsS(30)	Hlth(35)	Fin(40)	IT(45)	Tele(50)	Util(55)
eRm	1.358***	1.192***	0.996***	0.943***	0.851***	0.935***	0.742***	1.275***	1.051***	0.731***
	(0.046)	(0.087)	(0.023)	(0.044)	(0.041)	(0.067)	(0.025)	(0.072)	(0.116)	(0.078)
dOil	0.071***	−0.013	−0.005	−0.048*	−0.019	−0.028	−0.003	−0.044	−0.094	−0.100***
	(0.026)	(0.050)	(0.013)	(0.025)	(0.024)	(0.039)	(0.014)	(0.042)	(0.059)	(0.037)
Constant	0.0003	−0.003	0.0003	−0.0001	0.004*	−0.0001	−0.002	0.003	0.004	−0.0002
	(0.003)	(0.005)	(0.001)	(0.002)	(0.002)	(0.004)	(0.001)	(0.004)	(0.006)	(0.004)
Observations	410	410	410	410	410	410	410	410	227	219
Adjusted R^2	0.700	0.321	0.827	0.530	0.516	0.326	0.692	0.435	0.269	0.284
Note:										*p<0.1; **p<0.05; ***p<0.01

Panel B: Size Portfolios

	1(small)	2	3	4	5	6	7	8	9	10(large)
eRm	0.812***	0.902***	0.976***	0.984***	1.004***	0.974***	1.081***	1.077***	1.189***	0.984***
	(0.050)	(0.041)	(0.034)	(0.035)	(0.037)	(0.032)	(0.033)	(0.032)	(0.035)	(0.042)
dOil	−0.033	−0.006	0.017	0.038*	−0.005	0.013	−0.0001	−0.043**	−0.028	0.026
	(0.029)	(0.024)	(0.020)	(0.021)	(0.021)	(0.019)	(0.019)	(0.019)	(0.020)	(0.024)
Constant	0.014***	0.006**	−0.001	−0.001	0.003	0.0003	−0.003	−0.004*	−0.007***	−0.007***
	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
N	410	410	410	410	410	410	410	410	410	410
Adjusted R^2	0.394	0.545	0.673	0.668	0.651	0.702	0.732	0.732	0.746	0.591

Note: Data till 2014.

6 Testing the CAPM using Fama and MacBeth on the OSE crossection

We use the method of Fama and MacBeth (1973) to investigate asset pricing in the OSE crossection.

6.1 Introduction

Let us introduce some notation
r_{jt} is the return on stock j at time t.
r_{mt} is the return on a stock market index m at time t.
r_{ft} is the risk free interest rate over the same period.

Define the *excess return* as the return in excess of the risk free return.

$$er_{jt} = r_{jt} - r_{ft}$$

$$er_{mt} = r_{mt} - r_{ft}$$

The CAPM specifies

$$E[r_{jt}] = r_{ft} + (r_{mt} - r_{ft})\beta_{jm},$$

where β_{jm} can be treated as a constant.

This can be rewritten as

$$E[r_{jt}] - r_{ft} = (r_{mt} - r_{ft})\beta_{jm}$$

or, in excess return form

$$E[er_{jt}] = E[er_{mt}]\beta_{jm}$$

Consider now estimating the crossectional relation

$$(r_{jt} - r_{ft}) = a_t + b_t\beta_{j\hat{m}} + u_{jt} \;\; j = 1, 2, \ldots, N$$

or in excess return form

$$er_{jt} = a_t + b_t\beta_{j\hat{m}} + u_{jt} \;\; j = 1, 2, \ldots, N$$

Comparing this to the CAPM prediction

$$er_{jt} = er_{mt}\beta_{jm}$$

we see that the prediction of the CAPM is:

$$E[a_t] = 0$$

$$E[b_t] = (E[r_m] - r_f) > 0$$

To test this, average estimated a_t, b_t:

Test whether

$$E[a_t] = 0, \quad \frac{1}{T}\sum_{t=1}^{T} a_t \to 0$$

$$E[b_t] > 0, \quad \frac{1}{T}\sum_{t=1}^{T} b_t > 0$$

To do these tests we need an estimate of $\beta_{j\hat{m}}$. The "usual" approach is to use time series data to estimate $\beta_{j\hat{m}}$ from the "market model"

$$r_{jt} = \alpha_j + \beta_{jm}r_{mt} + \varepsilon_{jt}$$

on data *before* the crossection.

6.2 The mechanics of doing this type of analysis

We will be replicating the Fama MacBeth type of analysis in R.

The mechanics of doing someting like this is a bit involved, one need to loop over estimations.

program 2 R program producing the table for the one factor CAPM on industry portfolios.

```
library(stargazer)
library(zoo)

source ("../../../data/read_ose_data.R")
eR <- IndustryRets[,1:8] - Rf
eRm <- eRmew
head(eR)
head(eRm)

n  <- length(eRm)
B <- NULL
Rsqrs <- NULL

for (n2 in 61:n) {
    n1 <- n2-60
    regr <- lm(eR[n1:(n2-1),]~eRm[n1:(n2-1)])
    betai <- regr$coefficients[2,]
    eRi <- eR[n2,]
    attributes(eRi)      <- NULL
    attributes(betai)    <- NULL
    regr <- lm(eRi ~ betai )
    b <- regr$coefficients
    B <- rbind(B,b)

    rsqr <- summary(regr)$adj.r.squared
    Rsqrs <- c(Rsqrs,rsqr)
}

head(B)
colMeans(B)
t.test(B[,1])
t.test(B[,2],alternative=c("two.sided"))
t.test(B[,2],alternative=c("greater"))

test <- colMeans(B)

p1 <- t.test(B[,1])$p.value
p2 <- t.test(B[,2],alternative=c("greater"))$p.value
test <- rbind(test,c(p1,p2))
colnames(test)<- c("constant","beta");
rownames(test)<- c("average","p.value");
print(test)

diagn <- c(nrow(B),mean(Rsqrs))
names(diagn) <- c("n","mean R2")

tabl1 <- stargazer(test,float=FALSE,summary=FALSE)
tabl2 <- stargazer(diagn,float=FALSE,summary=FALSE)
cat(tabl1,tabl2,file="../../results/2014_12_fama_macbeth_ose/fm_indus_portf_capm.tex",
    sep="\n")
```

6.3 Econometric issues

So far have not gone into the econometrics of this type of analysis, simply done ordinary tests.

However, there are econometric issues in this type of analysis.

Best known: Errors in Variables, since betas are estimated

Solution used by Fama and MacBeth (1973): Group stocks into portfolios, reducing estimation error in betas.

A recent overview of econometrics of panel data in finance, including Fama Macbeth: Petersen (2008)

6.4 FM analysis results

Results of running the analysis is shown in table 21

Table 21 Fama Macbeth analyses of the Norwegian crossection

Results of separate Fama and MacBeth analyses on the OSE crossection. Panel A uses Industry portfolios (sorted by GICS). Panel B uses portfolios sorted by size. Panel C portfolios sorted by Book/Market. Panel D portfolios sorted by average spread. Panel A: Industry Portfolios

	constant	beta
average	0.008	0.002
p.value	0.105	0.353

n	mean R2
372	0.104

Panel B: Size Portfolios

	constant	beta
average	0.013	-0.004
p.value	0.0004	0.864

n	mean R2
372	0.053

Panel C: B/M portfolios

	constant	beta
average	0.009	0.0004
p.value	0.092	0.474

n	mean R2
360	0.040

Panel D: Spread portfolios

	constant	beta
average	0.013	-0.005
p.value	0.012	0.810

n	mean R2
360	0.086

6.5 Expanding the explanatory factors: Oil Price

We return to the question to how oil prices interact with the stock market. In 1986 Chen, Roll and Ross published a paper where they did a Fama MacBeth type of analysis of US stock market crossections, asking whether changes in oil prices was a risk factor.

We will do a similar, albeit simplified, analysis on the Norwegian Oil Market.

Chen, Roll and Ross use the following explanatory variables

- US Inflation
- US Treasury bill rate (short term)
- US industrial production
- US Long term treasury rates
- Low-Grade bonds (Baa)
- Stock market return
- US Consumption (per capita)
- Oil Prices

They investigate to what degree these alternative "pricing factors" can explain the crossection of asset returns.

For the Norwegian case, we will limit ourself to

- β – Stock market beta
- $dOil$ – change in Oil prices

The change of oil prices is the log difference in the (dollar) oil price. To analyze whether the oil price is important for the crossection, we add it to beta and investigate whether it adds explanatory power to the CAPM.

Specifically, estimate

$$er_{it} = a + b_\beta \widehat{\beta}_{it}^m + b_{ip} \widehat{\beta}_{it}^{oil} + e_{it},$$

where the betas are estimated using a MM type regression on data before t, for example five years.

$$er_{i\tau} = \alpha_i + \beta_{it}^m er_{m\tau} + \beta_{it}^{oil} dOil_\tau + \varepsilon_{i\tau}$$

using observations $\tau = t - 61, \cdots, t - 1$.

Table 22 gives the results.

Table 22 Fama Macbeth analyses of the Norwegian crossection

Panel A: Industry Portfolios

	constant	beta	oil
average	0.004	0.004	0.010
p.value	0.409	0.225	0.565

n	mean R2
343	0.129

Panel B: Size Portfolios

	constant	beta	oil
average	0.007	0.003	0.051
p.value	0.290	0.347	0.0002

n	mean R2
343	0.067

Panel C: B/M portfolios

	constant	beta	oil
average	0.011	-0.001	0.0003
p.value	0.075	0.564	0.978

n	mean R2
331	0.047

Panel D: Spread portfolios

	constant	beta	oil
average	0.006	0.002	0.025
p.value	0.276	0.330	0.064

n	mean R2
331	0.121

7 Multivariate Tests of the CAPM under normality

When we for example use the Black et al. (1972) approach, testing for $\alpha_i = 0$ in

$$r_{it} - r_{ft} = \alpha_i + (r_{mt} - r_{ft}) + \varepsilon$$

on an equation by equation basis, this is inefficient.

Want to aggregate the tests used in e.g. Black et al. (1972) into a single test statistic. If we are willing to make distributional assumptions, in this case multivariate normality, can use Maximum Likelihood methods to construct an aggregate test.

This was developed in a sequence of papers: Gibbons (1982), MacKinlay (1987) and Gibbons, Ross, and Shanken (1989).

The test statistic we will calculate was developed in Gibbons et al. (1989).

7.1 Multivariate test of the CAPM - Gibbons - Ross and Shanken (1989)

Gibbons et al. (1989) uses the setup of Gibbons (1982) to construct a test statistic to answer only one question, whether the market portfolio m is mean variance efficient.

7.2 How to test for aggregate MV efficiency

Let us first show intuitively how such a construction of a test statistic is done.

Consider the estimation of the two following models:

Unconstrained model

$$r_{jt} = \alpha_j + \beta_j r_{mt} + e_{jt}$$

Constrained model

$$r_{jt} = r_{zt}(1 - \beta_j) + \beta_j r_{mt} + e_{jt}$$

The constrained model is a special case of the unconstrained model.

If the CAPM is true, and m *is* MV efficient, the constrained model is the true model. Hence, our estimate of α_j in the unconstrained model should be approximately equal to $r_{zt}(1 - \beta_j)$ (the intercept in the constrained model)

All the multivariate tests of MV efficiency does is to compare the fit of these two models. If the difference is large (according to some statistical metric), reject MV efficiency. Otherwise accept it.

The difference between the methods lies in how to measure the (statistical) difference in fit of the two models. Such test statistics relies on using Maximum Likelihood to do the estimation, and having made the distributional assumption that all errors are multivariate normal. Define:

$$r_t = \begin{bmatrix} r_{1t} \\ \vdots \\ r_{nt} \end{bmatrix} \quad \alpha_t = \begin{bmatrix} \alpha_{1t} \\ \vdots \\ \alpha_{nt} \end{bmatrix} \quad \beta_t = \begin{bmatrix} \beta_{1t} \\ \vdots \\ \beta_{nt} \end{bmatrix} \quad \text{and } e_t = \begin{bmatrix} e_{1t} \\ \vdots \\ e_{nt} \end{bmatrix}$$

The model is then written as

$$r_t = \alpha_t + \beta_t r_{mt} + e_t$$

with the distributional assumption

$$e_t \sim N(\mathbf{0}, V_t)$$

where V_t is the covariance matrix $E[e_t e_t'] = V_t$.

We find the estimates by maximising the log-likelihood ℓ_T with respect to the parameters of interest.

$$\ell_T = -\left(\frac{NT}{2}\right) \ln(2\pi) - \frac{T}{2} \ln\left|\widehat{V}_e\right| - \frac{1}{2} \sum_{t=1}^{T} \hat{e}_t' \widehat{V}_e^{-1} \hat{e}_t$$

We calculate the same function, but now using the estimates $\widehat{V}_e^c$ from the restricted model

$$\ell_T^c = -\left(\frac{NT}{2}\right) \ln(2\pi) - \frac{T}{2} \ln\left|\widehat{V}_e^c\right| - \frac{1}{2} \sum_{t=1}^{T} e_t^{c\prime} (\widehat{V}_e^c)^{-1} e_t^c$$

The test statistic we use to test whether m is MV efficient is then

$$-2(\ell_T^c - \ell_T) = T(\ln|\widehat{V}_e^c| - ln|\widehat{V}_e|)$$

It can be shown that this converges to a χ^2 distribution, and we use this to make probability statements about the outcome.

7.3 The GRS statistic

The general expression in terms of likelihoods above can be simplified substantially in the case of the CAPM with a risk free rate r_{ft}.

$$E[r_{it}] = r_{ft} - \beta_i(E[r_{mt} - r_{ft}])$$

The calculation can then be done in terms of *excess returns*, returns above the risk free rate.

Let us use the notation in chapter 5 of Campbell, Lo, and MacKinlay (1997), and go through the construction of the GRS statistic.

Define Z_t as a $(N \times 1)$ vector of excess returns for N assets (or portfolios of assets). For these N assets, the excess returns can be described using the excess-return market model.

$$\mathbf{Z}_t = \boldsymbol{\alpha} + \boldsymbol{\beta} Z_{mt} + \boldsymbol{\epsilon}_t$$
$$E[\boldsymbol{\epsilon}_t] = \mathbf{0}$$
$$E\left[\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t{}'\right] = \boldsymbol{\Sigma}$$
$$E[Z_{mt} = \mu_m$$
$$E[(Z_{mt} - \mu_m)^2] = \sigma_m^2$$
$$\text{cov}(Z_{mt}, \boldsymbol{\epsilon}_t) = 0$$

$\boldsymbol{\beta}$ is the $(N \times 1)$ vector of betas, Z_{mt} is the time period t market portfolio excess return, and $\boldsymbol{\alpha}$ and $\boldsymbol{\epsilon}_t$ are $(N \times 1)$ vectors of asset return intercepts and disturbances, respectively.

The maximum likelihood estimates are

$$\hat{\boldsymbol{\alpha}} = \hat{\boldsymbol{\mu}} - \hat{\beta}\hat{\mu}_m$$
$$\hat{\boldsymbol{\beta}} = \frac{\sum_{t=1}^T (\mathbf{Z}_t - \hat{\boldsymbol{\mu}})(Z_{mt} - \hat{\mu_m})}{\sum_{t=1}^T (Z_{mt} - \hat{\mu}_m)^2}$$
$$\hat{\boldsymbol{\Sigma}} = \frac{1}{T}\sum_{t=1}^T (\mathbf{Z} - \hat{\boldsymbol{\alpha}} - \hat{\boldsymbol{\beta}} Z_{mt})(\mathbf{Z} - \hat{\boldsymbol{\alpha}} - \hat{\boldsymbol{\beta}} Z_{mt})'$$

where

$$\hat{\boldsymbol{\mu}} = \frac{1}{T}\sum_{t=1}^T \mathbf{Z}_t,$$
$$\hat{\mu}_m = \frac{1}{T}\sum_{t=1}^T Z_{mt}$$

and

$$\hat{\sigma}_m^2 = \frac{1}{T}\sum_{t=1}^T (Z_{mt} - \hat{\mu}_m)^2$$

These are the same as the OLS estimators.

We want to test the null hypothesis

$$\mathbf{H}_0: \quad \boldsymbol{\alpha} = \mathbf{0}$$

agains the alternative

$$\mathbf{H}_A: \quad \boldsymbol{\alpha} \neq \mathbf{0}$$

The GRS statistic J_i

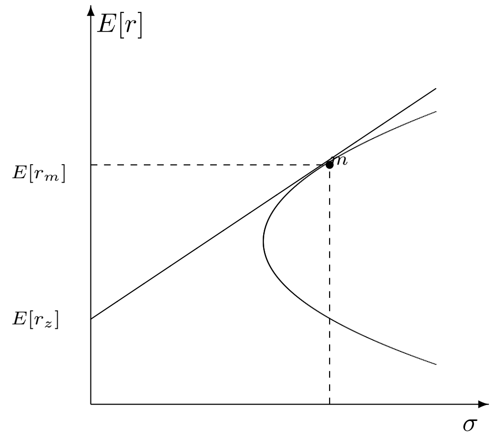
$$J_1 = \frac{(T - N - 1)}{N}\left[1 + \frac{\hat{\mu}_m^2}{\hat{\sigma}_m^2}\right]^{-1} \hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\alpha}}$$

is under the null unconditionally distributed central F with N degrees of freedom in the numerator and $T - N - 1$ degrees of freedom in the denominator.

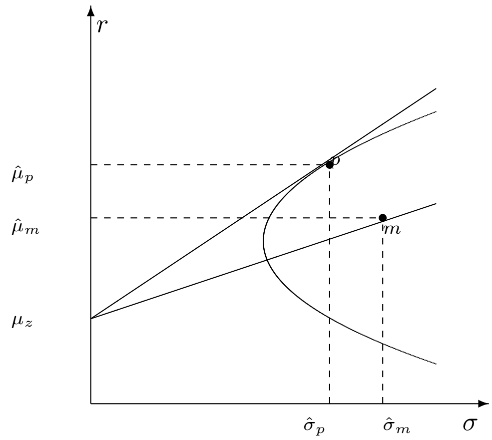
7.4 The Geometric Intuition of the GRS statistic

Let us look at some geometric intuition:

We are interested in a portfolio m. What we would like to know is whether m was on the MV frontier in the ex ante case:



In *ex post* MV space, we can always form the *ex post* efficient frontier:



Here m is the ex post outcome for the portfolio m and p is an ex post frontier portfolio. Intuitively, the test statistic measures the difference in the slope of the two lines in the picture. If this difference is large, we think that the market portfolio is not ex ante efficient.

This is shown algebraically by Gibbons et al. (1989), who show that the GRS statistic J_1 can alternatively be calculated as

$$J_1 = \frac{(T-N-1)}{N}\left(\frac{\frac{\hat{\mu}_q^2}{\hat{\sigma}_q^2} - \frac{\hat{\mu}_m^2}{\hat{\sigma}_m^2}}{1 + \frac{\hat{\mu}_m^2}{\hat{\sigma}_m^2}}\right)$$

where the portfolio denoted by q denotes the *ex post* tangency portfolio constructed from the N included assets *plus* the market portfolio.

7.5 Estimating the Gibbons, Ross and Shanken (1989) statistic on the OSE crossection

The results of estimating the GRS statistic J_1 on four different OSE portfolios are shown in table 23.

```
indir <- "../../../data/"
```

Table 23 Estimates of the GRS statistic on the OSE Crossection. 1980–2014

Estimates of the GRS statistic J_1 for portfolios at the Oslo Stock Exchange cross section.

Portfolio Sort	J_1	p-value
Industry(8 portfolios)	0.7650	0.6339
Size(10 portfolios)	3.4832	0.0002
Book/Market (10 portfolios)	2.7087	0.0032
Liquidity (Relative Spread)(10 portfolios)	2.4510	0.0075

```
filename <- paste0(indir,"read_ose_data.R")
source (filename)


eR  <- SizeRets-Rf
eRm <- eRmew
                                   # make sure data is aligned
data <- merge(eR,eRm,all=TRUE)
eR <- data[,1:10]
eRm <- data[,11]
names(eRm) <- "eRm"

head(eR)
tail(eR)
head(eRm)


SharpeMarket <- mean(eRm)/sd(eRm)
print(SharpeMarket)

T <- length(eRm)
N <- ncol(eR)
                                   #  do all at once
regr <- lm(eR~eRm)

alpha <- as.matrix(regr$coefficients[1,])
print(alpha)

Sigma <- cov(as.matrix(regr$residuals))
SigmaInv <- solve(Sigma)

J1 <- (T-N-1)/N * ( t(alpha) %*% SigmaInv %*% alpha ) / (1 + SharpeMarket^2)
print(J1)
pf(J1,N,(T-N-1),lower.tail=FALSE)
```

8 Estimating the CAPM by GMM

The analysis of this section builds on MacKinlay (1987).

We consider the CAPM, or another linear model where excess returns are linear functions of factors.

In the single factor CAPM:

$$E[er_i] = \alpha + \beta er_m$$

more generally, we write

$$E[er_i] = \alpha + \mathbf{bf}$$

where $\mathbf{f}$ is a set of observable factors, the typical restrictions imposed by the model is that the coefficients α are jointly zero for a crossection of assets.

Formulating this as a crossectional GMM estimation lets us directly test this. We can estimate all coefficients, and then perform a *single* test of whether all the coefficients are zero, unlike the equation by equation tests one will do when using the BJS framework.

First, illustrate the joint estimation using eight industry portfolios

Table 24 Estimating CAPM on eight industry portfolios (texreg output)

Results from estimating

$$E[eR_i] = E[\alpha + beR_{mt}]$$

using GMM, where eR_i is the excess portfolio return, and eR_m the excess market return.

	Model 1
Intercept	
Energy10_(Intercept)	0.000 (0.003)
Material15_(Intercept)	−0.003 (0.003)
Industry20_(Intercept)	0.000 (0.001)
ConsDisc25_(Intercept)	−0.000 (0.002)
ConsStapl30_(Intercept)	0.004 (0.003)
Health35_(Intercept)	−0.000 (0.004)
Finan40_(Intercept)	−0.002 (0.002)
IT45_(Intercept)	0.004 (0.005)
Market	
Energy10_erm	1.382 (0.080)***
Material15_erm	1.188 (0.130)***
Industry20_erm	0.994 (0.029)***
ConsDisc25_erm	0.926 (0.050)***
ConsStapl30_erm	0.843 (0.044)***
Health35_erm	0.926 (0.099)***
Finan40_erm	0.741 (0.038)***
IT45_erm	1.260 (0.116)***
Criterion function	0.000
Num. obs.	410

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 25 Testhing hypothesis that all intercepts are zero

```
> R <- cbind(diag(8),matrix(0,8,8))
> c <- rep(0,8)
> print(R,c)
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13] [,14]
[1,]    1    0    0    0    0    0    0    0    0     0     0     0     0     0
[2,]    0    1    0    0    0    0    0    0    0     0     0     0     0     0
[3,]    0    0    1    0    0    0    0    0    0     0     0     0     0     0
[4,]    0    0    0    1    0    0    0    0    0     0     0     0     0     0
[5,]    0    0    0    0    1    0    0    0    0     0     0     0     0     0
[6,]    0    0    0    0    0    1    0    0    0     0     0     0     0     0
[7,]    0    0    0    0    0    0    1    0    0     0     0     0     0     0
[8,]    0    0    0    0    0    0    0    1    0     0     0     0     0     0
     [,15] [,16]
[1,]     0     0
[2,]     0     0
[3,]     0     0
[4,]     0     0
[5,]     0     0
[6,]     0     0
[7,]     0     0
[8,]     0     0

> linearHypothesis(res,R,c,test="F")
Linear hypothesis test

Hypothesis:
Energy10_((Intercept) = 0
Material15_((Intercept) = 0
Industry20_((Intercept) = 0
ConsDisc25_((Intercept) = 0
ConsStapl30_((Intercept) = 0
Health35_((Intercept) = 0
Finan40_((Intercept) = 0
IT45_((Intercept) = 0

Model 1: restricted model
Model 2: er ~ erm

  Df  Chisq Pr(>Chisq)
1
2  8 5.0435     0.7529
```

Table 26 Testing for whether intercepts are zero

Testing the restriction that intercept is zero. We first estimate

$$E[eR_i] = E[\alpha + beR_{mt}]$$

using GMM, where eR_i is the excess portfolio return, and eR_m the excess market return. The test statistic below is the joint test of the hypothesis that $\alpha_i = 0$ for all assets. This test is done for four portfolio sorts: Industry, Size, B/M and Relative Spread.

	Chisq	d.f.	p-value
Industry	0.620	8	0.761
Size	3.657	10	0.0001
Book / Market	2.929	10	0.001
Relative Spread	2.728	10	0.003

program 3 R program producing the first table.

```
library(zoo)
library(stargazer)
library(texreg)

source("../../../data/read_ose_data.R")

eR <- IndustryRets[,1:8]-Rf
eRm <- eRmew
# take intersection to align the data
data <- merge(eR,eRm,all=FALSE)
er  <- as.matrix(data[,1:8])
erm <-as.matrix(data[,9])
library(gmm)
res <- gmm(er~erm,x=erm)
summary(res)
texreg(res,
       file="../../results/2015_01_industry_portfolios/gmm_eight_industries_texreg.tex",
       single.row=TRUE,
       table=FALSE,
       digits=3,
       dcolumn=TRUE,
       use.packages=FALSE,
       groups=list("Intercept"=1:8,"Market"=9:16))
res$coefficients
library(car)
R <- cbind(diag(8),matrix(0,8,8))
c <- rep(0,8)
print(R,c)
linearHypothesis(res,R,c,test="F")
```

9 Estimating m directly on the Norwegian Crossection

Consider the moment condition

$$E[\mathbf{mR}] = 0$$

where **R** is an excess return and m a stochastic discount factor.

If we assume a parameterization of **m** we can estimate the parameters of this parameterization using GMM. Let us assume **m** is a linear function of factors f

$$\mathbf{m} = c + \mathbf{bf}$$

This can be estimated using GMM directly from data on factors **f**.

Note though that c needs to be forced away from zero, otherwise the whole system is not identified, you can always force it to zero by setting

$$c = 0$$

$$\mathbf{b} = \mathbf{0}$$

The typical solution is to set $c = 1$, to a constant. The rest is then estimated. Let us consider two different choices of factors. The first is the excess return on the market portfolio, eR_m. The second is the three Fama and French factors. In table 27 we show results estimating the model using eight industry portfolios of OSE data. With this specification only the market portfolio is a significant explanatory factor in the cross-section. This can be due to the portfolio sample, the industry portfolios have limited cross-sectional variation. We therefore redo the estimation using ten size based portfolios.

Table 27 Estimating m on eight industry portfolios

Results from estimating $E[\mathbf{mR}_t] = 0$ for two different parameterizations of **m**. In panel A we use: $m = 1 + b_1 er_{mt}$. In panel B we use: $m = 1 + b_1 er_{mt} + b_2 SMB_t + b_3 HML_t$. The estimation is done using eight industry portfolios on the OSE.

Panel A: One factor

	Model 1
Theta[1]	-3.487 $(1.099)^{**}$
Criterion function	1227.075
Num. obs.	410

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Panel B: Three factors

	Model 1
Theta[1]	-3.007 $(1.325)^{*}$
Theta[2]	1.644 (4.005)
Theta[3]	0.638 (2.445)
Criterion function	1158.262
Num. obs.	393

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

In table 28 we show the results of estimating using the crossection of ten size-based portfolios.

Table 28 Estimating m on ten size portfolios

Results from estimating $E[\mathbf{mR}_t] = 0$ for two different parameterizations of $\mathbf{m}$. In panel A we use: $m = 1 + b_1 er_{mt}$. In panel B we use: $m = 1 + b_1 er_{mt} + b_2 SMB_t + b_3 HML_t$. The estimation is done using ten size sorted portfolios on the OSE.

Panel A: One factor

	Model 1
Theta[1]	-4.629 (1.074)***
Criterion function	8379.835
Num. obs.	410

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Panel B: Three factors

	Model 1
Theta[1]	-3.966 (1.183)***
Theta[2]	-4.621 (1.351)***
Theta[3]	-8.935 (3.515)*
Criterion function	4072.636
Num. obs.	393

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

9.1 Appendix - R program

program 4 R program producing the first table, estimating a one factor model for industry portfolios.

```
library(texreg)
source("../../../data/read_ose_data.R")

eR <- IndustryRets[,1:8]-Rf
eRm <- eRmew
# take intersection to align the data
data <- merge(eR,eRm,all=FALSE)
er  <- as.matrix(data[,1:8])
erm <-as.matrix(data[,9])

X <- cbind(er,erm)
g <- function (parms,X) {
  b <- parms[1];
  f <- as.vector(X[,9])
  m <- 1 + b * f
  e <- m * X[,1:8]
  return (e);
}
library(gmm)
t0=c(0.1);
res <- gmm(g,X,t0,method="Brent",lower=-10,upper=10)
summary(res)
texreg(res,
       file="../../results/2015_01_industry_portfolio/gmm_m_industry_one_fact.tex",
       single.row=TRUE,
       table=FALSE,
       digits=3,
       use.packages=FALSE,
       dcolumn=TRUE)
```

10 Estimating risk premia in a factor setting

In a theoretical factor model one will assume that expected return for a stock in excess of the risk free return in equilibrium can be expressed as

$$E[er^i] = \sum_j \lambda_j \beta_j^i \tag{4}$$

where $E[er^i]$ is expected excess return for stock i, $j \in \{1, .., J\}$ the number of factors affecting returns, β_j^i is the exposure to risk factor j for stock i and λ_j is the risk premium for risk factor j common to the whole market.

There are various methods to estimate risk premia for one or more factors, and testing whether a model can price a collection of assets. The traditional method uses two steps. The first step is the method developed by Black et al. (1972), time series regressions of the type

$$er_t^i = a^i + \sum_{j=1}^{J} \beta_j^i f_{jt} + \varepsilon_t^i \tag{5}$$

where er_t^i is the excess return for stock i, a^i a constant term, and β_j^i the estimated exposure to factor f_j of stock i. The estimated factor exposures measures the sensitivity of the return of an asset to movements in the factors. When a factor is expressed as a return series, for example as the return of a portfolio of large companies less the return of a portfolio of small companies, the factor model can be tested by testing the restriction that all the constant terms, a^i, equals zero. If this is rejected the model is rejected.

In this estimation we do not use the restriction of constant risk premia across assets. The next step in the the two step procedure is therefore to estimate factor risk premia, and test whether the model is able to price stocks/portfolios correctly. Given the estimates from (5) the risk premium linked to factor j can be estimated by a cross-sectional regression

$$er^i = \lambda_0 + \sum_{j=1}^{J} \lambda_j \beta_j^i + \varepsilon^i \tag{6}$$

where λ_0 is a constant term, an λ_j is the risk premium of factor j. Finally one will perform statistical tests on λ_j to investigate whether the risk premia of the various factors are significantly different from zero.

In this section we show results of such estimations of the system

$$E[\mathbf{er}] = \alpha + \beta \mathbf{f}$$

and

$$E[\mathbf{er}] = \beta \lambda$$

The estimate of λ in these regression has an interpretation as the *risk premium* associated with that particular factor.

We first show results for a single factor version,

$$E[er] = \alpha + \beta er_m$$

$$E[er] = \lambda \beta$$

The implementation of this is a two step one. We first estimate

$$E[er] = \alpha + \beta er_m,$$

using GMM to estimate the system

The GMM estimates are then used as input in the second stage estimation (using GMM) of

$$E[er] = \lambda \beta$$

The resulting λ is an estimate of the risk premium on the various factors.

10.1 Single factor specification

10.1.1 Size Portfolios

We show results using 10 size sorted portfolios

Table 29 Estimate of Factor Premia, Size Portfolios

Estimates of the system $E[er] = \alpha + \beta er_m$ and $E[er] = \lambda\beta$. Panel A: Estimates of first equation. Panel B: Estimates of second equation.

Panel A. CAPM estimate

	CAPM
α_1	$0.014\ (0.003)^{***}$
α_2	$0.006\ (0.002)^{**}$
α_3	$-0.001\ (0.002)$
α_4	$-0.001\ (0.002)$
α_5	$0.003\ (0.002)$
α_6	$0.000\ (0.002)$
α_7	$-0.003\ (0.002)$
α_8	$-0.004\ (0.002)^{*}$
α_9	$-0.007\ (0.002)^{***}$
α_{10}	$-0.007\ (0.002)^{**}$
β_1	$0.800\ (0.075)^{***}$
β_2	$0.900\ (0.053)^{***}$
β_3	$0.982\ (0.050)^{***}$
β_4	$0.997\ (0.042)^{***}$
β_5	$1.002\ (0.059)^{***}$
β_6	$0.979\ (0.045)^{***}$
β_7	$1.081\ (0.060)^{***}$
β_8	$1.066\ (0.040)^{***}$
β_9	$1.175\ (0.049)^{***}$
β_{10}	$0.993\ (0.065)^{***}$
Num. obs.	410

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Panel B. Estimate of Factor Premia

	Factor Premia
λ_1	0.008^{*}
	(0.003)
Num. obs.	410

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

program 5 R program producing the analysis of size portfolios

```
library(gmm)
source ("../../../data/read_ose_data.R")


eR  <- SizeRets-Rf
eRm <- eRmew

data <- merge(eR,eRm,all=FALSE)
er  <- as.matrix(data[,1:10])
erm <- as.vector(data[,11])
                                        #  ols regression coefficients
regr <- lm(er~erm)
                                        # first estimate corresponding system to the ols regression
X <- cbind(er,erm)
g1 <- function (parms,X) {
  a <- parms[1:10]
  b <- parms[11:20]
  mcond<-c()
  for (i in 1:10){
    e <- X[,i]- a[i]- b[i]*X[,11]
    mcond <- cbind(mcond,e)
    mcond <- cbind(mcond,e*X[,11])
  }
  return (mcond)
}
t1   <- as.matrix(c(regr$coefficients[1,],regr$coefficients[2,]))
res1 = gmm(g1,X,t1)
a <- res1$coefficients[1:10]
b <- res1$coefficients[11:20]
                                        # then estimate the crossectional restriction,
g2 <- function (parms,X) {
  lambda <- parms[1];
  mcond<-c()
  for (i in 1:10){
      e <- X[,i] - lambda * b[i]
      mcond <- cbind(mcond,e)
  }
  return (mcond);
}

lbound <- c(-0.1)
ubound <- c(0.1)
t2 <- c(0.01)
res2 <- gmm(g2,X,t2,method="Brent",upper=ubound,lower=lbound)
summary(res2)
```

10.1.2 Industry Portfolios

We show results using 8 industry sorted portfolios

Table 30 Estimate of Factor Premia, Industry Portfolios

Estimates of the system $E[er] = \alpha + \beta er_m$ and $E[er] = \lambda\beta$. Panel A: Estimates of first equation. Panel B: Estimates of second equation.

Panel A. CAPM estimate

	CAPM
α_1	0.000 (0.003)
α_2	−0.003 (0.003)
α_3	0.000 (0.001)
α_4	−0.000 (0.002)
α_5	0.004 (0.003)
α_6	−0.000 (0.004)
α_7	−0.002 (0.002)
α_8	0.004 (0.005)
β_1	1.382 (0.080)***
β_2	1.188 (0.130)***
β_3	0.994 (0.029)***
β_4	0.926 (0.050)***
β_5	0.843 (0.044)***
β_6	0.926 (0.099)***
β_7	0.741 (0.038)***
β_8	1.260 (0.116)***
Num. obs.	410

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Panel B. Estimate of Factor Premia

	Factor Premia
λ_1	0.011**
	(0.004)
Num. obs.	410

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

10.1.3 Spread Portfolios

We show results using 10 spread sorted portfolios

Table 31 Estimate of Factor Premia, Spread Portfolios

Estimates of the system $E[er] = \alpha + \beta er_m$ and $E[er] = \lambda\beta$. Panel A: Estimates of first equation. Panel B: Estimates of second equation.

Panel A. CAPM estimate

	CAPM
α_1	-0.005 $(0.002)^{*}$
α_2	-0.005 $(0.002)^{**}$
α_3	-0.003 (0.002)
α_4	-0.002 (0.002)
α_5	-0.002 (0.002)
α_6	-0.002 (0.002)
α_7	-0.001 (0.002)
α_8	0.003 (0.002)
α_9	0.009 $(0.002)^{***}$
α_{10}	0.010 $(0.003)^{***}$
β_1	1.048 $(0.060)^{***}$
β_2	1.078 $(0.049)^{***}$
β_3	1.124 $(0.043)^{***}$
β_4	0.945 $(0.042)^{***}$
β_5	0.990 $(0.053)^{***}$
β_6	0.951 $(0.035)^{***}$
β_7	0.957 $(0.042)^{***}$
β_8	0.912 $(0.053)^{***}$
β_9	0.871 $(0.057)^{***}$
β_{10}	0.847 $(0.095)^{***}$
Num. obs.	399

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Panel B. Estimate of Factor Premia

	Factor Premia
λ_1	0.010^{**}
	(0.004)
Num. obs.	399

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

10.1.4 B/M Portfolios

We show results using 10 book/market sorted portfolios

Table 32 Estimate of Factor Premia, B/M Portfolios

Estimates of the system $E[er] = \alpha + \beta er_m$ and $E[er] = \lambda\beta$. Panel A: Estimates of first equation. Panel B: Estimates of second equation.

Panel A. CAPM estimate

	CAPM
α_1	-0.003 (0.002)
α_2	0.001 (0.003)
α_3	-0.007 $(0.002)^{**}$
α_4	-0.000 (0.002)
α_5	-0.003 (0.002)
α_6	-0.002 (0.002)
α_7	0.004 $(0.002)^{*}$
α_8	0.004 $(0.002)^{*}$
α_9	0.004 (0.002)
α_{10}	0.007 $(0.003)^{**}$
β_1	0.984 $(0.071)^{***}$
β_2	1.137 $(0.113)^{***}$
β_3	1.122 $(0.056)^{***}$
β_4	0.974 $(0.032)^{***}$
β_5	1.016 $(0.051)^{***}$
β_6	1.021 $(0.051)^{***}$
β_7	1.067 $(0.050)^{***}$
β_8	1.084 $(0.060)^{***}$
β_9	1.071 $(0.060)^{***}$
β_{10}	1.086 $(0.063)^{***}$
Num. obs.	399

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Panel B. Estimate of Factor Premia

	Factor Premia
λ_1	0.010^{**}
	(0.004)
Num. obs.	399

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 33 Three factors

Panel A: Size Portfolios

	Factor Premia
λ_1	0.0098^{**}
	(0.0037)
λ_2	0.0046
	(0.0028)
λ_3	0.0178
	(0.0097)
Num. obs.	393

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Panel B: B/M Portfolios

	Factor Premia
λ_1	0.0111^{**}
	(0.0037)
λ_2	0.0173
	(0.0095)
λ_3	0.0086^{*}
	(0.0040)
Num. obs.	393

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Panel C: Relative Spread Portfolios

	Factor Premia
λ_1	0.0105^{**}
	(0.0036)
λ_2	0.0055
	(0.0047)
λ_3	0.0105
	(0.0100)
Num. obs.	393

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Panel D: Industry Portfolios

	Factor Premia
λ_1	0.0095^{**}
	(0.0037)
λ_2	-0.0063
	(0.0082)
λ_3	-0.0013
	(0.0056)
Num. obs.	393

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

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