Derivatives and Risk Management in Commodity Markets

Topic 6: Risk management

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1/13/2020

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Topics

- Arguments for and against hedging
- Hedging using futures (basis risk, optimal hedging ratios & hedging efficiency
- Risk management using derivatives
- Risk management of options (using greeks)
- Value-at-risk

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Learning objectives: hedging using futures contracts

- Why should you hedge using futures?
- What are short and long hedges?
- What are the arguments for and against hedging?
- What is basis risk?
- What is cross hedging?
- What do we mean by «rolling a futures contract»?

Hedging strategies using futures

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Hedging strategies using futures contracts

- Short hedges: short position in futures contract
  - appropiate when the hedger already owns an asset and expects to sell it some time in the future
  - oil producer

- Long hedges: long position in futures contracts
  - Appropriate when a company knows it will have to purchase a certain asset in the future and wants to lock in a price today

Arguments for and against hedging

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Arguments for/against hedging

- For:
- Many companies have no particular skill or expertise in predicting variables (interest rate, FX, commodity prices)
- Hedge in order to be able to focus on main acitivities

- Against
- Hedging and shareholders
  - Shareholders can do the hedging themselves
  - less expensive for company than individual
  - shareholders can diversify risks (hedging is unecessary)

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Arguments for/against hedging

- Against
- Hedging and competitors
  - If competitors do not hedge then the profits of hedgers will fluctuate more
    - non-hedgers will just pass through the floating price to their customers, no effect on their profitability

Basis risk, cross hedging, optimal hedge ratios and hedging efficiecy

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Basis risk

- There are certain difficulties with hedging

1. The asset whose price is to be hedged may not be exactly the same as the asset underlying the futures contract
2. The hedger may be uncertain as to the exact date when the asset will be bought or sold
3. The hedge may require the futures contract to be closed out before its delivery month

=> basis risk

Basis risk

- The basis

Basis = spot price of asset to be hedged - futures price of contract used

- If asset to be hedged = underlying asset at expiration of futures contract, then basis = 0
  - prior to expiration, basis may be positive or negative

Basis risk

- Example

S_1 = spot price at time t_1 = 2.50
S_2 = spot price at time t_2 = 2.00
F_1 = futures price at time t_1 = 2.20
F_2 = futures price at time t_2 = 1.90
b_1 = basis at time t_1
b_2 = basis at time t_2

$b_1 = S_1 - F_1 = 2.50 – 2.20 = 0.30$
$b_2 = S_2 – F_2 = 2.00 – 1.90 = 0.10$

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Basis risk

- Example 1
- Hedger knows the asset will be sold at t_2, he therefore shorts futures at t_1

- Price realised = S_2 + profit on futures position

$$S_2 + F_1 - F_2 = (S_2 - F_2) + F_1 = b_2 + F_1$$

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Cross hedge

- If asset that gives rise to the hedge exposure is different than the asset underlying the hedge, basis risk is usually higher

- Example
- Norwegian is concerned about the futures price of jet fuel, but there are not futures contracts for jet fuel
- Need to find a correlated product, e.g. heating oil to hedge its exposure

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Hedge ratio

- Hedge ratio: the ratio of the size of the position taken in a futures contract to the size of the exposure
- If asset underlying futures = hedged asset, then the hedge ratio is 1.0
- If there is a cross hedge, then 1.0 is not always optimal
- Then, chose the hedge ratio that minimises the variance of the value of the hedged position
- Hedge effectiveness: proportion of the variance eliminated by hedging

Minimum variance hedge ratio

$$h^* = \rho \frac{\sigma_S}{\sigma_F}$$

- ΔS = change in spot S during the life of the hedge
- ΔF = change in futures S during the life of the hedge
- σ_S = standard deviation of ΔS
- σ_F = standard deviation of ΔF
- ρ = correlation between ΔS and ΔF
- h* = optimal hedge ratio (minimum variance of hedge position)

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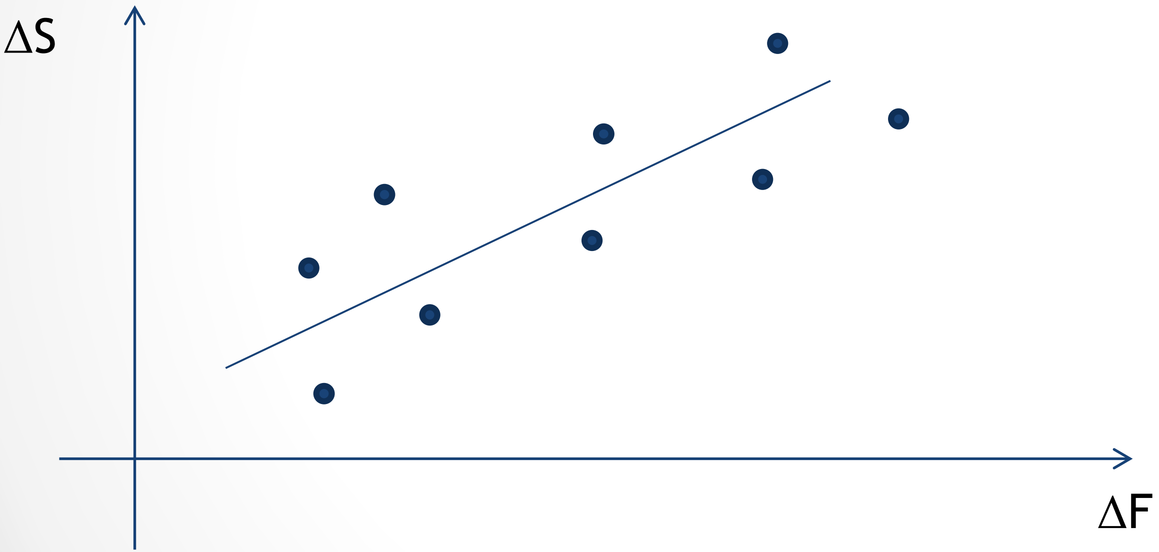
Calculating the Minimum Variance Hedge

- Regression analysis



- h^* is the slope of the regression line
- R^2 is the hedge effectiveness
- Observations of Δ F and Δ S from non-overlapping intervals

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Optimal number of contracts

- The number of futures contracts required is given by
- $N^* = \frac{h^* Q_A}{Q_F}$

- Q_A = Size of positions being hedged (units)
- Q_F = Size of one futures contract (units)
- N^* = Optimal number of futures contracts for hedging

Example

Month i	ΔF	ΔS
1	0.021	0.029
2	0.035	0.020
3	-0.046	-0.044
4	0.001	0.008
5	0.044	0.026
6	-0.029	-0.019
7	-0.026	-0.010
8	-0.029	-0.007
9	0.048	0.043
10	-0.006	0.011
11	-0.036	-0.036
12	-0.011	-0.018
13	0.019	0.009
14	-0.027	-0.032
15	0.029	0.023

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Hedging using forwards, futures and options

Example: hedging with forward contracts

- Risk Management: ”active use of derivatives and other techniques to manage risk and protect profitability
- Example: Oil Inc.
- Oil Inc. is an oil exploration and production company (crude oil) that plans to produce 100.000 barrels (bbl) of crude oil during the next year
  - assume that the company sells the production in exactly 1 year
- They will then get market price for their product if they sell them in the market place
- Today’s spot price is 40.5 $/bbl

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Example: Oil Inc.

- Oil Inc hopes that the price of crude will appreciate (increase) during the next year
- BUT, the price can also fall substantially
- If the price of crude oil is low enough the company will incur losses
- Should the company stop the production of crude oil if the price is too low?
  - The decision independent on level of fixed costs (these will have to be paid anyways)
  - The decision is dependent on level of variable costs
  - If price > variable costs => produce oil
  - If price < variable costs => do not produce oil

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Example: Oil Inc.

- Costs
  - Fixed: 33.0 $/bbl
  - Variable: 5.0 $/bbl

- The fixed costs have to be paid independent of production level and will not affect the decision of producing oil or not

- In this case it will be profitable to produce if the price of crude oil is higher than 5.0 $/bbl

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Example: Oil Inc.

- What is the Oil Inc's profitability if the price of crude oil is \$35.0, \$40.0, \$45.0 or \$50.0 per barrel in 1 year?

Crude oil price in 1 year	Fixed costs	Variable costs	Profit (unhedged)
35.0	-33.0	-5.0	-3.0
40.0	-33.0	-5.0	2.0
45.0	-33.0	-5.0	7.0
50.0	-33.0	-5.0	12.0

Example: Oil Inc.

- Oil Inc. can lock in the crude oil price in 1 year by entering into a forward contract to sell crude in 1 year (short forward)
- Assume that the futures/forward price for selling crude in 1 year is 42.0 $/bbl and that Oil Inc. promise to sell the entire production in 1 year
- What is the hedged profitability?
- Answer: Forward price – Fixed costs – Variable costs

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Example: Oil Inc.

- Oil Inc. can lock in the crude oil price in 1 year by entering into a forward contract to sell crude oil in 1 year (short forward)
- Assume that the price for selling crude oil in 1 year is 42.0 $/bbl and that Oil Inc. promise to sell the entire production in 1 year
- What is the hedged profitability?
- Answer: 42.0 – 33.0 – 5.0 = 4.0 $/bbl

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Example: Oil Inc.

Price of crude in 1 year	Fixed costs	Variable costs	Profit on short forward	Profit (hedged)
35.0	-33.0	-5.0	(42.0-35.0) = 7.0	(-3.0+7.0) = 4.0
40.0	-33.0	-5.0	(42.0-40.0) = 2.0	(2.0+2.0) = 4.0
45.0	-33.0	-5.0	(42.0-45.0) = -3.0	(7.0-3.0) = 4.0
50.0	-33.0	-5.0	(42.0-50.0) = -8.0	(12.0-8.0) = 4.0

Portfolio: Combination of a long position in the underlying asset and a short position in a forward contract on the underlying asset

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Example: Oil Inc.

Price of crude in 1 year	Fixed costs	Variable costs	Profit on short forward	Profit (hedged)
35.0	-33.0	-5.0	(42.0-35.0) = 7.0	(-3.0+7.0) = 4.0
40.0	-33.0	-5.0	(42.0-40.0) = 2.0	(2.0+2.0) = 4.0
45.0	-33.0	-5.0	(42.0-45.0) = -3.0	(7.0-3.0) = 4.0
50.0	-33.0	-5.0	(42.0-50.0) = -8.0	(12.0-8.0) = 4.0

Unhedged profit | profit on hedge | Hedged profit

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Oil Inc. revisited: Hedging using an option contract

- A downside when hedging with forward contracts is that the company looses the upside in the spot price
- Even if the price of crude oil increases Oil Inc will only receive 42.0 $/bbl
- A put option will provide the company with a price floor (insurance against a fall in the price of crude oil), and at the same time retain the upside (earn when the price of crude oil increases)
- Since we pay for the option 1 year before exercise, we have to take into account the interest rate (multiply option price today with the interest rate for 1 year)

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Oil Inc. revisited: Hedging using an option contract

- Assume that the strike price is 42.0 \$/bbl
- Assume that the option price is 0.877 \$/bbl
- Risk free interest rate is 5% med yearly compounding

= ln (1.05) = 4.879% with continuous compounding

- The value of the option in 1 year is

$$p_{T=1\,year} = 8.77 e^{0.04879 \times 1} = 9.21$$

- This is a cost which needs to be deducted

- We also have to add the profit from exercising the option

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Oil Inc. revisited: Hedging using an option contract

Crude oil price in 1 year	Fixed costs	Variable costs	Profit from exercising option	Hedged profit
35.0	-33.0	-5.0	max(42.0-35.0,0)-0.921 = 6.079	3.079
40.0	-33.0	-5.0	max(42.0-40.0,0)-0.921 = 1.079	3.079
45.0	-33.0	-5.0	max(42.0-45.0,0)-0.921 = -0.921	6.079
50.0	-33.0	-5.0	max(42.0-50.0,0)-0.921 = -0.921	11.079

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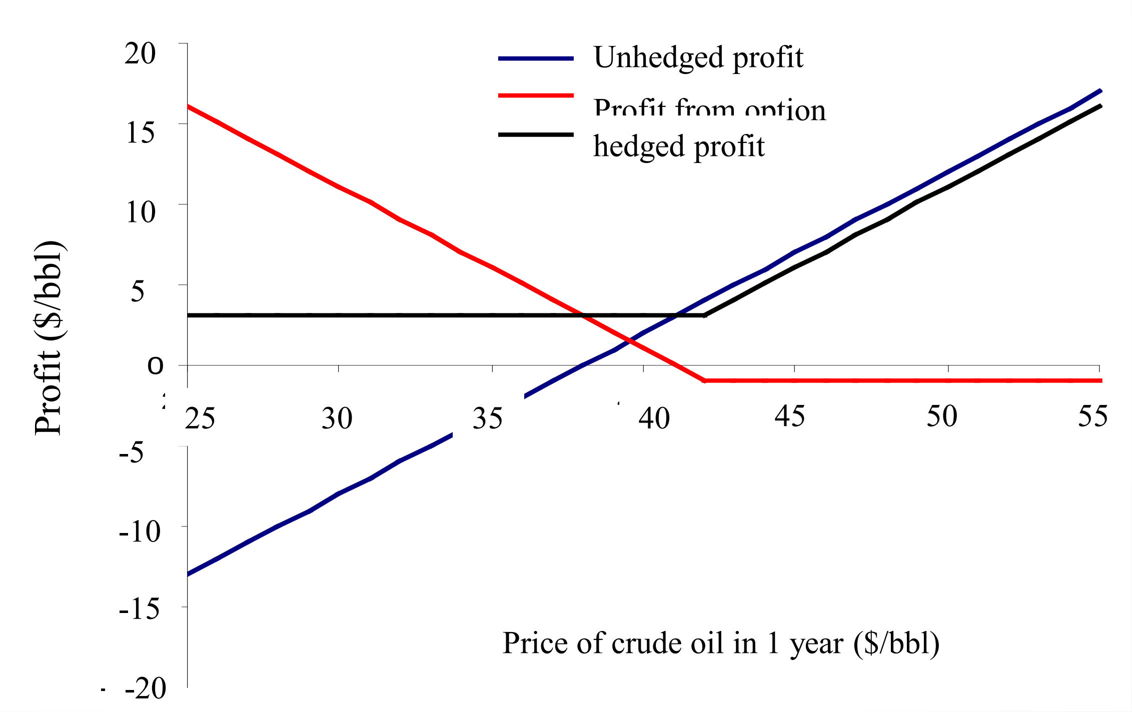
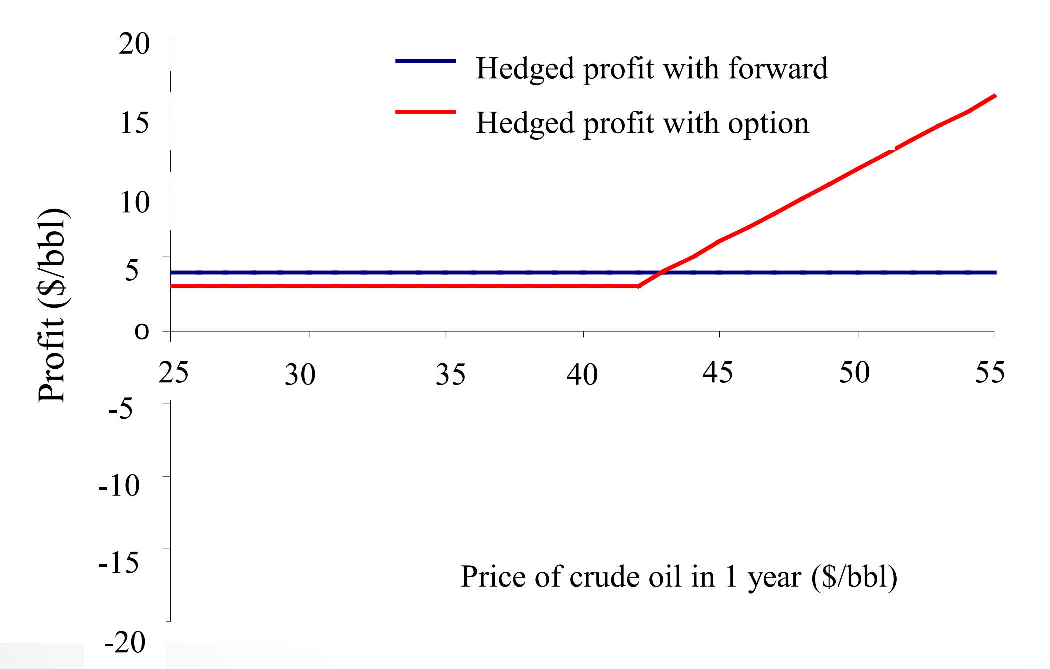
Oil Inc. revisited



Oil Inc. revisited



- Both the option and the forward contract insures against a fall in the price of crude oil
- The put option will let the company retain the upside if the price of crude oil increases, but at a cost of 0.877 $/bbl (today)
- A trade-off
- Depends on the market view of the company

Exam questions

- Autumn 2007 Question 1
- Autumn 2008 Question 5
- Spring 2013: Question 3

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Risk managing options (greeks and delta hedging)

Hedging

- You are exposed to market price risk if:
  - you have a position in the underlying
  - you have a position in an option
- How can you hedge your risk?
  - Naked position, covered position
  - Delta hedging (hedging using greeks)
  - synthetic derivatives
- How can you measure your exposure?
  - The Greeks
  - Value at Risk

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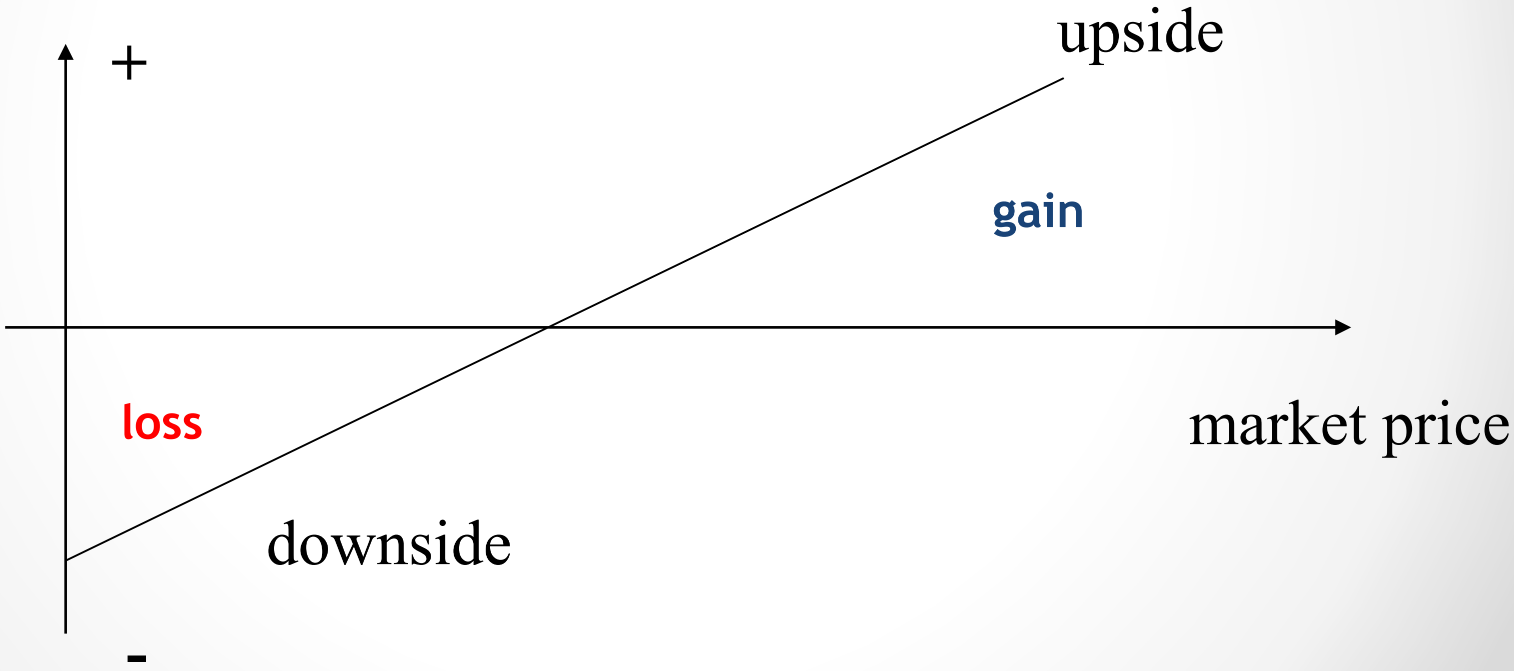
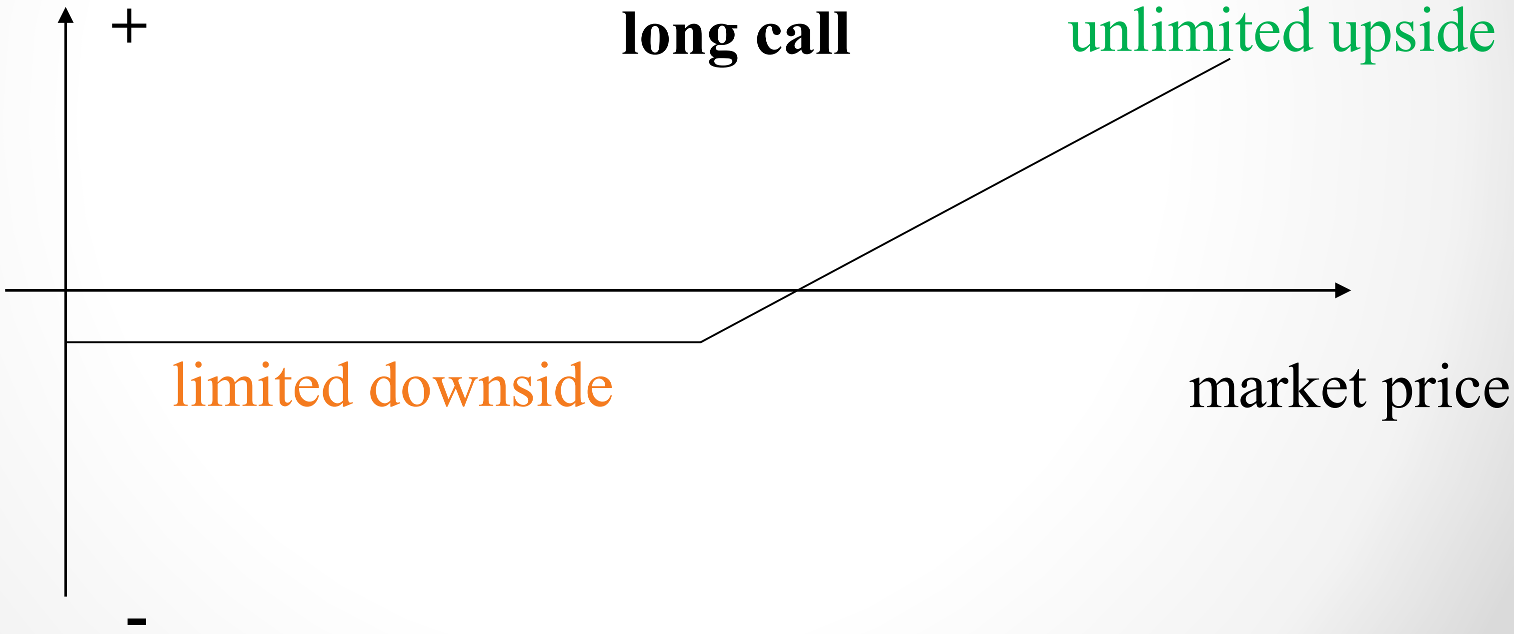
Price risk exposure

The market price risk when investing in the underlying, or a futures/forward contract looks like this



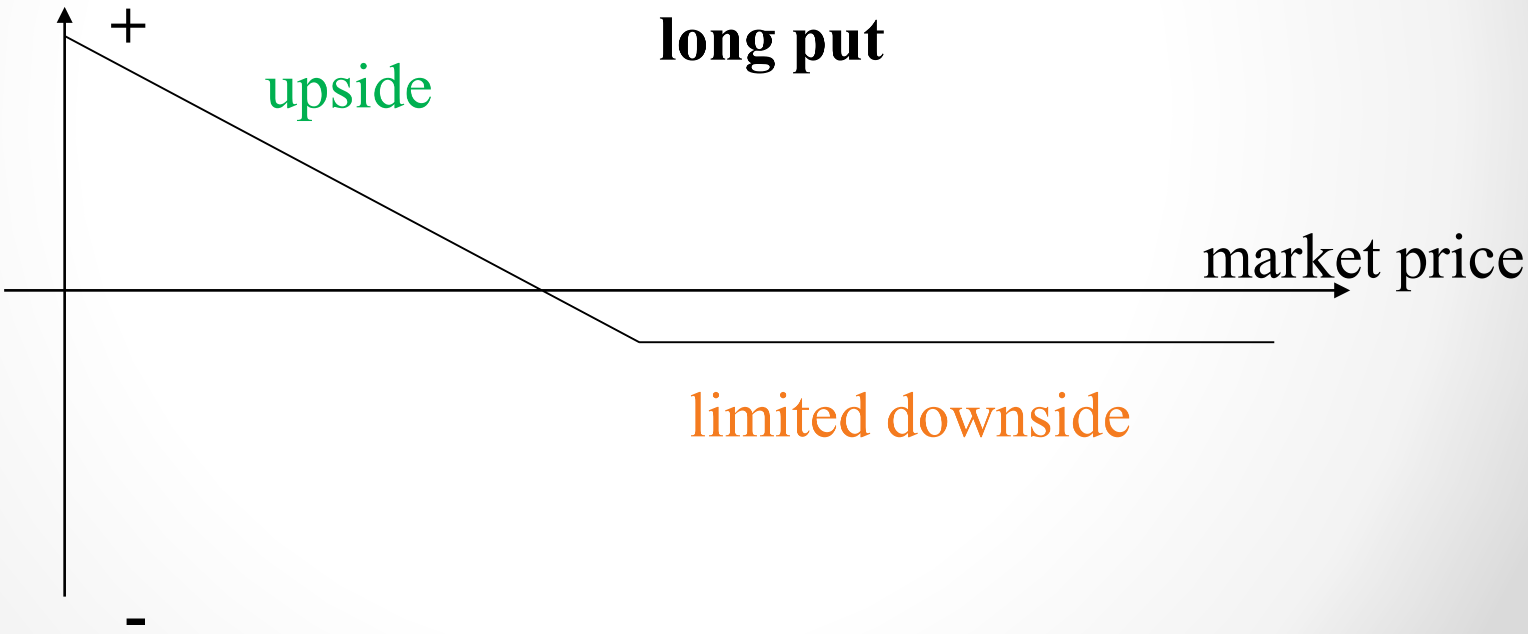
Price risk exposure

The market price risk when investing an options contract



Price risk exposure

The market price risk when investing an options contract



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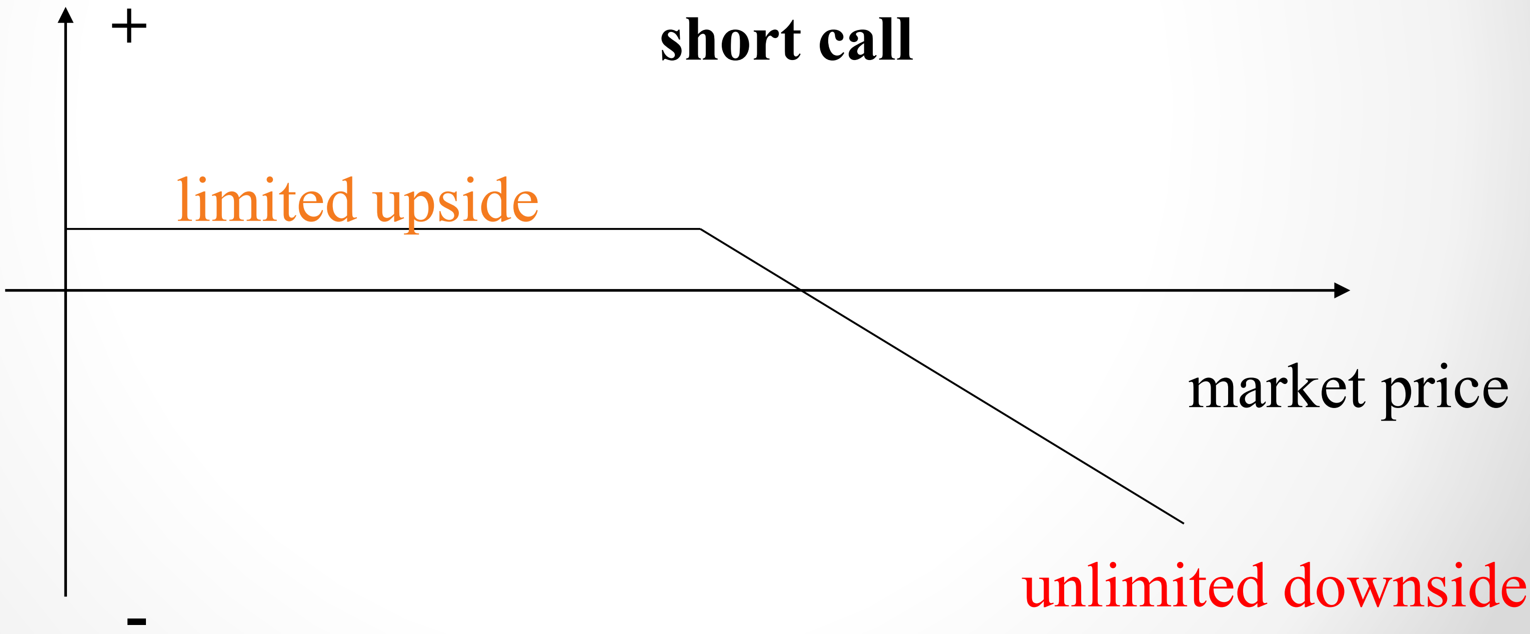
Price risk exposure

The market price risk when investing an options contract



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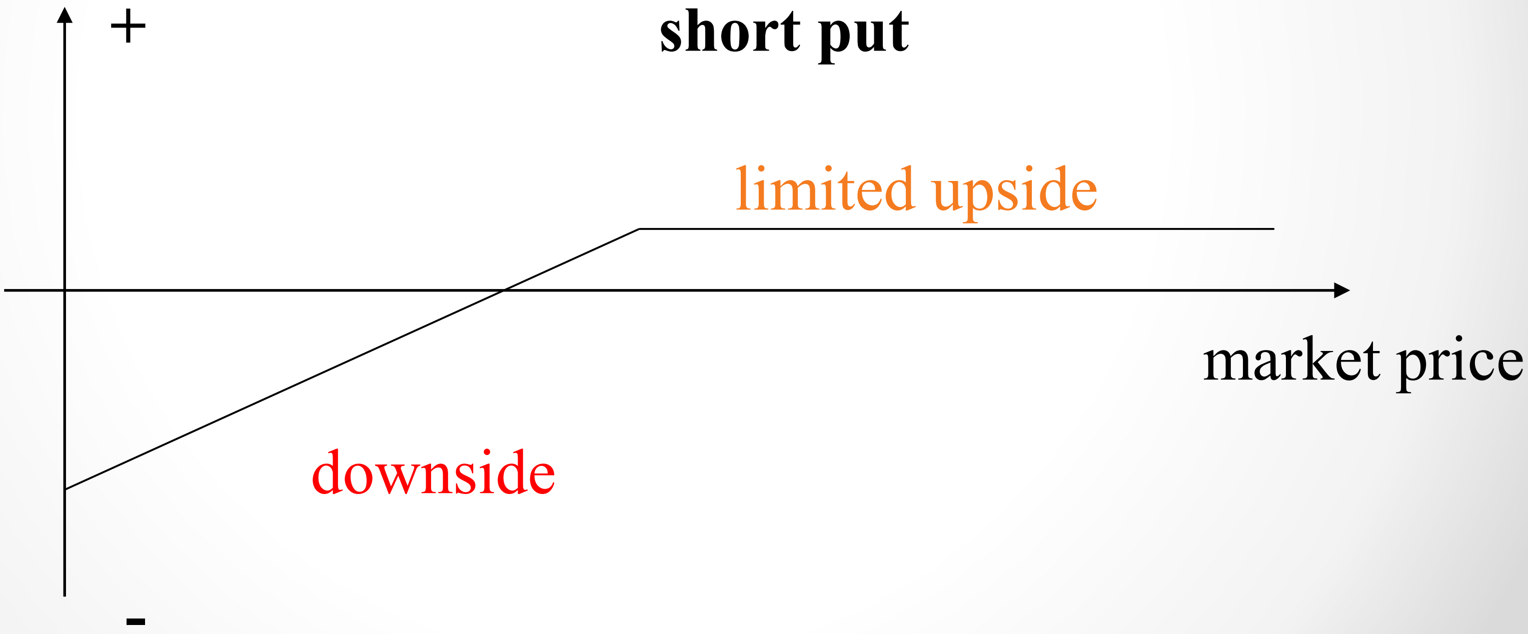
Price risk exposure

The market price risk when investing an options contract

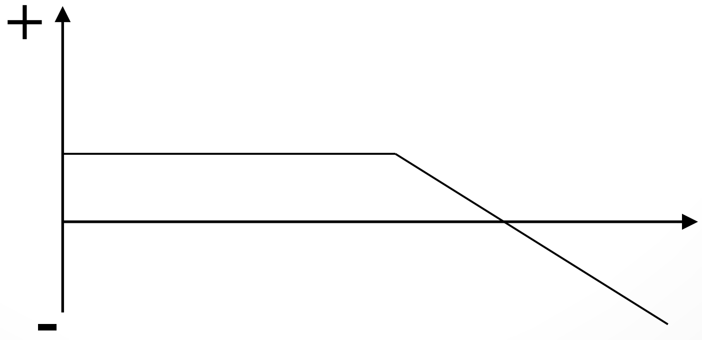


Hedging: example

- A financial institution has sold European call options on 100,000 shares

  $S0 = \$49$, $X = \$50$, $r = 5\%$, $\sigma = 20\%$, $T = 20/52$

- It received $300,000 for the options
- How can this institution hedge its risks?



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Hedging

1. Naked position (no hedging)

- Works well if price is below $50 after 20 weeks

- Is bad if the option is exercised and the institution has to buy 100000 shares in the market at higher price levels

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Hedging

2. Covered position

- The institution buys 100000 shares as soon as the option has been sold
  - works well if the option is exercised
  - bad if the options is not exercised (lower prices). Ex. if the stock price falls to $40, the financial institution loses $900000 on its stock position (more than the $300000 option premium)
- Neither the naked position or a covered position provides a good hedge

Hedging

3. Stop-loss strategy

- Buy 1 unit of stock if $S_t > X$ (Covered position)
- Sell 1 unit of stock if $S_t < X$ (Naked position)

- Objective: hold stocks if the option is in the money and not own money if the option is out of the money

- Unfortunately, this strategy does not work particularly well as a hedging scheme
  - expensive if the price crosses X many times

Hedging

4. Hedging using greeks (ex. delta hedging)

- You use the ’greeks’ to measure your risk exposure
- Try to achieve a balance between risk and hedge

- The aim is to be ’neutral’ (i.e. No residual risk/exposure)
  - portfolio delta = 0 (delta neutral)
  - portfolio gamma = 0 (gamma neutral)
  - portfolio vega = 0 (vega neutral)
  - etc…

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What are the 'Greeks'?

- The Greeks measure the risk of an option position
- When you buy an option, you are exposed to changes in different dimensions of risk
- Each of the greek letters measure a unique risk dimension
- Related to the value driver of the options
- Can be used in risk management

Greeks and value drivers

Δ, Γ

rho

Θ

$$c_0 = S_0 e^{-\delta T} N(d_1) - X e^{-rT} N(d_2)$$

ν

$$d_1 = \frac{\ln(S_0 / X) + (r - \delta + \tfrac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

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market price risk/volatility, interest rate risk, time risk

Delta

- The Delta (Δ) of an option is defined as the sensitivity of an option price to changes in the price of the underlying asset

$$\Delta = \frac{c_u - c_d}{S_u - S_d}$$ (binomial model, discrete time)

$$\Delta = \frac{\partial c}{\partial S}$$ (continuous time)

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The delta of a European call

- For a stock that does not pay dividends, it can be shown that the delta of a European call is:

$$\Delta(call) = N(d_1)$$

- Short position

$$-[\Delta(call)] = -[N(d_1)] < 0$$

- Long position

$$+[\Delta(call)] = +[N(d_1)] > 0$$

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The delta of a European call

- For a stock that does not pay dividends, it can be shown that the delta of a European call is:

$$\Delta(call) = N(d_1)$$

- A delta hedge for a short position in a European call requires a long position in $N(d_1)$ stocks at each point in time
- A delta hedge for a long position in a European call requires a short position in $N(d_1)$ stocks at each point in time

The delta of a European put

For a stock that does not pay dividends, it can be shown that the delta of a European put is:

$$\Delta(put) = N(d_1) - 1$$

- Delta is negative
- Short position

$$-[\Delta(put)] = -[N(d_1) - 1] > 0$$

- Long position

$$+[\Delta(put)] = +[N(d_1) - 1] < 0$$

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The delta of a European put

For a stock that does not pay dividends, it can be shown that the delta of a European put is:

$$\Delta(put) = N(d_1) - 1$$

- Delta is negative
- A delta hedge for a long position in a European put requires a long position in stocks at each point in time
- A delta hedge for a short position in a European put requires a short position in stocks at each point in time

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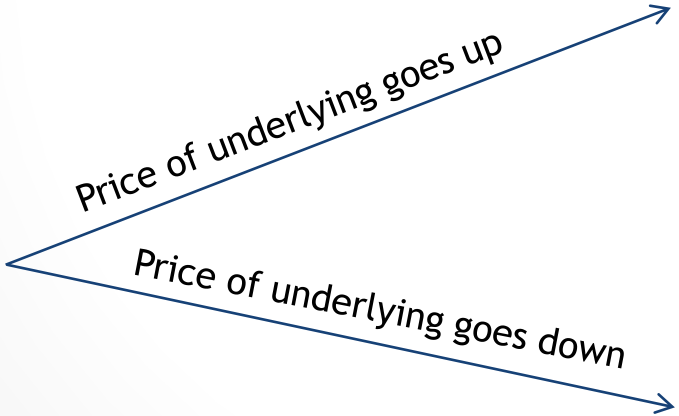
Cash flow effects of delta hedging

1. Long position in option (gives exposure to underlying asset)
2. Short position in stocks (hedge exposure to underlying asset)

Cash flow effects (payoff)



1. Long option (+)
2. Short stocks (-)

1. Long option (-)
2. Short stocks (+)

today

tomorrow

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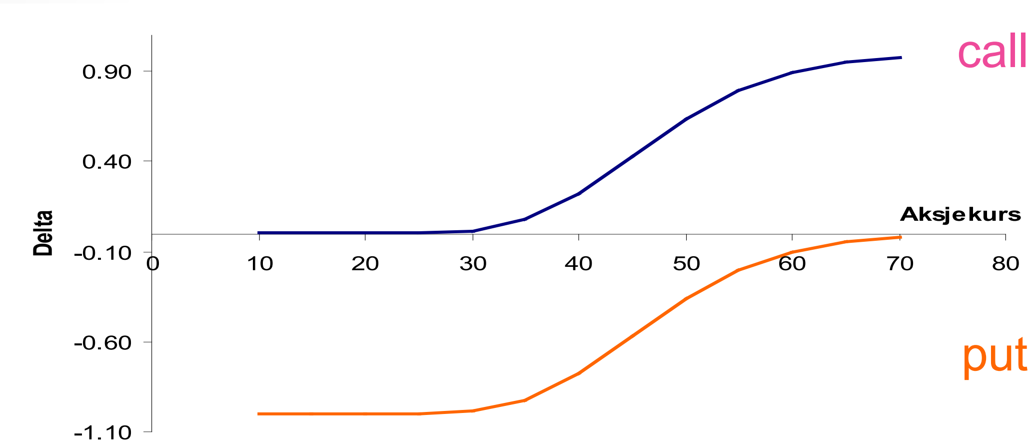
Illustration



$$\max(S_T - X, 0)$$

$$\max(X - S_T, 0)$$

Change in option delta as a function of stock price

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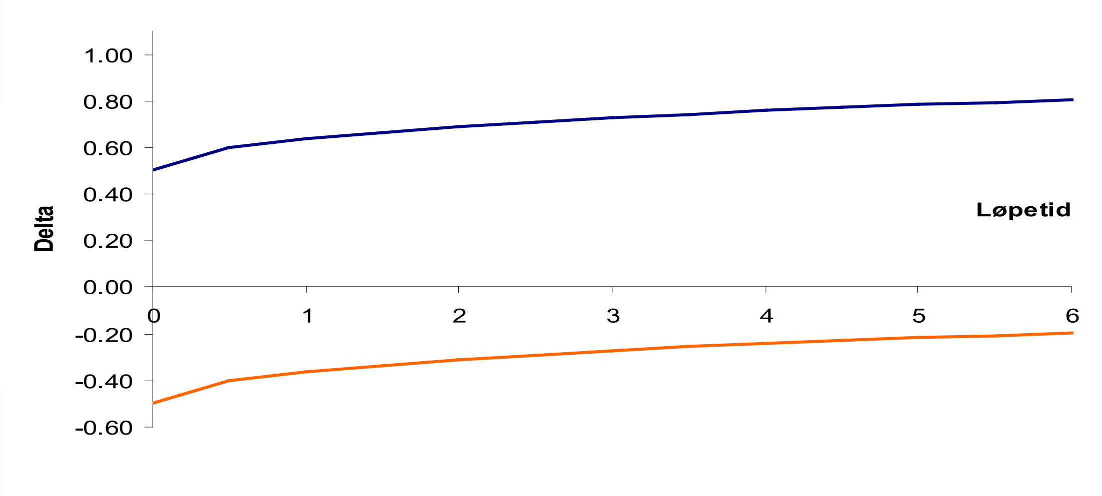
Illustration



Change in option delta as a function of time to maturity

Delta of other options

- Stocks that pay dividends, δ

$$\Delta(call) = e^{-\delta T} N(d_1)$$
$$\Delta(put) = e^{-\delta T}[N(d_1) - 1]$$

- Analoguous for options on other underlying assets

Delta of futures and forwards

- The Delta of a forward is 1.0

- A short forward on 1 share can be hedged by buying 1 share

  Delta (short position) = -1

  Delta (long position) = +1

- due to mark-to-market, the delta of a futures position will not be 1, but e^{rT} (but for simplicity we will assume also a delta of 1.0 for futures in this course)

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Delta of the underlying asset

- The delta of the underlying asset is 1.0

- If a exposure has a delta of -5, this means we can eliminate the delta (i.e. residual risk exposure) by buying 5 of the underlying asset

- Portfolio (or individual financial instrument) delta =-5
- Eliminate residual delta by buying 5 shares: 1.0 x 5 = 5
- New delta = -5 + 5 = 0 (i.e. Delta neutral)

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Delta of a portfolio

- The Delta of a portfolio of options or other derivatives based on the same underlying (spot price S) is:

$$\frac{\partial \Pi}{\partial S}$$

- The Delta of the portfolio can be calculated from the delta of each of the individual options in the portfolio

$$\Delta = \sum_{i=1}^{n} w_i \Delta_i$$

- w_i = number of option *i*, Δ_i = delta of option *i*

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Delta of a portfolio (2)

- An investor in USA has the following positions in Australian dollars:
  - 100,000 long call options with X=0.55, T=3months. Each of the options has Δ = 0.533
  - 200,000 short call options with X=0.56, T=5months. Each of the options has Δ = 0.468
  - 50,000 short put options with X=0.56, T=2months. Each of the options has Δ = -0.508

- The portfolio Delta is:

$$100{,}000 \times 0.533 - 200{,}000 \times 0.468 - 50{,}000(-0.508) = -14{,}900$$

The portfolio can be made delta neutral with a long position of 14,900 Australian dollars

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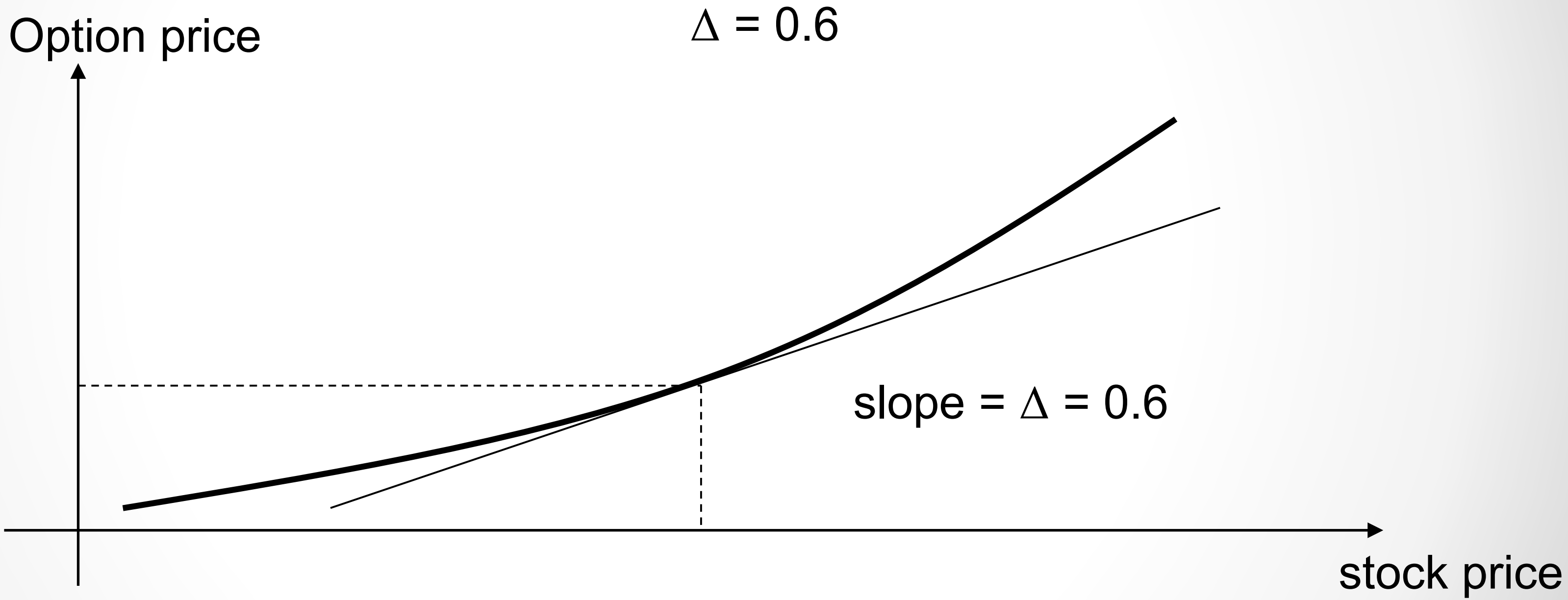
Delta hedging



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Δ = 0.6 means that when the price of the underlying changes by 10%, the option price will change with 6%

Delta hedging (1)

Example

- Assume that the stock price is 100 $/share and the option price is 10 $/share
- An investor has written 20 calls, and each options buys 100 shares (i.e. options to buy a total of 2000 shares)
- The delta of the option position is (portfolio delta):

  20 options x 100 shares x (-0.6 delta)

  = 2000 x 0.6 = -1200

Delta hedging (2)

Example

- The option portfolio delta is -1200
  - If the price of the stock increases with $1/share then the price of the option will increase with $0.60/share
  - The portfolio will lose
    - 100 x $0.60 = $60 per option
    - 60 x 20 = $1200 in total (20 options)
  - Vice versa if the stock price decreases with $1

- How can the investor hedge his risk?
  - naked and covered position
  - stop-loss strategy
  - delta hedging

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Delta hedging (3)

- How can the investor's option position be hedged (that is, how can the portfolio be made delta neutral)?

- The investor can buy shares (long position) to become delta neutral

- Delta of options portfolio = -1200
- Delta of stock investment = +1200
- Portfolio delta = 0

- How many shares should the investor buy?

  = +1200 delta/portfolio / +1 delta/share = 1200 shares

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Delta hedging (4)

- The profit (loss) on the option will partly be offset by a loss (gain) on the stock investment
- E.g., if the stock price increases with 1 NOK
  - the investor will gain 1200 NOK on the stock investment
  - the investor will lose 2000x0.6 = -1200 NOK on the short options position
- If the stock price increases with 1 NOK
  - the investor will lose 1200 NOK on the stock investment
  - the investor will gain 2000x0.6 = +1200 NOK on the short options position
- The total portfolio (stock + option) does not lose money, e.g. the portfolio is **delta neutral**

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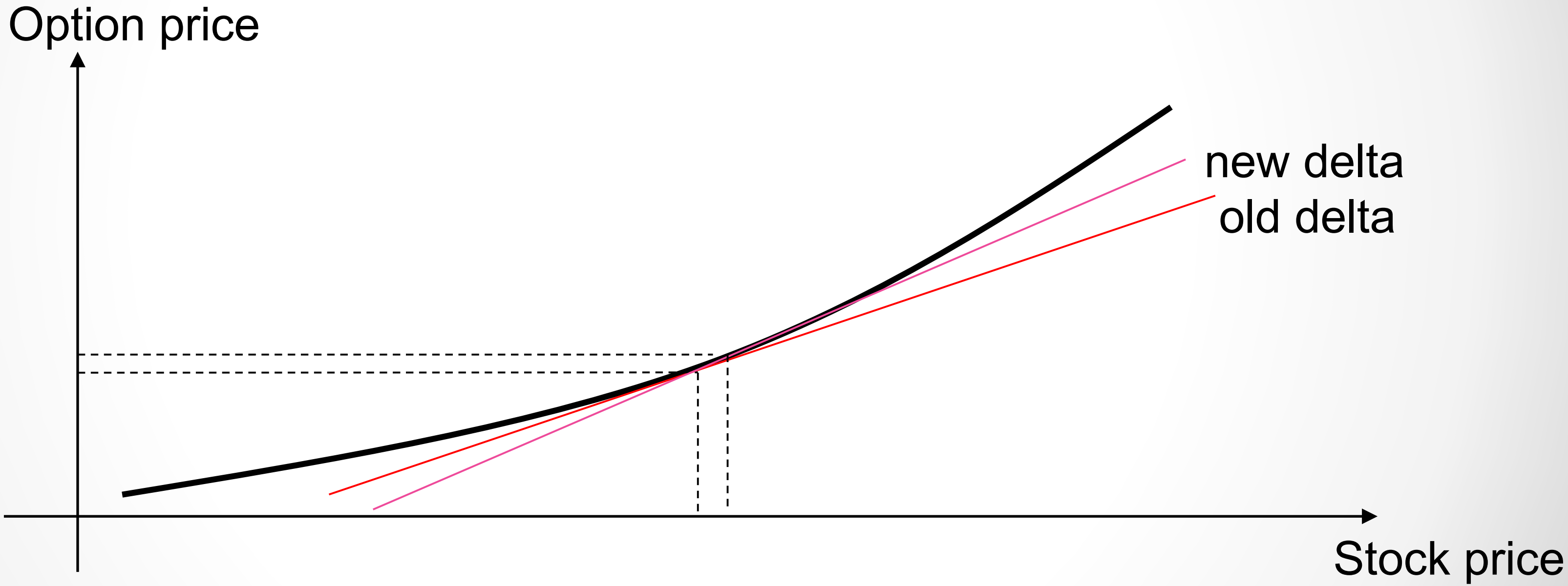
Delta hedging (5)

- BUT, it is important to be aware that the delta changes over time, such that an investor is only delta neutral for a short period of time (see curve)

- If an investor wants to be delta neutral all the time, he/she must continuously rebalance his/her portfolio/position

- An increase in the stock price increases delta

Delta hedging (6)



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Delta hedging (7)

- Assume that the stock price increases to 110 NOK
- Assume that the delta increases from 0.60 to 0.65
- To be delta neutral, an investor must buy more stocks
- (0.65-0.60) x 2000 = +100 stocks
- Assume that the stock price increases or falls by 1 kr. Show that the profit/loss of shares/options exactly offset each other. Show that the portfolio is delta neutral

Theta

- The Theta (θ) of a portfolio measures the sensitivity of the portfolio value to changes in time to maturity

$$\Theta(call) = \frac{-S_0 N'(d_1)\sigma}{2\sqrt{T}} - rXe^{-rT} N(d_2)$$

$$\Theta(put) = \frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} + rXe^{-rT} N(-d_2)$$

- where

$$N'(d_1) = \frac{1}{\sqrt{2\Pi}} e^{-x^2/2}$$

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Theta (2)

- Theta is usually negative for an option. This is because the value of an option decreases with decreasing time to maturity (everything else equal)

Gamma

- Gamma (Γ) of a portfolio of options measures the sensitivity of a delta with changes in the price of the underlying asset

$$\Gamma = \frac{\partial^2 \Pi}{\partial S^2} \qquad \Gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$$

- If the gamme is small, the delta will change slowly and rebalancing to keep the portfolio delta neutral is infrequent
- If the gamme is large, the delta is very sensitive to changes is the price of the underlying, and the investor must rebalance frequently to keep the portfolio delta neutral

Vega

- Vega (v) of a portfolio of options measures the portfolio value sensitivity to changes in the volatility of the underlying

$$\nu = \frac{\partial \Pi}{\partial \sigma} \qquad \nu = S_0 \sqrt{T} N'(d_1)$$

- If vega is high (low) the portfolio value is very sensitive to small (large) changes in volatility
- A position in the underlying has 0 vega

Rho

- Rho (rho) of a portfolio of options measures the portfolio value sensitivity to changes in the interest rate

$$rho = \frac{\partial \Pi}{\partial r}$$

$$rho(call) = XTe^{-rT} N(d_2)$$

$$rho(put) = -XTe^{-rT} N(-d_2)$$

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Delta, vega and gamma neutral

- An investor has a portfolio which has
  - zero delta (delta neutral)
  - gamma -5000
  - vega -8000

- A traded option (Option 1) has gamma 0.5, vega 2.0 and delta 0.6

- A second traded option (Option 2) has gamma 0.8, vega 1.2 and delta 0.5.

- Using the two traded options, make the portfolio delta, vega and gamma neutral.

Delta, vega and gamma neutral

- This means:

$$-5000 + 0.5w_1 + 0.8w_2 = 0 \text{ (gamma)}$$

gamma original portfolio

gamma Option 1

gamma Option 2

gamma new portfolio (gamma neutral)

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Delta, vega and gamma neutral

- This means:

$$-8000 + 2.0w_1 + 1.2w_2 = 0 \text{ (vega)}$$

vega original portfolio

vega Option 1

vega Option 2

vega new portfolio (vega neutral)

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Delta, vega and gamma neutral

- By solving: find w_1 and w_2 such that:

$$-5000 + 0.5w_1 + 0.8w_2 = 0 \text{ (gamma)}$$
$$-8000 + 2.0w_1 + 1.2w_2 = 0 \text{ (vega)}$$

- The solution is $w_1 = 400$ and $w_2 = 6000$

- The portfolio becomes gamma and vega neutral by buying 400 of Option 1 and 6000 of Option 2.

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Delta, vega and gamma neutral

- However, the new portfolio is only gamma and vega neutral, not delta neutral
- The new portfolio delta becomes:
  400x0.6+6000x0.5 = 3240
- Selling 3240 units of the underlying makes the portfolio also delta neutral
- By buying 400 units of Option 1, 6000 units of Option 2 and selling 3240 units of the underlying asset makes the portfolio delta, gamma and vega neutral (hedged)

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Exam question delta hedging

- Autumn 2007 Question 2
- Autumn 2008 Question 4
- Autumn 2009 Question 1c
- Autumn 2010 Question 4
- Autumn 2011 Question 2
- Autumn 2012 Question 5
- Spring 2013 Question 1d-e

Value-at-Risk

What is Value-at-Risk (VaR)

- A measure of a portfolio’s market price risk

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Greeks

- The greeks (delta, gamma, vega, etc) measure different risks in a portfolio of derivatives
- Financial institutions usually calculate each of these measures every day for each of the market variables they are exposed to
- Often hundreds or thousands of market variables
- Provide useful information to a trader about the sensitivity of a particular market variable
- but, it is not an appropriate measure of the total risk the financial institution is exposed to

The aim of VaR

- The aim of VaR is to provide one single number that sums up the total risk of a portfolio of financial assets
- Has become very popular among risk departments in companies and among fund managers

What is a VaR measure?

- When we use VaR we are interested in saying:

”We are *X%* certain that we will not lose more than *V* dollrs during the next *N* days”

The variable V is the VaR of a portfolio

V is a function of 2 parametres

- time to horizon (N days)
- confidence level (X%)

V is the loss level over N days that we are X% certain that will not be passed

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What is a VaR measure?

- Typically, N = 10 days and X = 99%
- Technically: When N is the time horizon and X% is the confidence level, VaR is the loss equivalent to the (100-X) percentile in the probability distribution over the changes in portfolio value the next N days
- E.g. if N = 5 and X = 97 VaR is the 3rd precentile in the distribution over portfolio value changes over the next 5 days

What is a VaR measure?

- VaR is a popular measure because it is easy to understand
- VaR ask the question: ”how bad can it get?”
- It is popular among managers that want all the greeks for all the market variables compressed into 1 single number

What is a VaR measure?

- Is VaR the best measure?

- Alternatives:
  - C-VaR (conditional Value-at-Risk): If things go bad, how much can we expect to lose?”
  - CFaR (cash flow at Risk)
  - Earnings at risk
  - Profit at risk

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Time horizon

- The N-day VaR is usually calculated as:
- N-day VaR = 1-day VaR x $\sqrt{N}$

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Calculation of VaR

- 2 methods
  - historical simulation
  - Model approach

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1. Historical simulation

- Historical simulation is a popular method of calculation VaR
- Involvs using historical data to try to say something about the future

- E.g. say we want to calcultae portfolio VaR (1-day, 99% confidence level) based on 500 days of historical data)
- First we identify the market variables that affect the portfolio
  - Currency, stock price, interest rate etc...

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1. Historical simulation

- Then we get data on the changes in these market variables the last 500 days
- This gives us 500 scenarios for what will happen from today to tomorrow (expected change)
- **Scenario 1**: the change from today to tomorrow is equal to the change in the market variable from day 1 to day 2 in the historical data set (from 500 days before today to 499 days before today)
- **Scenario 2**: the change from today to tomorrow is equal to the change in the market variavle from day 2 to day 3 in the historical data set (from 499 days before today to 498 days before today)
-
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1. Historical simulation

- For each scenario we calculate the value change between today and tomorrow
- This defines the probability distribution over daily changes in portfolio value
- The 5th worst daily change is the 1 percentile in the distribution
- The VaR estimate is the loss when we are at the 1 percentile
- With the assumption that the last 500 days is a good guide for the changes for the next 1 day, we are 99% certain that we will not lose an amount that is larger than our VaR estimate

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1. Historical simulation

Day	Market variable 1	Market variable 2	…..	Market variable n
0	20.33	0.1132	….	65.37
1	20.78	0.1159	….	64.91
2	21.44	0.1162	….	65.02
3	20.97	0.1184	….	64.90
….	….	….	….	….
498	25.72	0.1312	….	62.22
499 (yesterday)	25.75	0.1323	….	61.99
500 (today)	25.85	0.1343	….	62.10

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1. Historical simulation

Day	Market variable 1	Market variable 2	.....	Market variable n
0	20.33	0.1132	....	65.37
1	20.78	0.1159	....	64.91
2	21.44	0.1162	....	65.02
3	20.97	0.1184	....	64.90
....	....	....	....	....
498	25.72	0.1312	....	62.22
499 (yesterday)	25.75	0.1323	....	61.99
500 (today)	25.85	0.1343	....	62.10

1. Historical simulation

- Changes from day 0 to day 1:

$$v_m = \frac{v_i}{v_{i-1}}$$

vm = the value of market variable today

v_i = the value of market variable day i

v_{i-1} = the value of market variable day i-1

- Market variable 1: 20.78 / 20.33 = 1.0221
- Market variable 2: 0.1159 / 0.1132 = 1.0239
- Market variable 3: 64.91 / 65.37 = 0.9930

1. Historical simulation

- Scenario 1 for changes in market variable 1 from today to tomorrow:

- Market variable 1: 1.0221 x 25.85 = 26.42

- Market variable 2: 1.0239 x 0.1343 = 0.1375

- Market variable 3: 0.9930 x 62.10 = 61.66

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1. Historical simulation

- Scenario 1 for changes in market variable 1 from today to tomorrow:
- Market variable 1: 1.0221 x 25.85 = 26.42
- Market variable 2: 1.0239 x 0.1343 = 0.1375
- Market variable 3: 0.9930 x 62.10 = 61.66

change in price day 0 to day 1

Price today

Price tomorrow (Scenario 1)

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1. Historical simulation

Scenarioes generated for tomorrow (day 501)

Scenario	Market variable 1	Market variable 2	.....	Market variable n	Portfolio value	Changes in value
1	26.42	0.1375	....	61.66	23.71	0.21
2	26.67	0.1346	....	62.21	23.12	-0.38
3	25.28	0.1368	....	61.99	22.94	-0.56
....	....	....	....	....	....	....
499	25.88	0.1354	....	61.87	23.63	0.13
500	25.95	0.1363	....	62.21	22.87	-0.63

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1. Historical simulation

- We are interested in the 1-percentile in the ditribution over value changes
- We rank changes in value from most negative to most positive
- We chose the 5th worst value change

The next day....

- after day 501 we can repeat the procedure
- We again look at the last 500 days, but day 0 in the new selection is equivalent to day 1 in the old, and the last day is day 501
- We repeat the procedure for every day (we always consider the last 500 days)
- Do you see any problem with this approach??

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2. Model approach

- The model approach is the main alternative to historical simulations

- We will consider
  - A portfolio with only one asset (stock)
  - A portfolio with two asset (2 stocks)
  - Linear model
  - linear model for option portfolios
  - Quadratic model for option portfolios

2. Model approach

- We will first consider the portfolio consisting of only 1 asset

- We have a portfolio consisting of 10 mill NOK i nMicrosoft stocks
- N = 10
- X = 99

- We are interested in the loss level for a 10-day period, that we are 99% certain will not be exceded

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2. Model approach

- We assume the volatility in the Microsoft stock is 2% daily (equivalent to 32% annually)

- The volatility measured in NOKs is:

  2% x 10 mill NOK = 200 000 NOK

- We assume that the average return is 0% (daily return)

- We assume the the daily returns on the portfolio value is normally distributed with average 0 and a standard deviation of 200 000 NOK

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2. Model approach

- From a normal distribution table we find that the 1% percentile is equal to 2.33 standard deviations from the mean: $N(-2.33) = 0.01$
- This means that there is a 1% probability that a normally distributed variable will decrease in value more than 2.33 standard deviations
- Or, we are 99% certain that a normal distributed variable will not decrease in value more than 2.33 standard deviations

2. Model approach

- The 1-day 99% VaR for our portfolio (1 stock) is then

  2.33 x 200.000 = 466 000 NOK

- The 10-day 99% portfolio VaR becomes

  466 000 x $\sqrt{10}$ = 1 473 621 NOK

- We are 99% certain that we will not lose more than 1 473 621 kroner i during the next 10 days

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2. Model approach

- Assume that we have a portfolio consisting of 5 mill NOK in AT&T stocks
- The AT&T volatility is 16% per year (=1% per day)

- The standard deviations in daily changes/returns is 1% x 5 mill NOK = 50 000
- 1-day 99% VaR is 50 000 x 2.33 = 116 500 NOK
- 10-day 99% VaR is 116 500 x $\sqrt{10}$ = 368 405 NOK

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2. Model approach: Portfolio (2 stocks)

- We will now calculate the VaR of a portfolio consisting of 10 million Microsoft shares and 5 million AT&T shares
- We assume that the logreturns of these two stocks are bivariate normal distributed with a correlation of 0.3
- From statistics we know that if two variables X and Y have standard deviations of σ_X and σ_Y , and have a correlation coeffisient of ρ_{XY}, then the standard deviation of X+Y (portfolio) is given by

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho_{XY}\sigma_X\sigma_y}$$

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2. Model approach: Portfolio (2 stocks)

- 1-day volatility Microsoft (σ_X) = 200 000
- 1- dags volatility AT&T (σ_Y) = 50 000
- The correlation between returns on Microsoft and AT&T (ρ_{XY}) is 0.30

- Then the standard deviation of the change in the portfolio of Microsoft and AT&T (σ_{X+Y}) is:

$$\sigma_{X+Y} = \sqrt{(200000)^2 + (50000)^2 + 2(0.3)(200000)(50000)}$$
$$= 220227$$

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2. Model approach: Portfolio (2 stocks)

- 1-day 99% VaR becomes:

  220 227 x 2.33 = 513 129 NOK

- 10-day 99% VaR becoms:

  513 129 x $\sqrt{10}$ = 1 622 657 NOk

- We are 99% certain that we will not lose more than 1 622 657 NOK (of a total portfolio valus of 15 mill) during the next 10 days

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The benifit of diversification

- 10-day 99% VaR the portfolio consisting of only Microsoft stocks = 1 473 621
- 10-day 99% VaR the portfolio consisting of only AT&T stocks = 368 405
- 10-day 99% VaR the portfolio consisting Microsoft and AT&T aksjer = 1 622 657
- The amount (1 473 621 + 368 405) – 1 622 657 = **219 369** represents the benifit of diversification. If AT&T and Microsoft were perfectly correlation the benfit would have been 0.
- Credit crisis and correlation?

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Linear model

- The previous examples are simple illustrations of what is called the linear method for VaR calculation

- Assume that we have a portfolio worth P consisting of n assets with invested amount α_i in asset *i* $(1 \leq i \leq n)$.
- We define Δx_i as the return on teh asset during 1 day
- The change (in NOK) in the investment in asset i during 1 day is $\alpha_i \Delta x_i$

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Linear model

- The change in portfolio value (ΔP) during 1 day is:

$$\Delta P = \sum_{i=1}^{n} \alpha_i \Delta x_i$$

- In the previous example 10 mill was invested in Microsoft and 5 mill in AT&T, i.e. $\alpha_1 = 10$ and $\alpha_2 = 5$ (in millions), and

$$\Delta P = 10\Delta x_1 + 5\Delta x_1$$

- To calculate VaR we only need the standard deviations (we assume the the average return is 0 over 1 day)

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Linear model

- To calculate the standard deviations in ΔP we define ρ_{ij} as the daily volatility in asset *i*, and σ_i as the correlation coefficient of returns of asset *i* and asset *j*

- This means that σ_i is the standard deviation of Δx_i, and ρ_{ij} is the correlation coefficient between Δx_i og Δx_j.

- The variance of ΔP (σ_P^2) is given as:

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \alpha_i \alpha_j \sigma_i \sigma_j$$

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Linear model

- The standard deviation of changes over N days is $\sigma_P\sqrt{N}$, and the 99% VaR for a N-day time horizon is 2.33

- In the example (in millions):

$$\sigma_1 = 0.02 \quad \sigma_2 = 0.01 \quad \rho_{1,2} = 0.30 \quad \alpha_1 = 10 \quad \alpha_2 = 5$$

$$\begin{aligned}\sigma_P^2 &= 1(10)^2(0.02)^2 + 1(5)^2(0.01)^2 + 0.3(10)(5)(0.02)(0.01) \\ &\quad + 0.3(5)(10)(0.01)(0.02) \\ &= 0.0485\end{aligned}$$

$$\sigma_P = \sqrt{0.0485} = 0.220$$

(remember: we calculated approx 220 000 NOKs earlier)

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Linear model

- 10-day 99% VaR becomes

  $2.33 \times 0.220 \times \sqrt{10} = 1.623$ million NOK

- which is the same as calculated earlier

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Linear model and options

- What if the portfolio contains options?
- We will now examine how the linear model can be used on a portfolio consisting of options on a single stock which price is S_0
- The option delta (portfolio) is given by δ

$$\delta = \frac{\Delta P}{\Delta S}$$

$$\Updownarrow$$

$$\Delta P = \delta \Delta S$$

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Linear model and options

- ΔP is the change in portfolio value over 1 day and ΔS is the change in stock price (underlying) over 1 day
- The change in portfolio value is a linear function of the change in the underlying
- We define Δx as the percentage change in the stock price during 1 day

$$\Delta x = \frac{\Delta S}{S}$$

Linear model and options

- An approximation of the relationship between ΔP and Δx is

  $\Delta S = S\delta\Delta x$

- When we have positions in several underlying market variables, including options, we can derive a approximately linear relationship between ΔP og Δx

$$\Delta P = \sum_{i=1}^{n} S_i \delta_i \Delta x_i$$

- where S_i is the value of market variable *i* and Δ_i si the portfolio delta with respect to market variable *i*

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Linear model and options

- This is similar to an equation we have seen earlier

$$\Delta P = \sum_{i=1}^{n} \alpha_i \Delta x_i$$

- where $\alpha_i = S_i \delta_i$

Example

- A portfolio consists of options on Microsoft and AT&T
- The options (portfolio) on Microsoft have a delta of 1000
- The options (portfolio) on AT&T have a delta of 20000
- The Microsoft stock price is 120
- The AT&T stock price is 30

- What is the 5-day 95% VaR?

Example

- According to the approximation we can calculate

$$\Delta P = \sum_{i=1}^{n} S_i \delta_i \Delta x_i$$

$$\begin{aligned}\Delta P &= 120(1000)\Delta x_1 + 30(20000)\Delta x_2 \\ &= 120000\Delta x_1 + 600000\Delta x_2\end{aligned}$$

- where Δx_1 and Δx_2 are the returns on Microsoft and AT&T during 1 day and ΔP is the change of value in the portfolio

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Example

- Assuming that the daily volatilites for Microsoft and AT&T are 2 og 1%, respectively, the correlation is 30%, the standard deviation in ΔP (in thousand NOK) is

$$\sigma_P = \sqrt{(120\times 0.02)^2 + (600\times 0.01)^2 + 2\times 120\times 0.02 \times 600\times 0.01\times 0.3}$$
$$= 7.099$$

- Since N(-1.65)=0.05 the 5-day 95% VaR is

1.65 x $\sqrt{5}$ x 7.099 = 26 193 NOK

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The quadratic model

- When a portfolio contains options, the linear model is only an approximation

- The linear model does not consider the portfolio gamma

- For a more precise VaR estimate than the linear model provides, we can use both delta and gamma to say something about the relation between ΔP and Δx_i

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The quadratic model

- Assume that *d* and *y* are the portfolio delta and gamma. It can be shown that the relationship between the change in portfolio value and the change in stock price (underlying) and delta and gamma can be represented as

$$\Delta P = \delta \Delta S + \tfrac{1}{2}\gamma(\Delta S)^2$$

- If we set

$$\Delta x = \frac{\Delta S}{S}$$

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The quadratic model

- We get

$$\Delta P = \delta S \Delta x + \tfrac{1}{2} S^2 \gamma (\Delta x)^2$$

- In general, a portfolio with *n* underlying market variables, with the instruments in the portfolio only dependent on one of the market variables, the equation becomes

$$\Delta P = \sum_{i=1}^{n} \delta_i S_i \Delta x_i + \sum_{i=1}^{n} \tfrac{1}{2} S_i^{\,2} \gamma_i (\Delta x_i)^2$$

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The quadratic model

- When several individual instruments in a portfolio can be dependent on more than 1 market variable, the equation becomes

$$\Delta P = \sum_{i=1}^{n} \delta_i S_i \Delta x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \tfrac{1}{2} S_i S_j \gamma_i \gamma_j \Delta x_i \Delta x_j$$

- Where a cross-gamma is defined as

$$\gamma_{ij} = \frac{\partial^2 P}{\partial S_i \partial S_j}$$

Other things….

- Monte Carlo simulations
- Stress testing and back testing
- Principal component analysis (PCA)

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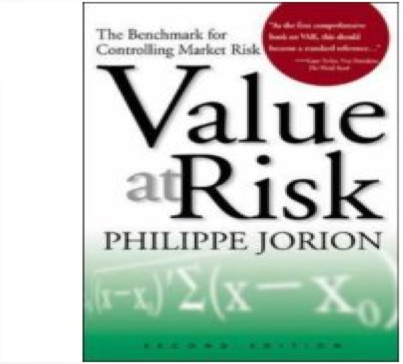
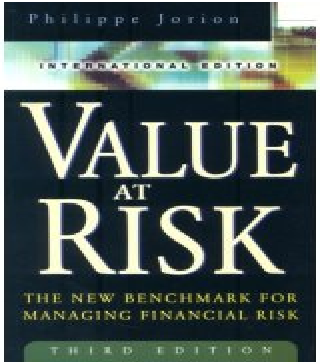
Read more about VaR

- Philippe Jorion: Value at Risk
  - The Benchmark for controlling market risk
  - The new benchmark for managing financial risk





Critisism of VaR

- www.fooledbyrandomness.com/jorion

Exam questions VaR

- Autumn 2007 Question 3

Relevant literature

- Brooks, C., Prokopczuk, M. and Y. Wu (2013). Commodity futures prices: More evidence on forecast power, risk premia and the theory of storage. The Quarterly Review of Economics and Finance 53, 73-85.
- Brennan, M. (1958). The supply of storage. American Economic Review, 48(1), 50–72.
- Brennan, M., & Schwartz, E. (1985). Evaluating natural resource investments. Journal of Business, 58, 135–157.
- Fama, E., & French, K. (1987). Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage. Journal of Business, 60(1), 55–73.
- Pindyck, R. (2001). The dynamics of commodity spot and futures markets: A primer. Energy Journal, 22(3), 1–30.
- Asche, F., Misund, B. and A. Oglend (2018). The case and cause of salmon price volatility. Marine Resource Economics 34(1), 23-38.
- Misund, B. and R. Nygård (2018). Big Fish: Valuation of the world's largest salmon farming companies. Marine Resource Economics 33(3), 245-261.
- Misund, B. (2018). Volatilitet i laksemarkedet. Samfunnsøkonomen 2:41-54.
- Misund, B. (2018). Common and fundamental risk factors in shareholder returns of Norwegian salmon producing companies.
- Misund, B. (2018). Valuation of salmon farming companies. Aquaculture Economics & Management 22(1), 94-111.
- Misund, B. & A. Oglend (2016). Supply and demand determinants of natural gas price volatility in the U.K.: A vector autoregression approach. Energy 111, 178-189.
- Asche, F., Misund, B. & A. Oglend (2016). Determinants of the futures risk premium in Atlantic salmon markets. Journal of Commodity Markets, 2(1), 6-17.
- Misund, B. & F. Asche (2016). Hedging efficiency of Atlantic salmon futures. Aquaculture Economics & Management 20(4), 368-381.
- Asche, F., Misund, B. & A. Oglend (2016). The spot-forward relationship in Atlantic salmon markets. Aquaculture Economics & Management 20(2), 222-234.
- Asche, F., Misund, B. and A. Oglend (2016). Fish Pool Priser – Hva Forteller de oss om fremtidige laksepriser? Norsk Fiskeoppdrett nr.8 2016, p.74-77.
- Symeonidis, L., Prokopczuk, M. Brooks, C. and E. Lazar (2012). Futures basis, inventory and commodity price volatility: An empirical analysis. Economic Modelling 29, 2651-2663.

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