# Derivatives and Risk Management in Commodity Markets 

## Topic 2: Pricing of forwards and futures

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## Topics

- Derivation of forward prices on
- Stocks that do not pay dividends
- Stocks that do pay dividends
- Investment commodities
- Consumption commodities
- The forward curve


## Determination of forward and futures prices: Objectives

- Know the difference between consumption and investment assets
- Know how to derive the value of a forward contract on an investment asset using the no-arbitrage principle
- When the underlying does not pay a yield (e.g. Dividend)
- When the underlying pays a yield (known fixed income and as a rate)
- Differences between forward and futures contracts?
- Know how to price a forward contract on
- Stocks, stock indices, currencies
- How are the spot and forward prices related for consumption assets?


## Determination of Forward and Futures prices

- Examine how forward/futures prices are related to the spot price of the underlying asset
- Investment asset vs Consumption asset
- Short selling
- Forward price for an investment asset
- Known income \& known yield
- Valuing forward contracts
- Forward prices vs futures prices
- Forward prices for:
- Stock indices
- Currencies
- Commodities
- The spot-forward relationship: Cost of carry / Risk premiums


## Short selling

- Short selling: selling an asset that is not owned
- Example:
- An investor borrows 500 IBM shares in April and sells in the market at 120 \$/share
- Later, the investor buys 500 IBM shares in the market for 100 \$/share
- Cash flow
$+120 \times 500=\$ 60000$
- $100 \times 500=\$ 50000$

$$
=+\$ 10000
$$

## No-arbitrage principle for valuation

- Over the following slides we are going to derive the value of a forward contract using the No-Arbitrage principle


## Assumptions and notation

We will assume that the following are true for some market participants:

1. The market participants are subject to no transaction costs when they trade
2. The market participants are subject to the same tax rate on all net trading profits
3. The market participants can borrow money at the same riskfree rate of interest as they can lend money
4. The market participants take advantage of arbitrage of arbitrage opportunities as they occur

## Assumptions and notation

## Notation

T: Time until delivery date in a forward or futures contract
$\mathrm{S}_{0}$ : Price of an asset underlying the forward or futures contract today
$\mathrm{F}_{0, \mathrm{~T}}$ : Forward or futures price today
$r$ : Zero-coupon risk-free rate of interest per year (continuous compounding). LIBOR/NIBOR
q : yield (e.g. dividend yield) per year. (continuous compounding).

## Forward price for an investment asset

- Easy example: investment asset that provides the holder with no income
- non-dividend paying stock, zero-coupon bond
- Long forward to buy a non-dividend paying stock in 3 months
- Current stock price is $\$ 40$ and the 3 -month risk free interest rate is $5 \%\left(T=3 / 12, S_{0}=40, q=0, r=5 \%\right)$


## Forward price for an investment asset

- Case 1: the forward price is $\$ 43$
- An arbitrageur can then

1. borrow $\$ 40$ at the risk free rate ( $5 \%$ )
2. buy 1 share
3. short a forward contract to sell 1 share in 3 months at 43

| 0 |  |  |
| :--- | :--- | :--- |
|  |  |  |
| borrow money | +40 | $-40 \mathrm{e}^{(0.05 \times 3 / 12)}$ |
| buy 1 share | -40 | $+\mathrm{S}_{\mathrm{T}}$ |
| short 1 forward | 0 | $43-\mathrm{S}_{\mathrm{T}}$ |

Total

## Forward price for an investment asset

- Case 1: the forward price is $\$ 43$
- An arbitrageur can then

1. borrow $\$ 40$ at the risk free rate ( $5 \%$ )
2. buy 1 share
3. short a forward contract to sell 1 share in 3 months at 43

| 0 |  |  |
| :--- | :---: | :--- |
|  |  |  |
| borrow money | +40 | $-40 \mathrm{e}^{(0.05 \times 3 / 12)}$ |
| buy 1 share | -40 | $+\mathrm{S}_{\mathrm{T}}$ |
| short 1 forward | 0 | $43-\mathrm{S}_{T}$ |
| Total | 0 | $43-40 \mathrm{e}^{(0.05 \times 3 / 12)}$ |

## Forward price for an investment asset

- Case 2: the forward price is $\$ 43$
- An arbitrageur can then

1. borrow $\$ 40$ at the risk free rate ( $5 \%$ )
2. buy 1 share
3. short a forward contract to sell 1 share in 3 months at 43

| 0 |  |  |
| :--- | :---: | :--- |
|  |  |  |
|  |  |  |
| borrow money | +40 | $-40 \mathrm{e}^{(0.05 \times 3 / 12)}$ |
| buy 1 share | -40 | $+\mathrm{S}_{\mathrm{T}}$ |
| short 1 forward | 0 | $43-\mathrm{S}_{\mathrm{T}}$ |
| Total | 0 | +2.50 |

## Forward price for an investment asset

- Case 2: the forward price is $\$ 39$
- An arbitrageur can then

1. short 1 share
2. invest $\$ 40$ at the risk free rate ( $5 \%$ )
3. buy a forward contract to buy 1 share in 3 months at 39

| 0 |  |  |
| :---: | :---: | :---: |
|  |  |  |
| short 1 share | +40 | $-\mathrm{S}_{T}$ |
| invest money | -40 | $+40 \mathrm{e}^{(0.05 \times 3 / 12)}$ |
| long 1 forward | 0 | $\mathrm{~S}_{T}-39$ |
| Total | 0 | $40 \mathrm{e}^{(0.05 \times 3 / 12)}-39$ |

## Forward price for an investment asset

- Case 2: the forward price is $\$ 39$
- An arbitrageur can then

1. short 1 share
2. invest $\$ 40$ at the risk free rate ( $5 \%$ )
3. buy a forward contract to buy 1 share in 3 months at 39

| 0 |  |  |
| :--- | :---: | :--- |
|  |  |  |
| short 1 share | +40 | $-\mathrm{S}_{\mathrm{T}}$ |
| invest money | -40 | $+40 \mathrm{e}^{(0.05 \times 3 / 12)}$ |
| long 1 forward | 0 | $\mathrm{~S}_{\mathrm{T}}-39$ |
| Total | 0 | +1.50 |

## Forward price for an investment asset

- If the forward price is $\$ 43$ then there is an arbitrage profit of \$2.50
- If the forward price is $\$ 39$ then there is an arbitrage profit of \$1.50
- At which price is there no arbitrage profits???


## Forward price for an investment asset

- If the forward price is $\$ 43$ then there is an arbitrage profit of \$2.50
- If the forward price is $\$ 39$ then there is an arbitrage profit of \$1.50
- At which price is there no arbitrage profits???
- Answer:
- When the forward price is $\$ 40.5$ or $40 e^{(0.05 \times 3 / 12)}$
- This is the No-Arbitrage Principle


## Generalisation

- The relationship between the spot price of the underlying and the forward/futures contract is:

$$
\begin{array}{ll}
F_{0, T}=S_{0} e^{r T} & (\text { now }=\text { time } 0) \\
F_{t, T}=S_{t} e^{r(T-t)} & \text { (now }=\text { time } \mathrm{t})
\end{array}
$$

## Generalisation

- Arbitrage profits can be realised if:

$$
\begin{aligned}
& F_{0, T}>S_{0} e^{r T} \\
& F_{0, T}<S_{0} e^{r T}
\end{aligned}
$$

## Example

Long forward to buy a non-dividend paying stock in 3 months.
Current stock price is $\$ 40$ and the 3-month risk free interest rate is $5 \%(T=3 / 12, S 0=40, q=0, r=5 \%)$

$$
F_{0, T}=S_{0} e^{r T}
$$

## Example

Long forward to buy a non-dividend paying stock in 3 months.
Current stock price is $\$ 40$ and the 3 -month risk free interest rate is $5 \%\left(T=3 / 12, S_{0}=40, q=0, r=5 \%\right)$

$$
\begin{aligned}
F_{0, T} & =40 e^{(0.05)(3 / 12)} \\
& =40.50
\end{aligned}
$$

## Forward on zero-coupon bond

- What is the price of a forward contract to buy a zero coupon bond (1-year maturity). The current spot price is 930, 4-month risk free rate is $6 \%$ (yearly, continuous compounding)?
- $\mathrm{S}_{0}=930$
- $\mathrm{T}=4 / 12$
- $\mathrm{r}=6 \%$


## Forward on zero-coupon bond

- What is the price of a forward contract to buy a zero coupon bond (1-year maturity). The current spot price is 930, 4-month risk free rate is $6 \%$ (yearly, continuous compounding)?
- $\mathrm{S}_{0}=930$
- $\mathrm{T}=4 / 12$
- $\mathrm{r}=6 \%$

$$
\begin{aligned}
F_{0,4 m o n t h s} & =930 e^{(0.06) 4 / 12} \\
& =948.79
\end{aligned}
$$

## Known income

- The previous example considered forward contracts on investment assets that pay no income
- What if the asset pays a predictable income?
- Stock paying dividends
- Bonds paying coupons
- Assumption: the dividend is known and is predictable


## Known income: example

- Consider a forward contract to buy a coupon bearing bond. The current price of the bond is $\$ 900$. The forward contract matures in 9 months. A coupon payment of $\$ 40$ is expected in 4 months. The 4 -month risk free interest rate is $3 \%$ and the 9 month risk free rate is $4 \%$ (continuously compounded).
- Case 1: The forward price is $\$ 910$
- Case 2: The forward price is $\$ 870$
- Is there arbitrage?
- What is the no-arbitrage price?


## Known income: example

- Case 1: The forward price is $\$ 910$
- An arbitrageur can then:

1. borrow 900 to buy bond
2. 39.60 (NPV of coupon) is borrowed at $3 \%$ (4-month rate)
3. the rest (900-39.60) is borrowed at $5 \%$ ( 9 -month rate)
4. short a forward contract

## Known income: example



## Known income: example

| 0 |  | 4months | 9 months |
| :---: | :---: | :---: | :---: |
| borrow npv of dividend | +39.60 | -40 |  |
| borrow the rest for 9 months | +860.40 |  | -886.60 |
| buy bond | -900 |  | $+\mathrm{S}_{\text {T }}$ |
| receive coupon short 1 forward | - 0 | +40 | 910-S ${ }_{\text {T }}$ |
| Total | 0 | 0 | 23.40 |

## Known income: example

- Case 2: The forward price is $\$ 870$
- An arbitrageur can then:

1. short 1 bond (receive 900)
2. 39.60 (NPV of coupon) is invested at $3 \%$ (4-month rate) to be able to pay coupon
3. the rest (900-39.60) is invested at $5 \%$ ( 9 -month rate)
4. buy a forward contract (long)

## Known income: example

| 0 | $\longrightarrow$ | 4months | 9months |
| :---: | :---: | :---: | :---: |
| short bond | +900 |  | - $\mathrm{S}_{\mathrm{T}}$ |
| invest npv of dividend | -39.60 | $+39.60 \mathrm{e}^{(0.03 \times 4 / 12)}$ |  |
| invest the rest for 9 months | $-900+39.60$ |  | $+860.4 \mathrm{e}^{(0.04 \times 9 / 12)}$ |
| pay coupon long 1 forward | d 0 | -40 | $\mathrm{S}_{\text {T- }} 870$ |
| Total | 0 | 0 | $+860.4 \mathrm{e}^{(0.04 \times 9 / 12)}-870$ |

## Known income: example



## Generalisation

- When an investment asset will provide income with a npv of I during the life of a forward contract then

$$
F_{0, T}=\left(S_{0}-I\right) e^{r T}
$$

## Generalisation

- Example
- Consider a forward contract to buy a coupon bearing bond. The current price of the bond is $\$ 900$. The forward contract matures in 9 months. A coupon payment of $\$ 40$ is expected in 4 months. The 4 -month risk free interest rate is $3 \%$ and the 9 month risk free rate is $4 \%$ (continuously compounded).
- What is the arbitrage free forward price?

$$
F_{0, T}=\left(S_{0}-I\right) e^{r T}
$$

## Generalisation

- Example
- Consider a forward contract to buy a coupon bearing bond. The current price of the bond is $\$ 900$. The forward contract matures in 9 months. A coupon payment of $\$ 40$ is expected in 4 months. The 4 -month risk free interest rate is $3 \%$ and the 9 month risk free rate is $4 \%$ (continuously compounded).
- What is the arbitrage free forward price?

$$
F_{0, T}=\left(900-40 e^{-0.03 \times 4 / 12}\right) e^{0.04 \times 9 / 12}
$$

$=886.60$

## Example: forward on stock

- Consider a 10 -month forward contract on a stock with a price of $\$ 50$. The risk free rate is $8 \%$. A dividend of $0.75 \$$ will be paid after 3, 6 and 9 months.
- What is the arbitrage free forward price?


## Example: forward on stock

- Consider a 10-month forward contract on a stock with a price of $\$ 50$. The risk free rate is $8 \%$. A dividend of $0.75 \$$ will be paid after 3, 6 and 9 months.
- $\mathrm{F}_{0,10 \text { months }}=$ ?
- $\mathrm{SO}=50$
- $\mathrm{T}=10 / 12$
- r=8\% (flat)
- dividend = 0.75
- I = ?


## Example: forward on stock

- Consider a 10 -month forward contract on a stock with a price of $\$ 50$. The risk free rate is $8 \%$. A dividend of $0.75 \$$ will be paid after 3, 6 and 9 months.

- I = NPV(8\%, 3, 6, 9)

$$
\begin{aligned}
I & =0.75 e^{-0.08 x 3 / 12}+0.75 e^{-0.08 x 6 / 12}+0.75 e^{-0.08 x 9 / 12} \\
& =2.162
\end{aligned}
$$

## Example: forward on stock

- Consider a 10 -month forward contract on a stock with a price of $\$ 50$. The risk free rate is $8 \%$. A dividend of $0.75 \$$ will be paid after 3, 6 and 9 months.

$$
F_{0, T}=\left(S_{0}-I\right) e^{r T}
$$

## Example: forward on stock

- Consider a 10-month forward contract on a stock with a price of $\$ 50$. The risk free rate is $8 \%$. A dividend of $0.75 \$$ will be paid after 3, 6 and 9 months.

$$
\begin{aligned}
F_{0, T} & =(50-2.162) e^{0.08 \times 10 / 12} \\
& =51.14
\end{aligned}
$$

## Known yield

- Consider the case where the asset underlying the forward contract provides a known yield instead of a known cash income
- as a percentage of the asset's price
- notation: q
- Example: Consider a 6-month forward contract on a stock that is expected to pay a $2 \%$ dividend yield per 6 -months. The risk free rate is $10 \%$ (yearly, continuous compounding). The spot price is 25 .
- What is the value of the forward contract?


## Known yield

- Define q as the average yield per annum on an asset during the life of a forward contract (continuous compounding). It can be shown that:

$$
F_{0, T}=S_{0} e^{(r-q) T}
$$

## Known yield

- Example: Consider a 6-month forward contract on a stock that is expected to pay a $2 \%$ dividend yield per 6-months. The risk free rate is $10 \%$ (yearly, continuous compounding). The spot price is 25 .
- $\mathrm{S}_{0}=25$
- $r=10 \%$
- T = 6/12
- dividend yield (semi-annual payment) $=2 \%$
- q = ??


## Known yield

- dividend yield (semi-annual payment) $=2 \%$
- dividend yield (annual, annual compounding)
$=(1.02)^{2}-1=4.04 \%$
- dividend yield (annual, continuous compounding)
$=\ln (1.0404)=3.96 \%$
$q=0.0396$


## Known yield

- Example: Consider a 6-month forward contract on a stock that is expected to pay a $2 \%$ dividend yield per 6-months. The risk free rate is $10 \%$ (yearly, continuous compounding). The spot price is 25 .

$$
F_{0, T}=S_{0} e^{(r-q) T}
$$

## Known yield

- Example: Consider a 6-month forward contract on a stock that is expected to pay a $2 \%$ dividend yield per 6-months. The risk free rate is $10 \%$ (yearly, continuous compounding). The spot price is 25 .

$$
\begin{aligned}
F_{0, T} & =25 e^{(0.10-0.0396) x 6 / 12} \\
& =25.77
\end{aligned}
$$

## Valuing forward contracts



Value when entering into a forward contract (time 0):

0
Value at later stage (marking-to-market):
f
Value at maturity:

$$
\mathrm{S}_{\mathrm{T}}-\mathrm{F}_{0, \mathrm{~T}}
$$

## Valuing forward contracts

- Notation:
- X: the original forward price
- $F_{0,7}$ : today's forward price for delivery at time $T$
- $f=$ the value of the forward contract today
- If today is the day that the contract is first negotiated, then $F_{0, T}=X$
- As time passes $\mathrm{F}_{0, \mathrm{~T}}$ becomes either smaller or larger than X


## Valuing forward contracts

- The general result is that f is calculated as:

$$
f=\left(F_{0, T}-X\right) e^{-r T}
$$

## Valuing forward contracts

- Example: A long forward on a non-dividend paying stock was entered into some time ago. It currently has 6 months left to maturity. The risk free rate is $10 \%$, the spot price is 25 and the delivery price (original contract) is 24:
- $\mathrm{S}_{0}=25$
- $X=24$
- $r=10 \%$
- $\mathrm{T}=6 / 12$
- $\mathrm{F}_{0,6 / 12}=$ ?
- $f=$ ?


## Valuing forward contracts

- Example: A long forward on a non-dividend paying stock was entered into some time ago. It currently has 6 months left to maturity. The risk free rate is $10 \%$, the spot price is 25 and the delivery price (original contract) is 24 :
- $F_{0,6 / 12}=25 e^{(0.10 \times 0.5)}=26.28$


## Valuing forward contracts

- Example: A long forward on a non-dividend paying stock was entered into some time ago. It currently has 6 months left to maturity. The risk free rate is $10 \%$, the spot price is 25 and the delivery price (original contract) is 24 :
- $f=(26.28-24) e^{-0.1 \times 0.5}=2.17$


## Valuing forward contracts

$$
\begin{aligned}
f & =\left(F_{0, T}-X\right) e^{-r T} \\
& =\left(S_{0} e^{r T}-X\right) e^{-r T} \\
& =S_{0} e^{r T} e^{-r T}-X e^{-r T} \\
& =S_{0}-X e^{-r T}
\end{aligned}
$$

## Valuing forward contracts

- No dividend paying stock

$$
f=S_{0}-X e^{-r T}
$$

- known income

$$
f=S_{0}-I-X e^{-r T}
$$

- knoypn=ysisgld $q T-X e^{-r T}$


## Forward contracts on stock indices

- The price of a forward contract on a stock index paying a dividend yield of q is:

$$
F_{0, T}=S_{0} e^{(r-q) T}
$$

- $\mathrm{F}_{0, \mathrm{~T}}=$ forward price on stock index
- $\mathrm{S}_{0}=$ spot price of index
- r = risk free interest
- $q=$ dividend yield on index
- T = time to maturity


## Forward contracts on currencies

- The price of a forward contract on a currency is:

$$
F_{0, T}=S_{0} e^{\left(r-r_{f}\right) T}
$$

- $\mathrm{F}_{0, \mathrm{~T}}=$ forward price on currency
- $\mathrm{S}_{0}=$ spot price of currency
- $r=$ domestic risk free interest rate
- $r_{f}=$ foreign risk free interest rate
- $\mathrm{T}=$ time to maturity


## Forward contracts on currencies

- The underlying in a currency forward contract is a certain number of units of the foreign currency
- E.g. USD/EUR
- $\mathrm{S}_{0}$ is the current spot price in dollars for 1 unit of EUR
- $F_{0, T}$ is the forward price in dollars of 1 unit of EUR


## Forward contracts on currencies

- Example
- Suppose the 2-year interest rate in Australia is 5\% and the 2year interest rate in the US is 7\%, the spot exchange rate (USD/AUD) is 0.6200 .

$$
F_{0, T}=S_{0} e^{\left(r-r_{f}\right) T}
$$

## Forward contracts on currencies

- Example
- Suppose the 2-year interest rate in Australia is 5\% and the 2year interest rate in the US is 7\%, the spot exchange rate (USD/AUD) is 0.6200 .

$$
\begin{aligned}
F_{0,2 \text { year }} & =0.6200 e^{(0.07-0.05) x 2} \\
& =0.6453
\end{aligned}
$$

## Forward contracts on currencies

- Example
- Suppose the 2-year interest rate in Australia is 5\% and the 2year interest rate in the US is 7\%, the spot exchange rate (USD/AUD) is 0.6200 .
- What is the 2 year forward rate?


## The forward (futures) curve



## Contango and backwardation

Forward price

## Contango

## Backwardation

## Exercise: Build your own forward curve

- Data:
- www.cmegroup.com for commodities
- www.cboe.com for equity index futures
- finance.yahoo.com for equities, commodities etc..
- What is the shape of the forward curve?
- How do the shapes of forward curves for Natural Gas prices compare to Crude oil prices?


## Exercise:Trading profits / convergence / Hedging

- Data: US Energy futures historical data:
- Gas prices
- http://www.eia.gov/dnav/ng/ng_pri_fut_s1_d.htm
- Crude oil prices
- http://www.eia.gov/dnav/pet/pet_pri_fut_s1_d.htm


## Commodities: the spot forward relationship

- Agenda
- Revisit investment assets
- Consumption assets or assets which cannot fully be stored (failure of no-arbitrage principle)
- The spot-forward relationship
- Contango/Backwardation vs Normal Contango/Backwardation
- The Rational Expectations Hypothesis
- Risk premium (hedging pressure theory)
- Theory of Storage


## Investment vs Consumption assets

- Two theories for explaining the link between commodity spot and forward prices
- For stocks (and assets that behave like stocks) the spot forward relationship is:

$$
F_{t, T}=S_{t} e^{(r-q)(T-t)}
$$

- For commodities things are a bit more complicated due to the nature of commodities


## Commodities vs stocks

- Commodities are diverse with respect to characteristics
- Metals:
- Compact
- can be stored
- Do not easily degrade
- Agriculture
- Less easily stored
- Quality tends to decrease if stored
- Energy
- Can be stored but requires storage facilities
- Does to degrade easily


## Investment assets vs Consumption assets

- Investment asset: an asset that is (can be) held for investment (by a significant number of investors)
- Stocks, bonds, gold, silver
- Consumption asset: an asset that is held primarily for consumption
- Copper, oil, pork bellies
- We can use arbitrage arguments to determine the forward and futures price of an investment asset from its spot price
- not possible for consumption assets


## No-arbitrage?

- If a commodity can
- Easily be stored and storage facilities are not a restriction
- Does not degrade
- Short-selling is possible
- Then the no-arbitrage principle could be used to price the forward and futures prices
- If the no-arbitrage principle cannot be used, pricing futures and forward contracts is much more difficult


## Pricing forwards on consumption commodities

- In the absence of storage costs and income from the underlying asset (investment asset), the forward price of a commodity is (where $\mathrm{U}=\mathrm{PV}$ (storage costs) = negative income)

$$
F_{t, T}=\left(S_{t}+U\right) e^{r(T-t)}
$$

- Treating storage costs as negative yield gives

$$
F_{t, T}=S_{t} e^{(r+u)(T-t)}
$$

## Pricing forwards on consumption commodities

- Example:
- Price 1-year futures on an investment asset that provides no income
- It costs $\$ 2$ per unit to store the asset (payment at the end of the year)
- Spot price = \$450
- Risk free rate = 7\%

1. Price the 1 -year futures on the underlying asset
2. Explain how to generate arbitrage profits if the market price of the futures contract is higher than your value
3. Explain how to generate arbitrage profits if the market price of the futures contract is lower than your value

## Answers

1. The futures price

$$
U=\text { present value of }
$$

$$
\begin{aligned}
& U=2 e^{-0.07 x 1}=1.865 \\
& F_{t, T}=(450+1.865) e^{0.07 x 1}=484.63
\end{aligned}
$$

storage costs

## Answers

- What if $\quad F_{t, T}>\left(S_{t}+U\right) e^{r(T-t)}$
- Then
- 1. Borrow an amount $\mathrm{SO}+\mathrm{U}$ at the risk-free rate and use it to purchase one unit of the commodity and to pay storage costs (i.e. Synthetic long forward)
- 2. Short a forward/futures contract on one unit of the commodity
- What if $F_{t, T}<\left(S_{t}+U\right) e^{r(T-t)}$
- Then
- Sell the commodity, save the storage costs, and invest the proceeds at the risk-free interest rate (i.e. Synthetic short forward)
- Take a long position in a forward/futures contract


## Consumption assets

- The no-arbitrage argument does not hold for consumption assets
- Individuals and companies keep commodity in inventory for consumption value, not investment value. They are reluctant to sell the commodity and buy forward contracts because forward contracts cannot be consumed
- Therefore

$$
F_{t, T} \leq\left(S_{t}+U\right) e^{r(T-t)}
$$

- or

$$
F_{t, T} \leq S_{t} e^{(r-u)(T-t)}
$$

## Introducing the convenience yield

- Therefore, holding the physical commodity has a benefit compared to holding just the financial paper (futures contract)

$$
\begin{aligned}
F_{t, T} & \leq\left(S_{t}+U\right) e^{r(T-t)} \\
F_{t, T}+\text { Benefit } & =\left(S_{t}+U\right) e^{r(T-t)} \\
F_{t, T} e^{y(T-t)} & =\left(S_{t}+U\right) e^{r(T-t)} \\
F_{t, T} & =S_{t} e^{(r+u-y)(T-t)}
\end{aligned}
$$

## Convenience yield

- Convenience yield = Benefit of holding the physical commodity as opposed to the the financial paper (futures contract)
- E.g. Refinery
- Physical crude oil is necessary as input for the refinery process
- Benefits
- Can keep the refinery process going
- Inventory in case of shortages
- Futures cannot be used in the refinery process
- Not the case for investment assets/commodities


## Cost of carry

- The relationship between the futures price and the spot price is called the cost of carry
- Cost of carry: c = r - q + u (interest rate - income + storage costs)

$$
\begin{aligned}
& F_{t, T}=S_{t} e^{(r-q+u-y)(T-t)} \\
& F_{t, T}=S_{t} e^{(c-y)(T-t)}
\end{aligned}
$$

- Practical issue: how to calculate the convenience yield?


## Current spot vs expected spot

- Current spot = spot price observed now

$$
S_{t}
$$

- Expected spot = spot price some time in the future (unobservable)

$$
S_{T} \quad \text { or } \quad E\left[S_{T}\right]
$$

- How does the Futures price relate to the expected spot price?


## Theory 1: Rational expectations theory

- According to the rational expectations theory, the futures price is an unbiased predictor of the future spot price

$$
F_{t, T}=E\left[S_{T}\right]
$$

## Theory 2: Hedging pressure theory (risk premium theory)

- According to the Hedging pressure theory, the futures price is a biased predictor of the future spot price

$$
F_{t, T}=E\left[S_{T}\right]+\text { premium }
$$

- The premium can be negative or positive, hence we can observe both

$$
\begin{array}{ll}
F_{t, T}>E\left[S_{T}\right] & \text { Normal contango } \\
F_{t, T}<E\left[S_{T}\right] & \text { Normal backwardation }
\end{array}
$$

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