Derivatives and Risk Management in Commodity Markets

Topic 5: Non-standard options & option structures

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Topics

- Structured products
- Spread options
- Real options

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Structured products

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What are structured products

- A structured product is also known as a market linked investment
- A pre-packaged investment strategy based on derivatives
  - Single security
  - Basket of securities
  - Options, indices, commodities, debt issuance, FX, Swaps
- No uniform definition of a structured product
- A key feature is a «principal guarantee» function which offers protection of the principal if held to maturity

Principal guarantee: example

- An investor invests 100 dollars today in a 5-year structured product in which the issuer guarantees that the investor will receive at least the principal (100 dollars) in 6 year's time
- The Issuer recieved 100 dollars and

1. invests in a risk free bond that provides sufficient interest to grow to 100 after the 5-year period
2. Invests the remainder (less fees) in the options needed to perform the investment strategy

In theory, investors can do this themselves

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Equity-linked note

- An equity-linked note have a future value (B_T)at maturity, which can be expressed in the following way if there is a guarantee that the principal will be paid back in full (100%).

$$B_T = B_0\left(1 + max\left(\frac{q_T - q_0}{q_0}, 0\right)\right)$$

B_0 = original investment amount
q_0 = exercise price of an option
q_T = value(price) of the underlying asset at time T (maturity)

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Equity-linked note

- Can be expressed as

$$B_T = B_0 + \frac{B_0}{q_0} max(q_T - q_0, 0)$$

Equity-linked note

- Can be expressed as

$$B_T = B_0 + \frac{B_0}{q_0} max(q_T - q_0, 0)$$

Value of product at maturity

Initial investment (principal)

Ratio (=number of options)

Call option payoff

Equity-linked note

- The equity-linked note has characteristics of a package (structured product) consisting of a riskfree investment (e.g. Zero-coupon bond) and call options on the underlying asset (e.g. Call option on stock index)

- Valuation: assume value additivity

Total value of product = value of part A + value of part B +,---,+value of part N

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Valuation: value-additivity

$$B_T = B_0 + \frac{B_0}{q_0} max(q_T - q_0, 0)$$

Value at maturity

$$V_0(B_T) = V_0(B_0) + V_0(\frac{B_0}{q_0} max(q_T - q_0, 0))$$

Value today

$$V_0(B_T) = V_0(B_0) + \frac{B_0}{q_0} \times V_0(max(q_T - q_0, 0))$$

$$V_0(B_T) = V_0(B_0) + \frac{B_0}{q_0} \times c_0$$

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Valuation

$$V_0(B_T) = V_0(B_0) + \frac{B_0}{q_0} \times c_0$$

Value of total product

$$V_0(B_0) = e^{-rT} B_0$$

Value of risk free component (zero-coupon bond)

$$\frac{B_0}{q_0} \times c_0 = \frac{B_0}{q_0} \times \text{Black} - \text{Scholes value}$$

Option component

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Expected return from an equity-linked note

- The expected return from an equity-linked note is dependent on the return factor

$$B_T = B_0 + \underbrace{\frac{B_0}{q_0}} max(q_T - q_0, 0)$$

This ratio determines how much of the return in the underlying asset passes through to the investor of the product

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Expected return from an equity-linked note

$$B_T = B_0 + \frac{B_0}{q_0} max(q_T - q_0, 0)$$

- If the strike price (q_0) is high, then the ratio is low
- High strike price -> low option price

- If the strike price (q_0) is low, then the ratio is high
- Low strike price -> high option price

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Example

$$B_T = B_0 + \frac{B_0}{q_0} max(q_T - q_0, 0)$$

An investor invests 1000 USD in an 5-year equity-linked note. The note guarantees that the investor will receive minimum the same amount as he invested, plus a portion of a return on the SP500 index. The index is currently at 2000 USD. The volatility of the SP500 index is 20%, the risk free rate is 1%, the index pays no dividends. The Issuer buys ATM call options to be able to provide the investor with a return linked to the return of the underlying asset.
Question: What is the price of this product?

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Example

$$V_0(B_T) = V_0(B_0) + \frac{B_0}{q_0} \times c_0$$

$$V_0(B_0) = e^{-rT} B_0 = e^{-0.01\times5} 1000 = 951.23$$

$$\frac{B_0}{q_0} \times c_0 = \frac{1000}{2000} \times c_0 = 0.5 \times 396.13 = 198.06$$

$$V_0(B_T) = 951.23 + 198.06 = 1149.29$$

Black-Scholes value
S = 2000
X = 2000
T = 5
σ = 20%
δ = 0%
C_0 = 396.13

This is good for the investor: pays 1000, receives 1149.

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Example

- This scenario is unlikely, the Issuer will most likely try to find cheaper options in order to earn a profit.
- The issuer will earn a profit if 1000 – value of zero-coupon bond > 0. This means that the options have to be cheaper than 1000 – 951.23 = 48.77

- Can do this by choosing call options with higher strike price
- Can do this through ‘volatility reduction’

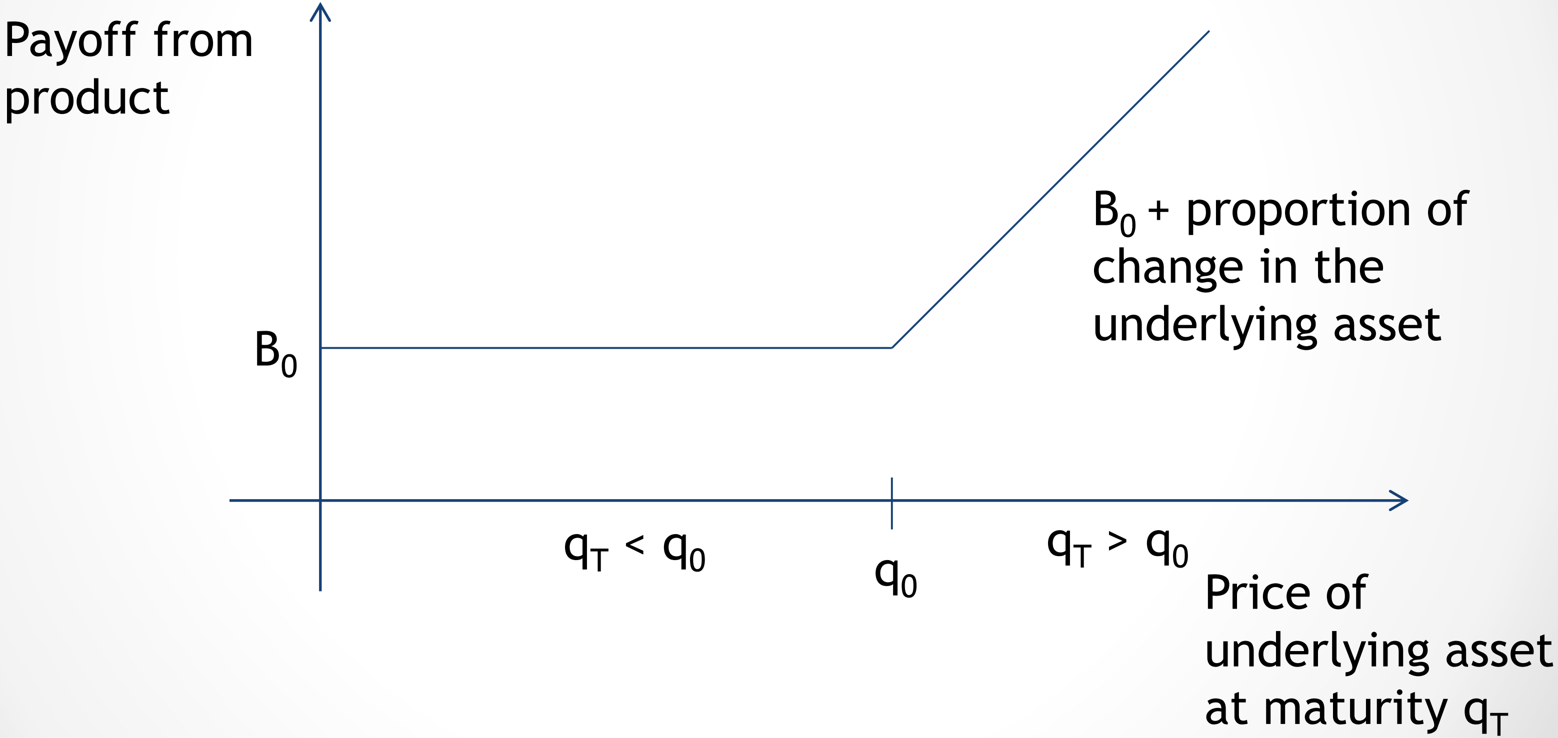
Volatility reduction

- The options can be made cheaper by ‘reducing’ the volatility

- 1. Choosing an index instead of a stock as the underlying asset (diversification)
- 2. Choosing an index in a foreign country as the underlying asset (diversification and effect of currency correlation): quanto

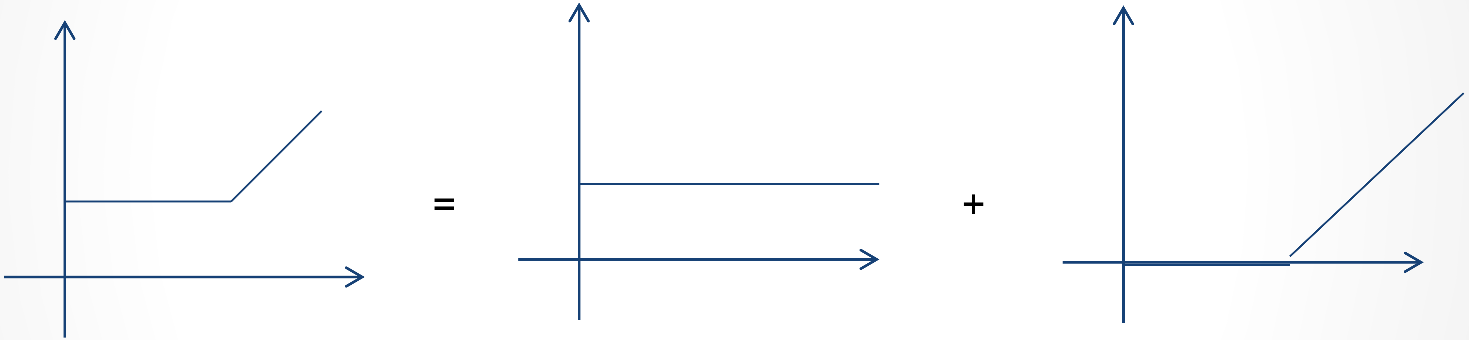
- Effect: can buy more options and increase the return factor

Payoff



Payoff from product
B_0
B_0 + proportion of change in the underlying asset
$q_T < q_0$
q_0
$q_T > q_0$
Price of underlying asset at maturity q_T

Value additivity



Payoff (structured product) = Payoff (zero-coupon bond) + Payoff (option)

Key principle

- Need to decompose structured product into subcomponents which can be valued individually

+ Value of Part A (e.g. Zero coupon bond)
+ Value of Part B (e.g. Individual or combinations of options)
+ Value of Part C

+++

+ Value of Part N

= Value of structured product

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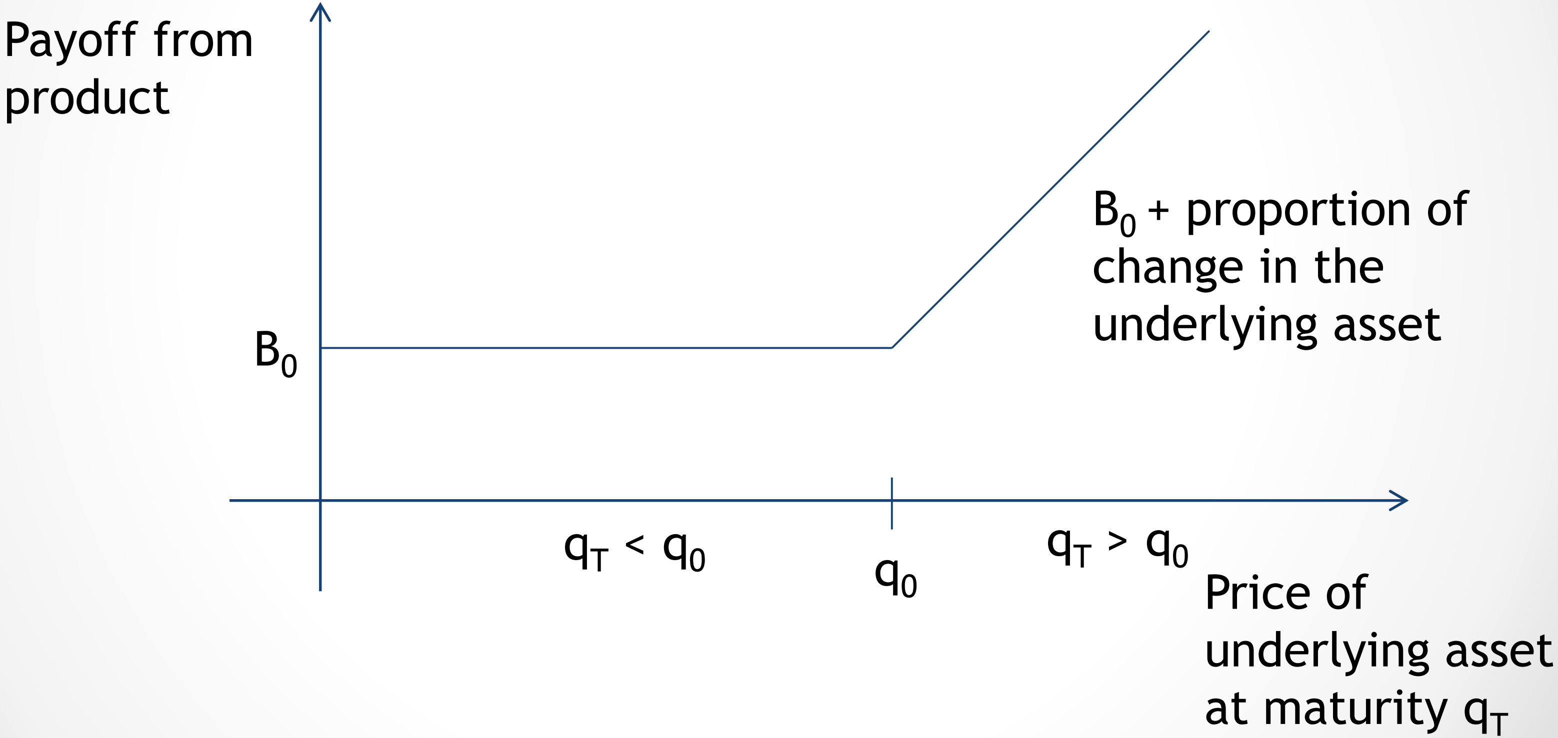
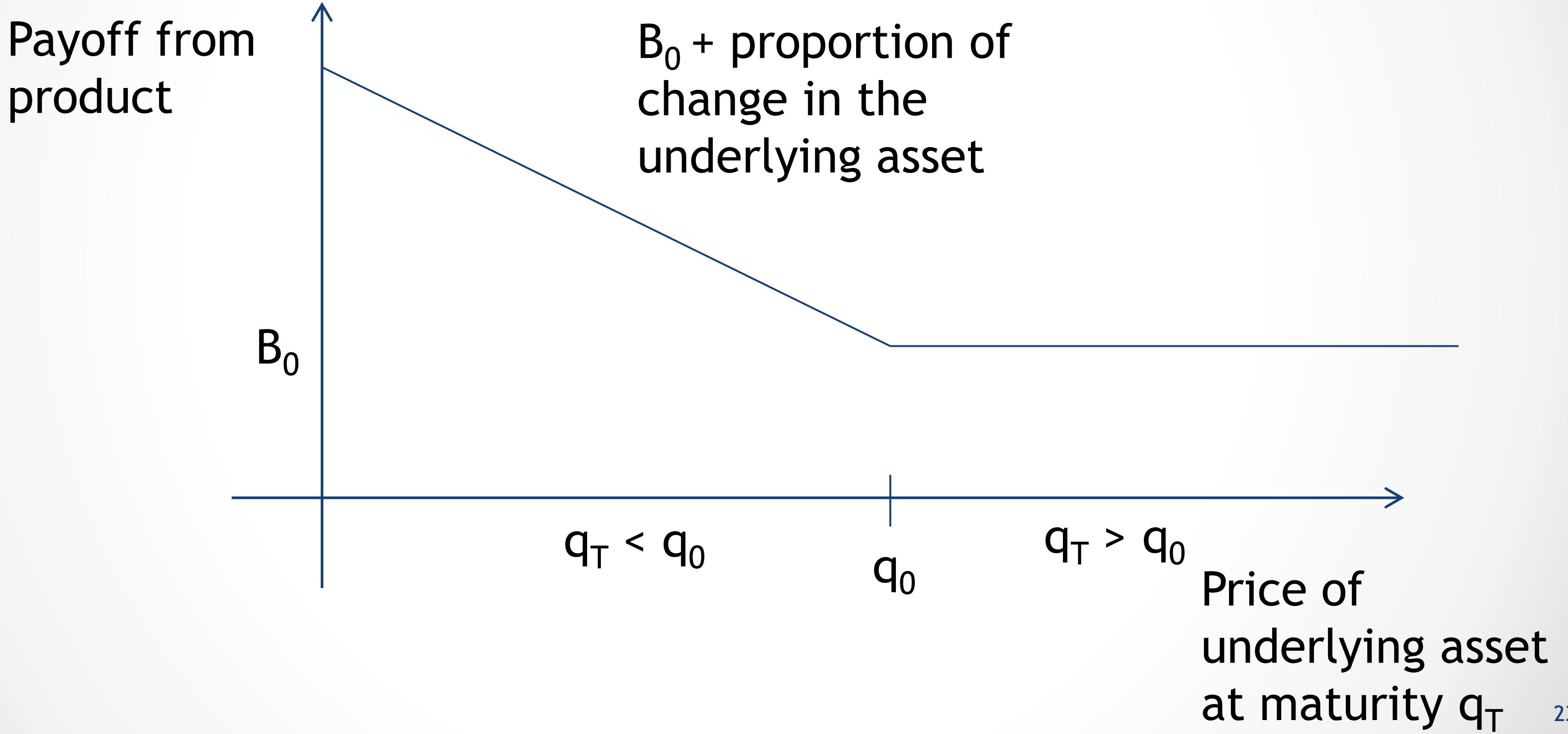
Example 1


Payoff from product
B_0
B_0 + proportion of change in the underlying asset
$q_T < q_0$
q_0
$q_T > q_0$
Price of underlying asset at maturity q_T

Example 1: decompose and value

- Decompose product to risk free element (zero coupon bond) and call option
- Value of product = value of zero coupon bond + call option value

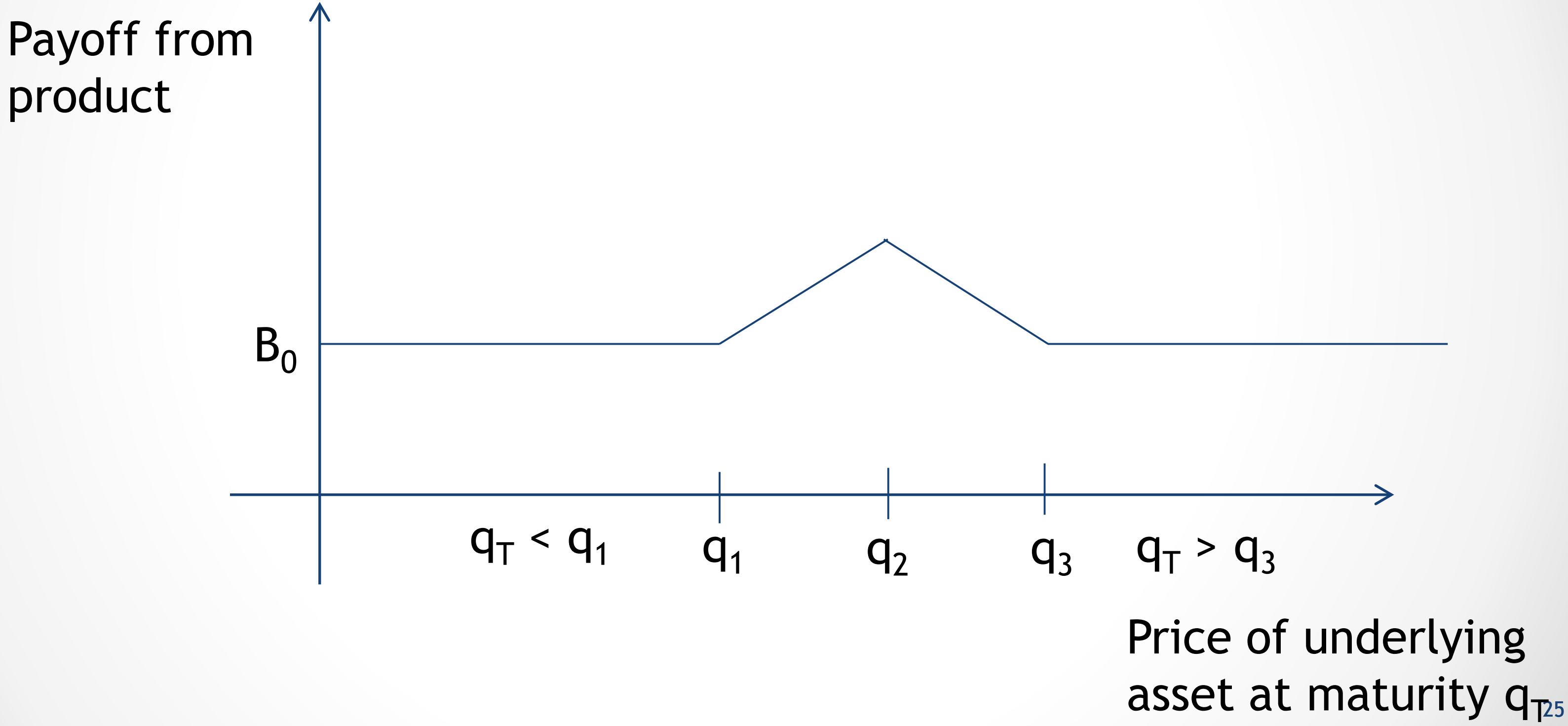
Example 2


Payoff from product
B_0 + proportion of change in the underlying asset
B_0
$q_T < q_0$
q_0
$q_T > q_0$
Price of underlying asset at maturity q_T

Example 2: decompose and value

- Decompose product to risk free element (zero coupon bond) and put option
- Value of product = value of zero coupon bond + put option value

Example 3



Payoff from product
B_0
$q_T < q_1$
q_1
q_2
q_3
$q_T > q_3$
Price of underlying asset at maturity q_T

Example 3: decompose and value

- Decompose product to risk free element (zero coupon bond) and option portfolio (butterfly spread)
- Value of product = value of zero coupon bond + put option value

+ Value (zero coupon bond)
+ Value (long call option with strike X1)
- 2 x Value (short call options with strike X2)
+ Value (long call option with strike X3)
= Total value of product

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Previous exams

- Autumn 2009 Question 4
- Autumn 2011 Question 3
- Spring 2013 Question 2

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Spread options

Agenda

- Spread options in the energy market
  - Locational (geographical) spread options
  - Time spread options
  - Cross-commodity spread options
  - Contract vs market spread options

- Valuing physical assets as spread options
  - Pipeline capacity
  - Gas-fired power plant
  - Oil refinery
  - Underground natural gas storage
  - LNG ships

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Spread options in the energy market

- The source of value for spread options are uncorrelated prices
  - Energy type spread option
    - Crude oil and gasoline
    - Gas and power
    - Coal and power
    - Coal vs gas vs power
  - Time spread options
    - Storage (gas, oil and electricity)
  - Geographical /locational spread option
    - LNG
    - Pipe gas

- Economics / rationale behind spreads
  - LOP (equilibrium: price differences should not exist, prices should be cointegrated)

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Economic intepretation of correlation between assets

- Law of one price (LOP): ”In an efficient market all identical goods must have only one price”
- If LOP does not hold, then arbitrage opportunities exist, however 3 conditions must be met
  - Products must be identical
  - Resale must be possible
  - There is no risk

- Price differences should only reflect the cost of transporting the goods between markets

Economic intepretation of correlation between assets

- In practice, price differences beyond transportation costs can exist for a shorter or longer period due to bottlenecks if the necessary transportation capacity is not available
  - An (arbitrage) opportunity for traders who sit on availbale capacity

- Price spreads can exist in several dimensions:
  - Geography
  - Time
  - Energy carrier
  - Price formula

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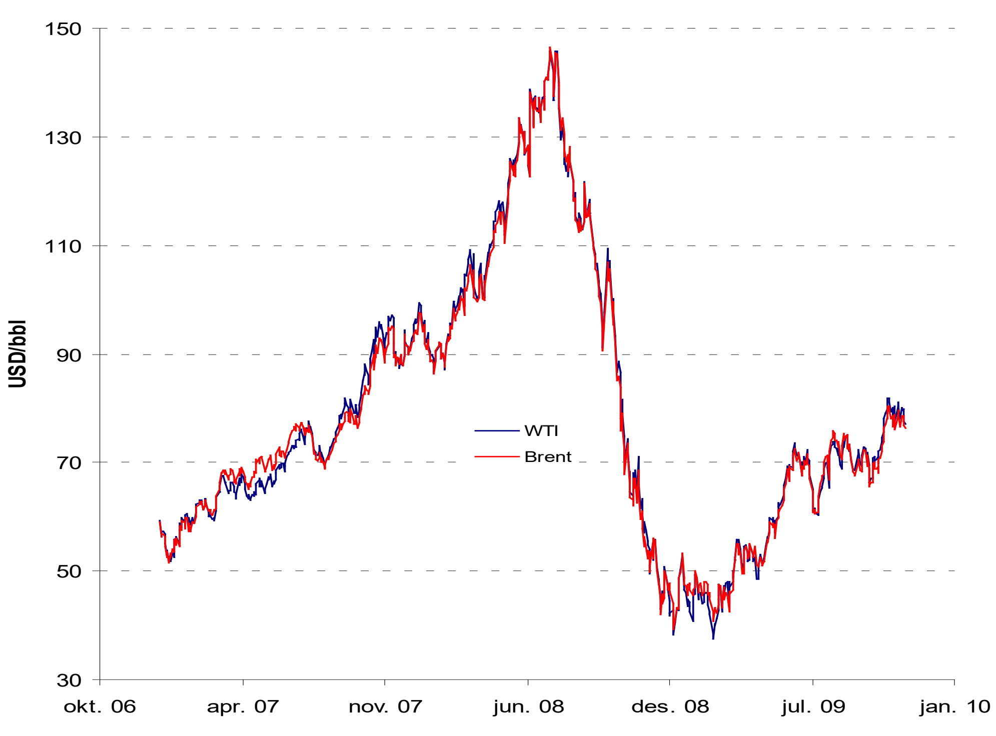
Locational spread option

- Option on a price spread between two identical underlying assets at different location

- Crude oil prices: WTI vs Brent
- Global gas prices: NBP vs HH
- Regional gas prices: NBP, TTF vs ZEE

Locational spread option

- WTI vs Brent
- Prices very correlated



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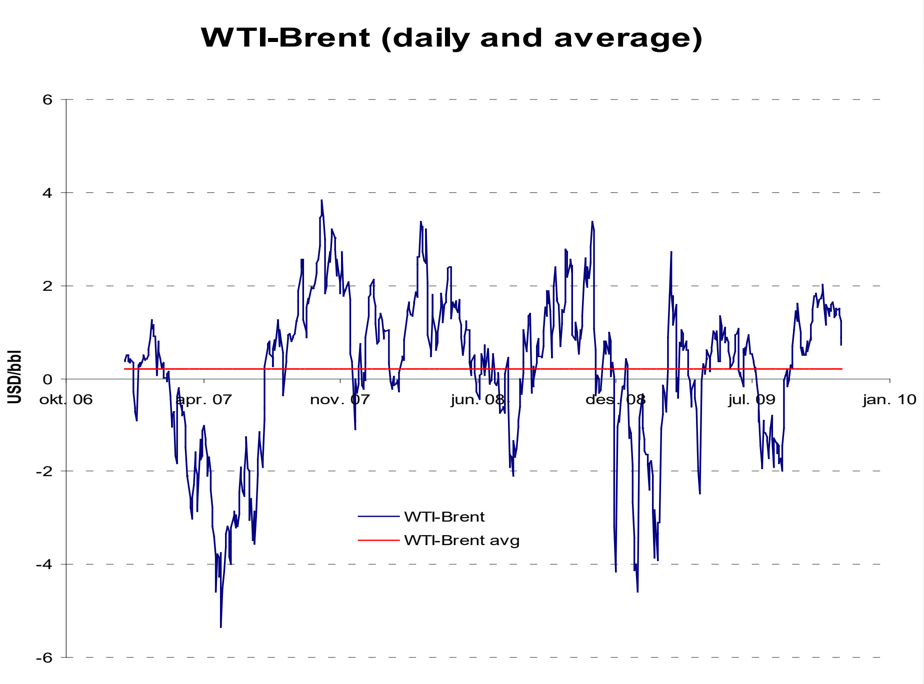
Locational spread option

- WTI vs Brent
- Substantial price differences can exist for months
- Mean reverting price spread?
- Global price for oil



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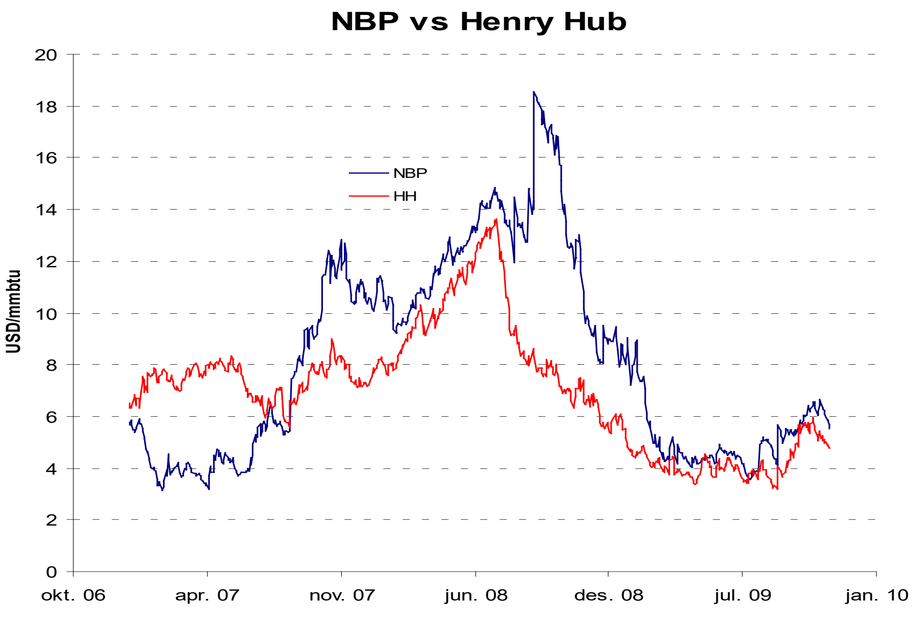
Locational spread option

- Price varies much more
- Global price for gas?



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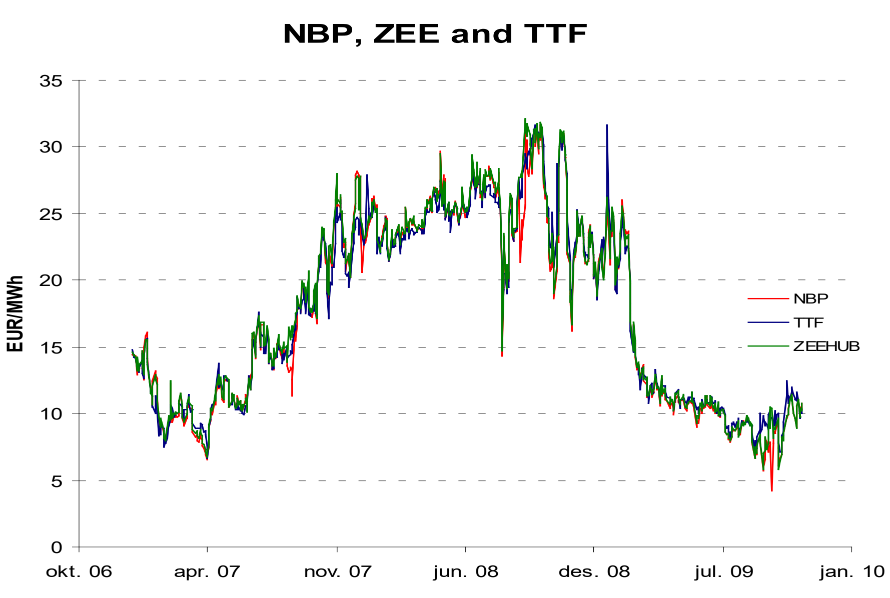
Locational spread option

- NBP vs TTF vs ZEE



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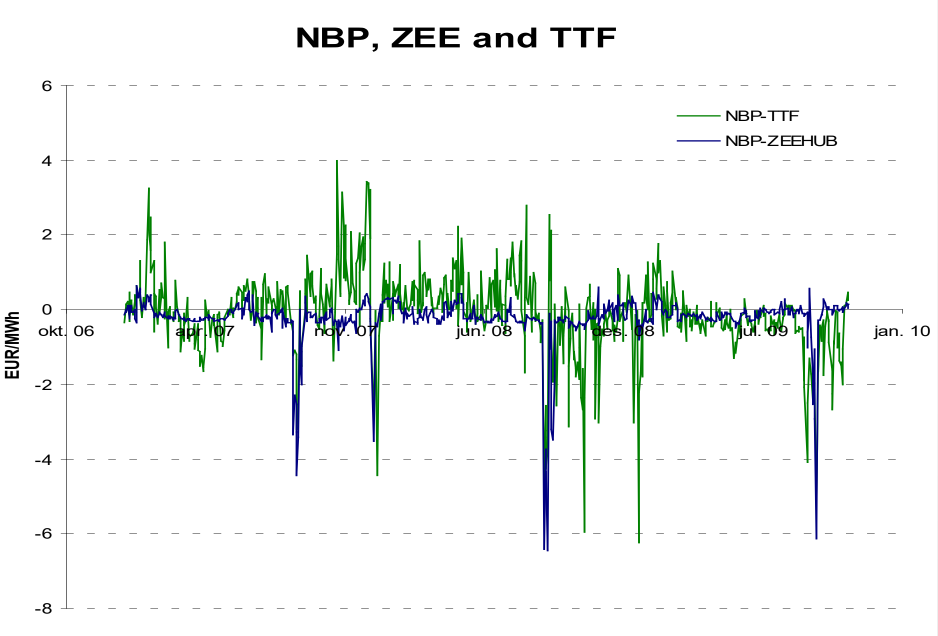
Locational spread option

- NBP vs TTF vs ZEE



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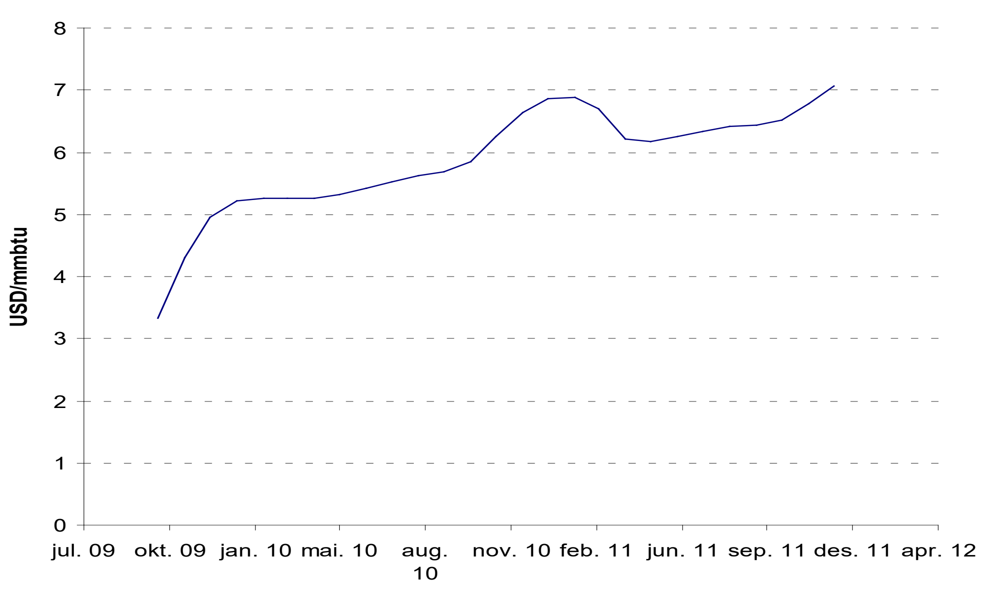
Time spread option

- Henry Hub
- Time spreads
  - Winter-summer spreads
  - Other time spreads
- Contango
  - Buy now, store, sell later



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Time spread option

- Example: Natural gas storage

- Important value driver is the price spread between summer and winter gas prices
- But storage also gives the owner many more time spread options (time spreads along the forward curve)
- Volatility and imperfect correlation create price time spreads
- Spread option on winter-summer spread:
- max (F_{winter} - F_{summer} - X, 0)

- Basket of spread options

$$C_{basket} = \sum \max(F_{it} - F_{jt} - X, 0)$$

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Time spread option

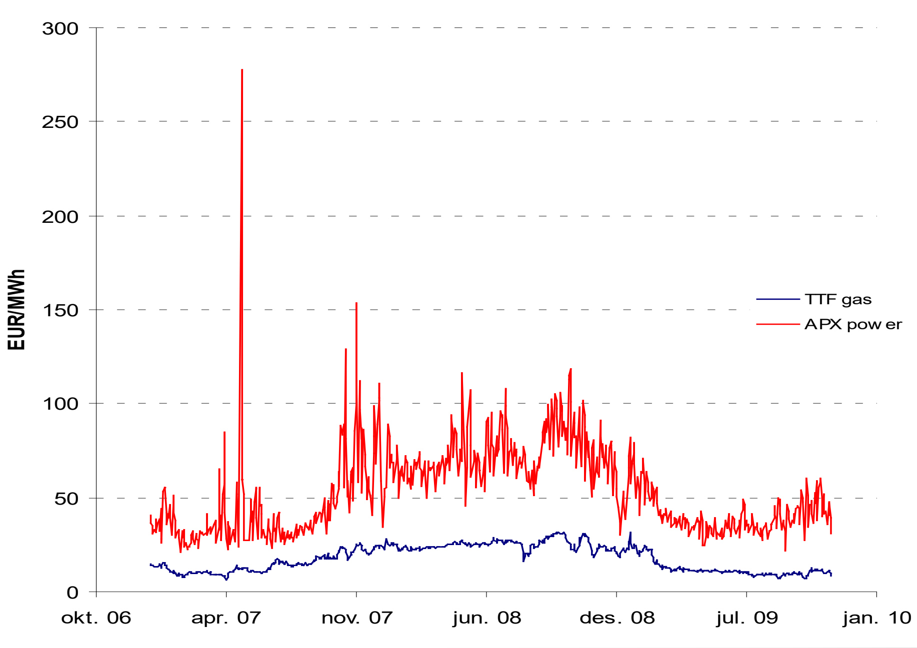
- Day 1 of storage operation
  - Observe forward curve – see spreads
  - Can exercise option, or wait

  - Day 2: spreads may have changed beacuse of volatility and imperfect correlation – a higher value

- Complex

Energy type spread option: spark spread

- TTF gas vs APX power
- Positive spark spread
- Need to factor in the conversion effeciency between fuel (e.g. gas) and power – Heat rate



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Spark spread

- The spark spread is defined as:

  P_{power} – F_{gas} x HR

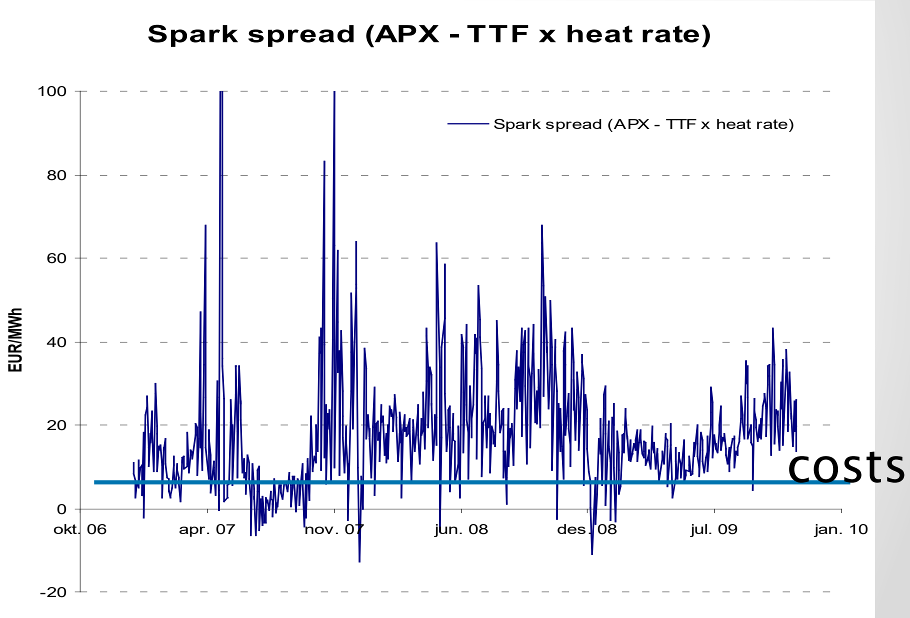
- HR = heat rate (energy conversion efficiency)

- Dirty spark spread
  P_{power} – F_{gas} x HR

- Clean spark spread
  P_{power} – F_{gas} x HR – emissions charge (CO_2)

Energy type: spread option

- TTF gas vs APX power
- Positive spark spread
- The spark spread needs to be above a certain threshold before it becomes profitable to run the power plant



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Spark spread option

- Every day, the owner of a flexible gas fired power plant has the option of selling gas outright or converting it to power

- Run power plant if Power price > gas price x heat rate + costs + emissions
- Do not run if Power price < gas price x heat rate + costs + emissions

$$\max(P_{power} - P_{gas} \times HR - X, 0)$$

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Spark spread option

- Example 1

$$\max(P_{power} - P_{gas} \times HR - X, 0)$$

- Price of electricity tomorrow = 50 EUR/MWh
- Price of gas tomorrow = 20 EUR/MWh
- Emissions (CO2) charge = 5 EUR/MWH
- Heat rate = 2 (i.e. 50% conversion efficiency)
- Variable costs = 2.5 EUR/MWh

- What is the clean and dirty spark spread?
- Should the power producer exercise the option?

Spark spread option

- Example 1

$$\max(P_{power} - P_{gas} \times HR - X, 0)$$

- What is the clean and dirty spark spread?

  Clean spark spread = power price – gas price x Heat rate – emissions
  Clean spark spread = 50 – 20 x 2 – 5 = 5 EUR/MWh
  Dirty spark spread = power price – gas price x Heat rate
  Dirty spark spread = 50 – 20 x 2 = 10 EUR/MWh
- Should the power producer exercise the option?
  - Yes
  - P_{power} – P_{gas} x HR – emissions – costs > 0
  - 50 – 20 x 2 – 5 -2.5 = 2.5 EUR/MWh

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Spark spread option

- Example 2

$$\max(P_{power} - P_{gas} \times HR - X, 0)$$

- Price of electricity tomorrow = 46 EUR/MWh
- Price of gas tomorrow = 20 EUR/MWh
- Emissions (CO2) charge = 5 EUR/MWH
- Heat rate = 2 (i.e. 50% conversion efficiency)
- Variable costs = 2.5 EUR/MWh

- What is the clean and dirty spark spread?
- Should the power producer exercise the option?

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Spark spread option

- Example 1

$$\max(P_{power} - P_{gas} \times HR - X, 0)$$

- What is the clean and dirty spark spread?
  Clean spark spread = power price – gas price x Heat rate – emissions
  Clean spark spread = 46 – 20 x 2 – 5 = 1 EUR/MWh
  Dirty spark spread = power price – gas price x Heat rate
  Dirty spark spread = 46 – 20 x 2 = 6 EUR/MWh
- Should the power producer exercise the option?
  - No
  - P_{power} – P_{gas} x HR – emissions – costs > 0
  - 46 – 20 x 2 – 5 -2.5 = -1.5 EUR/MWh

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Valuation of a basic spread option

- Based on Black-Scholes
- Although the Black-Scholes model is not the most appropriate for energy markets, valuable intuition can be gained about the unique behaviour of spread options even within a simple framwork.

- The payoff of a spread option:

$$\max(P_1 - P_2 - X, 0)$$

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Spread option (no strike price): Margrabe

- For the simplest case of the European spread option on forward contracts with no strike price (X=0), the value is described by the following formula developed by **Margabe**:

$$C_S = e^{-rT}[F_1 N(d_1) - F_2 N(d_2)]$$

- Where

$$d_1 = \frac{\ln(F_1 / F_2) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

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Spread option (no strike price): Margrabe

- This formula is almost identical to the Black-Scholes formula for the value of a call on a futures contract, with the price of the second contract put in the place of the strike price
- A substantial difference is that the volatility used in the formula is given by:

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho_{1,2}\sigma_1\sigma_2}$$

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Example

- What is the value of a spark spread option (for tomorrow) for a flexible gas fired power producer given:

- Power price: 100 EUR/MWh
- Gas price: 40 EUR/MWh
- Heat rate: 1.9
- Emissions charge: 0 EUR/MWh
- Variable operational costs = 0 EUR/MWh
- Volatility gas: 100% (annualised)
- Volatility power 500% (annualised)
- Correlation: 50%
- Risk free interest rate 5% (annualised, continuous compounding)

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Example

- First, calculate the volatility of the spark spread using the following formula:

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho_{1,2}\sigma_1\sigma_2}$$

$$\sigma = \sqrt{(5.0)^2 + (1.0)^2 - 2(0.5)(5.0)(1.0)}$$

$$\sigma = 458\%$$

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Example

- Then, calculate $N(d_1)$ and $N(d_2)$

$$d_1 = \frac{\ln(F_1 / F_2) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$

$$d_1 = \frac{\ln(100/40x1.9) + \frac{1}{2}(4.58)^2(1/250)}{(4.58)\sqrt{(1/250)}} = 1.0924$$

$$d_2 = d_1 - \sigma\sqrt{T} \qquad d_2 = 1.0924 - (4.58)\sqrt{(1/250)} = 0.5730$$

N(d1) = 0.8627
N(d2) = 0.7167

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Example

- Finally, calculate the value of the option

$$C_S = e^{-rT}[F_1 N(d_1) - F_2 N(d_2)]$$

$$C_S = e^{-0.05(1/250)}[100(0.8627) - (40x1.9)0.7167)]$$
$$= 31.79\ EUR/MWh$$

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Spread option (with strike price): Kirk model

- At present ther are no close-form formula for pricing spread options with nonzero strike price
- Several approaches can adress this valuation problem
- The simplest involves an approximation using the Margrabe formula with an adjusted volatility, and an adjustment as a function of the strike price

$$C_X = e^{-rT}[F_1 N(d_1) - (F_2 - X) N(d_2)]$$

$$d_1 = \frac{\left[\ln\left(\frac{F_1}{F_2 + X}\right) + \frac{1}{2}\sigma^2 T\right]}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

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Spread option (with strike price): Kirk model

This approximation performs quite well in comparison to other numerical techniques

$$\sigma = \sqrt{\sigma_1^2 + \left(\sigma_2^2 \frac{F_1}{F_2 + X}\right)^2 - 2\rho_{1,2}\sigma_1\sigma_2 \frac{F_1}{F_2 + X}}$$

Valuing spread options in a non Black-Scholes world

- Monte Carlo simulation

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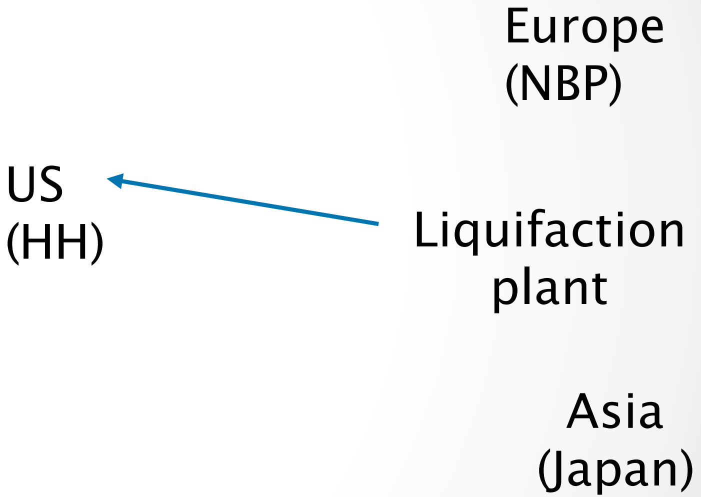
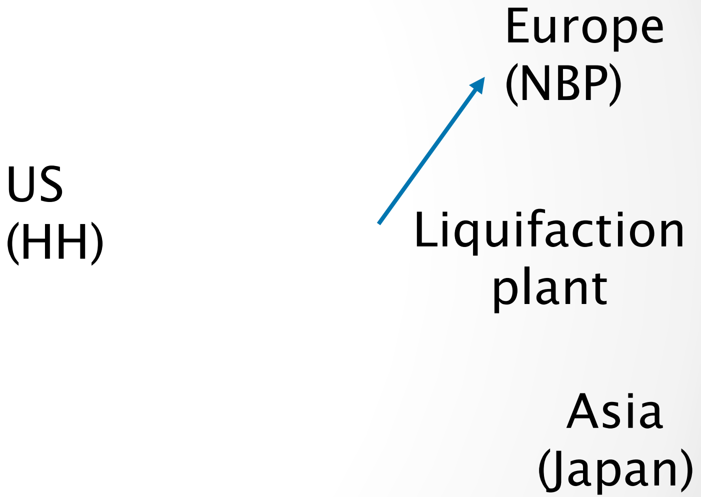
Example: valuation of LNG arbitrage

- The option is to change direction of LNG cargo
- **Sold to HH**



Example: valuation of LNG arbitrage

- The option is to change direction of LNG cargo
- Sold to HH
- **Option to re-route cargo and sell it at NBP if the price of gas is higher at NBP than HH**



Example: valuation of LNG arbitrage

- The option is to change direction of LNG cargo
- Sold to HH
- **Option to re-route cargo and sell it in Japan if the price of gas is higher in Japan than HH**

Europe
(NBP)

US
(HH)

Liquifaction
plant

Asia
(Japan)

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Example: valuation of LNG arbitrage

- What is the value of this spread option (i.e. re-route LNG cargo)?

- Can value this as a spread option between alternative market and base market (Henry Hub)

Alternative market is Europe (NBP)

$\max(F_{NBP}-F_{HH} - X, 0)$

Alternative market is Japan

$\max(F_{JAPAN}-F_{HH} - X, 0)$

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Example: valuation of LNG arbitrage

- Volatility HH: 100%
- Volatility NBP:150%
- Correlation: 50%
- Risk free interest rate
- Cost of re-reouting: 0 $/mmbtu (no strike price)
- Current price HH: 4 $/mmbtu
- Current price of NBP: 4.5 $/mmbtu
- Can re-route in 2 months time (time to maturity = 2 months, European option)

- What is the value of option to re-route the LNG cargo destined to Henry Hub to NBP?

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Example

- First, calculate the volatility of the price spread using the following formula:

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho_{1,2}\sigma_1\sigma_2}$$

$$\sigma = \sqrt{(1.5)^2 + (1.0)^2 - 2(0.5)(1.5)(1.0)}$$

$$\sigma = 132\%$$

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Example

- Then, calculate N(d1) and N(d2)

$$d_1 = \frac{\ln(F_1 / F_2) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$

$$d_1 = \frac{\ln(4.5/4.0) + \frac{1}{2}(1.32)^2(2/12)}{(1.32)\sqrt{(2/12)}} = 0.4892$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$d_2 = 0.4892 - (1.32)\sqrt{(2/12)} = 0.1488$$

N(d1) = 0.6876
N(d2) = 0.5591

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Example

- Finally, calculate the value of the option

$$C_S = e^{-rT}[F_1 N(d_1) - F_2 N(d_2)]$$

$$C_S = e^{-0.05(2/12)}[4.5(0.0.6876) - 4(0.5591)]$$
$$= 0.85\, USD / mmbtu$$

Real options

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What is a real option?

- Related to projects
- Investments in Property, buildings, factories and equipment
- Very often there are optionality/flexibility embedded in investmentdecisions
- Very important to identity the flexibility that is embedded in an investment opportunity
- Will increase the value of the project (>0)

Financial vs Real options

- Financial options (financial instruments)
  - Options on stocks, bonds, commodities, etc..
  - Traded options
- Real options (projects)
  - Non-traded options
  - Embedded options
  - Most investment projects contain some type of flexibility (optionality)
  - Hidden uncertainty
  - These options can increase the value of a project
  - Often ignored or valued wrongly
  - Flexibility has a positive nonzero value that should be quantified

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Flexibility: example

- A company is considering building a new factory for producing a new product. The company has the possibility to terminate the project if it turns out not to be profitable. The company also has the possibility to expand the factory if things turn out better than expected.

Flexibility: example

- A company is considering building a new factory for producing a new product. The company has the **possibility to terminate the project** if it turns out not to be profitable. The company also has the **possibility to expand** the factory if things turn out better than expected.

- ” **possibility to terminate the project”** => optionality

- ” **possibility to expand** ” => optionality

Types of real options

- Abandonment options
- Expansion options
- Contraction options
- Options to defer
- Options to extend

Abandonment options

- This is an option to sell or terminate a project
- An american option on the value of the project
- The strike price of the option is the liquidation value for the project minus termination costs

- An abandonment option counteracts the effect of extremely adverse outcomes and increases the value of the option as a whole

Expansion options

- This is an option to do additional investments and increase production once it is profitable to do so
- This is an american option on the value of increased capacity
- The strike price is the cost of increasing the capacity discounted to the exercise date
- The strike price is often dependent on the original investment
  - Overinvestment => reduced strike price

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Contraction options

- This is an option to reduce the size of the operation of the project
- It is an american option on the value of lost capacity
- The strike price is the present value of future saved cash flows, discounted to the exercise date

Options to defer

- One of the most important options a company has is the option to defer a project
- This is an american option on the value of the project

Option to extend

- It is often possible to extend the life time of an asset by paying a fixed sum
- This is a European option on the asset's future value

Options in the upstream oil and gas business

- The oil industry is characterised by large and long term projects
- Investments in billions dollars
- Demanding technology
- Investment decisions based on imperfact, little and uncertain information
- Long time horizon (>30 years)
- Long lead times (time from the first investment is carried out to the project generates a positive cash flow)
- Uncertain cash inflow (volatile oil prices)

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Options in the upstream oil and gas business

- What can oil and gas companies do to improve the NPV's of oil and gas projects?

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Real options in downstream energy markets

- In energy markets (unlike financial markets) spread options and other correlation products are very important
- Majority of generation assets can be seen as complex versions of spread options

Real options vs standard NPV analysis

- The traditional approach to valuing an investment project is the net present value method (NPV)
- The NPV of a project is the present value of the incremental cash flow
- The discount rate used to calculate the NPV is a risk adjusted cost of capital, and is selected so that it reflects the risk of the project

Example

- An investment costs 100 MNOK and will last for 5 years
- The expected income from the investment is 25 MNOK per year
- The risk adjusted discount rate is 12%

- The NPV for this cash flow is:

$$-100 + 25e^{-.12x1} + 25e^{-.12x2} + 25e^{-.12x3} + 25e^{-.12x4} + 25e^{-.12x5}$$

$$= -11.53$$

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Example

- The negative NPV is an indication that the project will reduce the value of the company from the perspective of the shareholders, and should not be carried out
- A positive NPV project should be initiated since it will increase the value of the company

The value of flexibility

- A problem with the traditional NPV approach is that many projects include options/optionality
- If a project includes one or more forms for flexibility, this is an indication that options are present
- An option value is always positive
- Flexibility therefore has a value/price that needs to be valued

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Example: including the value of flexibility

- We can invest in a machine that costs 10 MUSD and can produce 1 product per year for ever
- Each product costs 0.90 MUSD to produce
- The price for the product is 0.55 next year, and increases with 4% per year after that
- The risk free interest rate is 5% per year
- We can invest in the machine at any time
- There is no uncertainty

- What is the most amount of money you are willing to pay for the rights to this project?

Static NPV

- First we examine if it is profitable to invest today:

$$NPV^{today} = 0.55\left(\frac{1}{1.05} + \frac{1.04}{1.05^2} + \cdots\ldots\right)$$
$$-0.90\left(\frac{1}{1.05} + \frac{1}{1.05^2} + \cdots\ldots\right) - 10$$
$$= \frac{0.55}{1.04}\left(\frac{1}{1.05/_{1.04} - 1}\right) - \frac{0.90}{0.05} - 10 = 27$$

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Static NPV

- If the production of products should start up next year, we are willing to pay maximum 27 MUSD for the rights to the project
- This is the projects static NPV
- In the early years, the project looses money since the price is 0.55 and the cost is 0.90, in addtion to the opportunity cost of capital (5%)

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Static NPV

- If could therefore be profitable to wait with the start-up of the project, e.g. delay the investment with 5 years. The static NPV is then

$$NPV^{5yrs} = \frac{1}{1.055^5}\left[(1.04)^5\frac{0.55}{.01} - 28\right] = 30.49$$

- This analysis shows that is better to wait for 5 years instead of investing today

- But how to do this under uncertainty?

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Investment under uncertainty

- If we also include the uncertainty in the project's cash flow, the value of the implied put option will also affect the decision to delay the project
- We will use binomial trees to value the option
- We treat the option as an american option
- We pay the investment cost (=strike price) in order to receive an asset (the npv of future cash flows)

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A simple NPV problem

- An analyst is evaluating a project which will geenrate a simple cash flows, X, arriving at time T

- The project's return, volatility or covariance with the stock market is not directly observable

- The analyst must make some assumptions and simplifications, e.g. look at companies with comparable characteristics (e.g. to calculate the project beta)

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A simple NPV problem

- After assessing all data, the analyst calculates that the cash flow will be X_u is the economy does well with probability p and X_d if the economy does poorly
- The project needs investments of I_0 at time 0 and I_t at time T
- The analyst finds that comparable projects with similar risk profiles has an effective annual expected return of α.

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A simple NPV problem

- The value of the project, V is then:

$$V = \frac{pX_u + (1-p)X_d}{(1+\alpha)^T}$$

- If the choice is to invest today or never, we will invest in the project if:

$$V \geq I_0 + \frac{I_t}{(1+r)^T}$$

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Example

- Risk free rate is 6% (discrete)
- The expected market return, r_M is 10%
- The project beta is 1.25
- p=0.60
- T=1
- X_u = 120 and X_d = 80

- Should one invest in the project?

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Example

- First, we calculate the expected return on the project:

$$\alpha = 0.06 + 1.25(0.10 - 0.06) = 0.11$$

- The expected cash flow is:

$$E(X) = 0.60x120 + (1.00 - 0.60)x80 = 104$$

- The NPV of the project's cash flows is:

$$V = \frac{104}{1.00 + 0.11} = 93.694$$

Example

- If $I_0 = 10$ and I_t is 95, the NPV for the project is

$$NPV = V - I_0 - \frac{I_t}{(1+r)} = 93.694 - 10 - \frac{95}{(1+0.06)} = -5.929$$

- If we instead assume that the investment at time T, I_t has some optionaliy, we can try to assess the optionality embedded in the project

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Valuing the real options

- To calculate the option value we use binomial trees and risk neutral probabilities (risk neutral valuation)

- We can calculate the risk neutral probabilities for the project with the help of the forward price. Remember that the expected risk neutral price is equal to the forward price

- We have calculated V as the price of an asset that pays a simple cash flow at time T

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Valuing the real options

- The forward price is then

$$F_{0,T} = Ve^{rT}$$

- The risk neutral probabilities have to satisfy the relationship:

$$qX_u + (1-q)X_d = F_{0,T}$$

- This implies that

$$q = \frac{F_{0,T} - X_d}{X_u - X_d}$$

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Valuing the real options

- Using the same example as previously

$$F_{0,T} = 93.694e^{r*T} = 93.964(1.06) = 99.315$$

- The risk neutral probabilities then become

$$q = \frac{F_{0,T} - X_d}{X_u - X_d} = \frac{99.315 - 80}{120 - 80} = 0.4829$$

- The value of the project is then

$$\frac{0.4829 \times 120 + (1 - 0.4829) \times 80}{1.06} = 93.694$$

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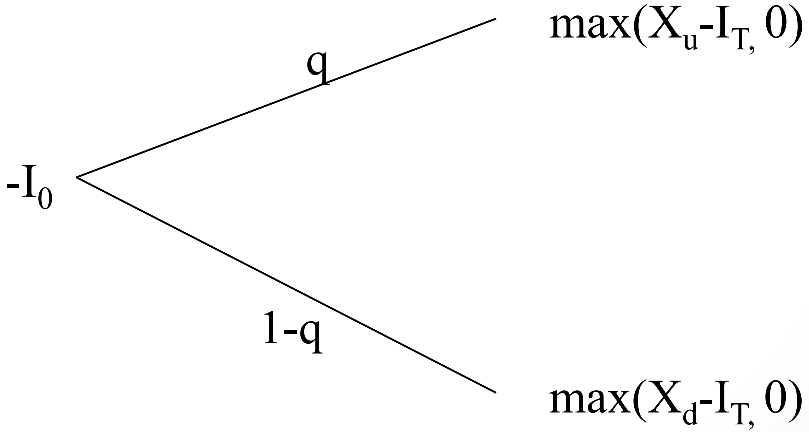
Valuing the real options

- We can now try to calculate the value of the project given that we have a right, but not an obligation to invest at time T



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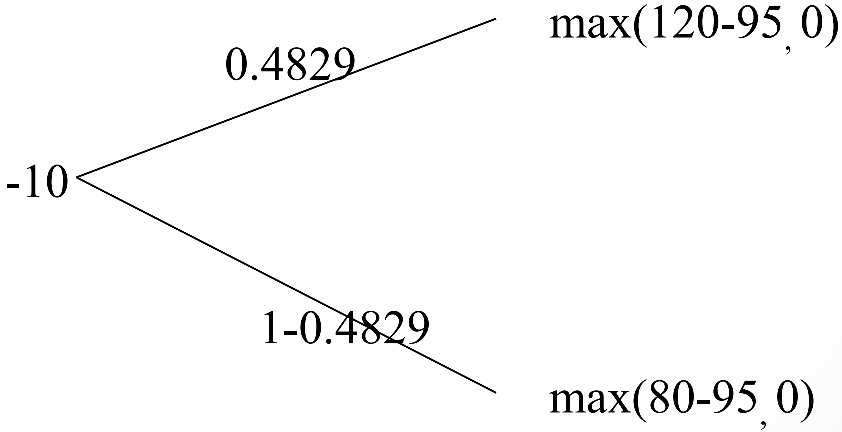

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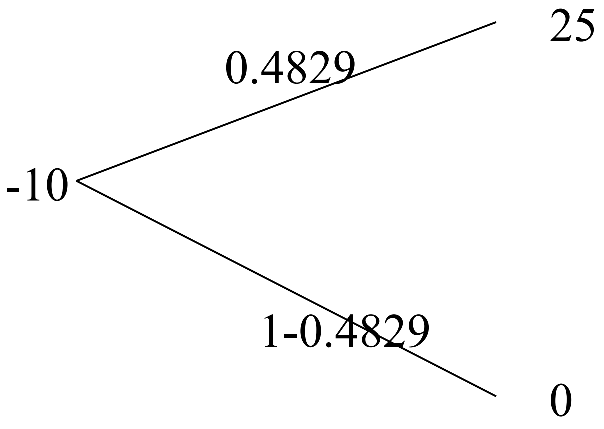

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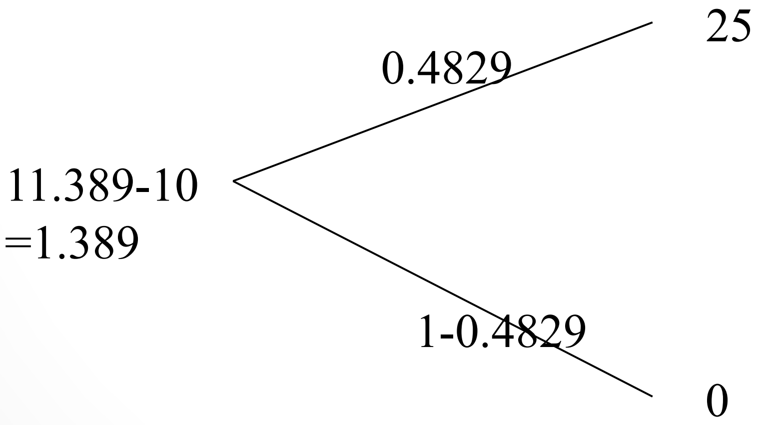

Valuation

- If we have risk neutral probabilities and the distribution of the cash flows we can value projects with options and non-linear cash flows
- When we value the options on stocks half the job is already done! The market has valued a present value of the company (=the underlying asset) for us (the market value)
- But when we value options on projects we need to calculate the present value of the project (=the underlying asset)ourselves since this cannot be observed (not market price)

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Another example (2-year investment horizon)

- We will now examine when to invest in a risky project
- Equal to an american option: we pay the investment cost (strike) and receive the asset (the present value of the future cash flows)
- The project costs 100 MUSD and generates a perpetual cash flow, 1 year after the investment
- The expect cash flow the first year is 18 MUSD and is expected to grow by 3% per year
- The risk free rate is 7%, the market's risk premium is 6% and the beta is 1.33

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Another example (2-year investment horizon)

- Cost of capital using CAPM

$$0.07 + 1.33(0.06) = 0.15$$

- To value the project we calculate the NPV of the cash flows

$$NPV = \frac{E(CF_1)}{r_{project} - growth\, rate} = \frac{0.18}{0.15 - 0.03} = 150$$

- The static NPV is 150 - 100 = 50

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Another example (2-year investment horizon)

- If we assume that we over the coming 2 years have the possibility to decide if we shall accept the project (invest or loose the investment opportunity)
- We need to assess the of waiting

Another example (2-year investment horizon)

- An important question is:

- What is the volatility of the project?
- We assume that the volatility is 50%

- We can then use the Cox-Ross-Rubinstein approach to calculate u and d (2-step tree)

$$u = e^{\sigma\sqrt{T}} = e^{0.50\sqrt{1}} = 1.65$$
$$d = e^{-\sigma\sqrt{T}} = e^{-0.50\sqrt{1}} = 0.61$$

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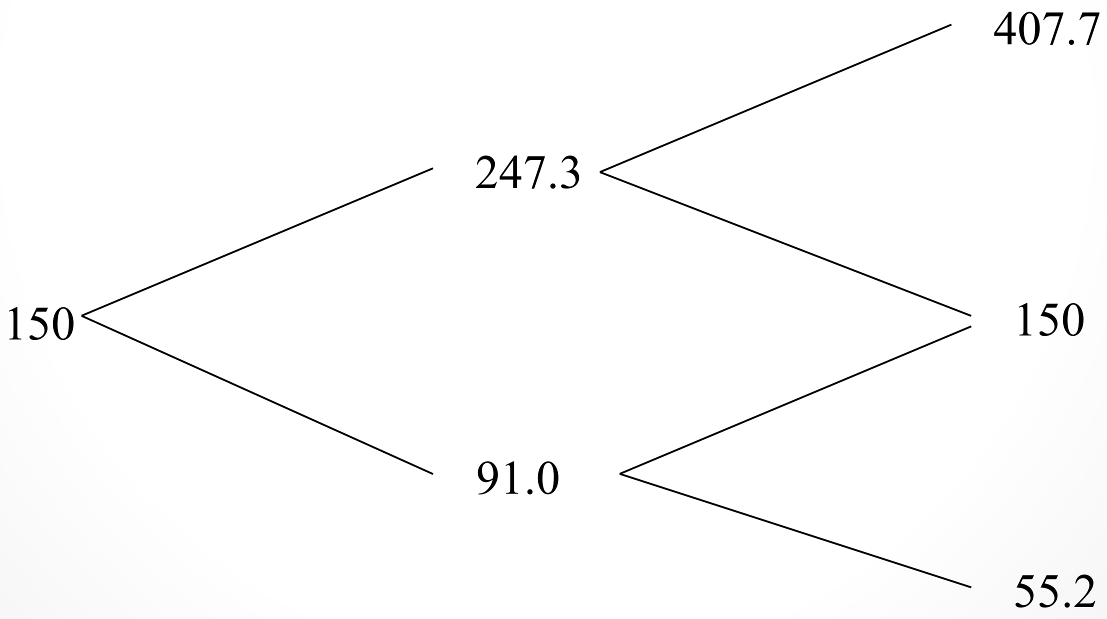
Another example (2-year investment horizon)

- Since the cash flows are lognormally distributed with 50% stdev, and the project value is proportional with the cash flows, the value of the project also be lognormally distributed with stdev 50%



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Another example (2-year investment horizon)

- Since replication of real options is not possible (in the same way as is possible for financial options) the prices are not arbitrage free (ref. principle of no arbitrage)

- Hence, the values are not *arbitrage free*, but rather *fair value* estimates

Another example (2-year investment horizon)

- Valuing the option

- The inputs are:
  - The project's initial present value, S_0=150
  - The investment cost, X=100
  - The risk free rate (continuous), r = ln(1.07) = 6.6766%
  - Volatility, σ = 0.50
  - Time to maturity, T=2
  - The dividend is 18/150 (receive cash flow 18 in 1 year given investment), constant. Continuous dividend rate is ln (1.12) = 0.1133

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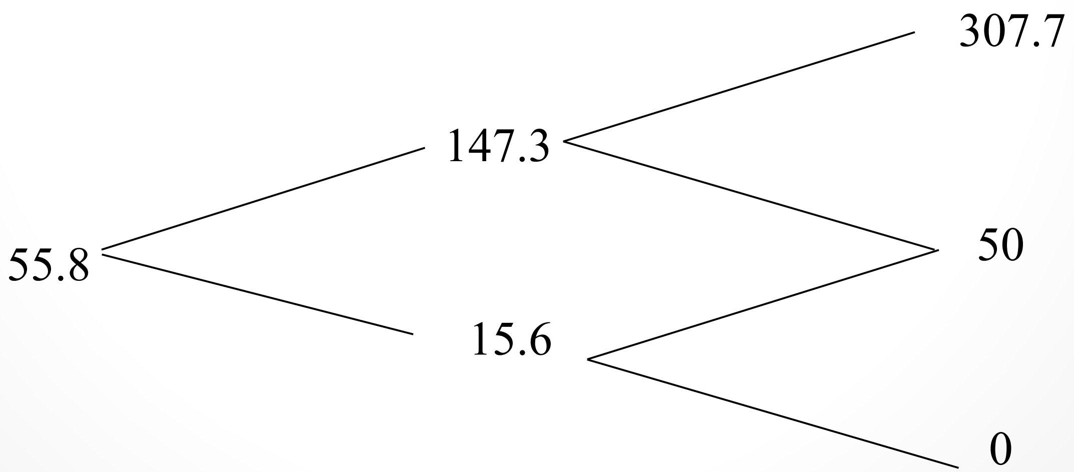
Another example (2-year investment horizon)

- The risk neutral probabilities are:

$$q = \frac{e^{0.0676-0.1133} - e^{-0.5}}{e^{0.5} - e^{-0.5}} = 0.335$$

- We can now calculate the value of the option in each node



Another example (2-year investment horizon)

- The value of the project including the option is 55.8, which is higher than the static value of 50

- The value of the option to postpone the project 2 years is 55.8 – 50 = 5.8 MUSD

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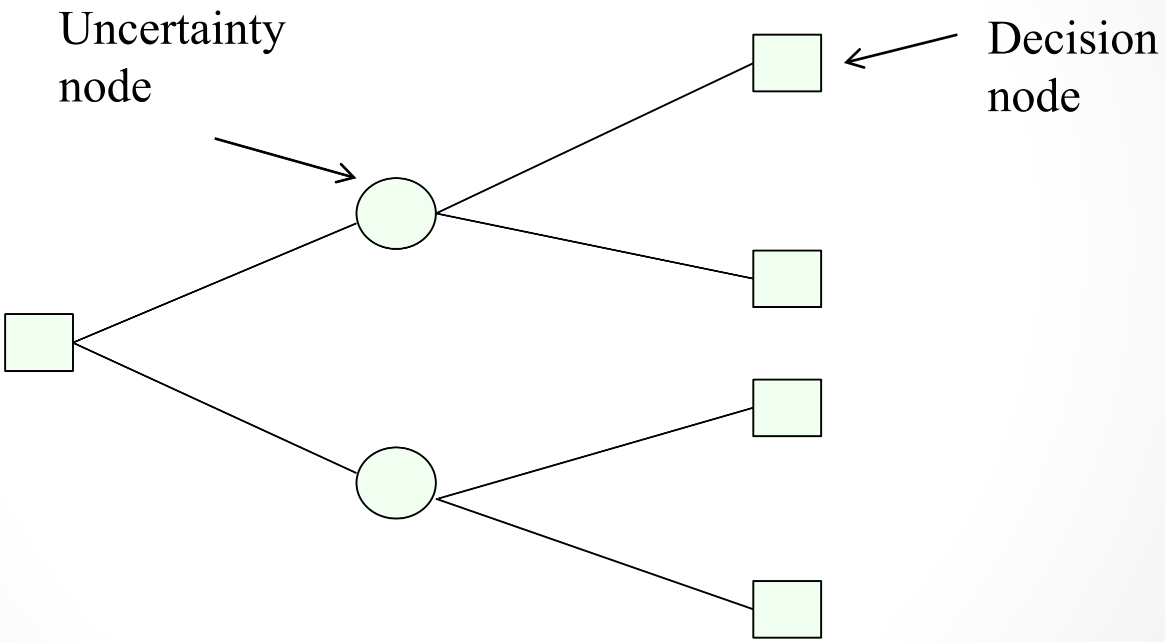
Valuation approach - Decision trees



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